State Separation for Code-Based Game-Playing Proofs

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Motivation

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- various nice guarantees:
    - constant-time code
    - memory safe
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- implementation of TLS in $\mathrm{F}^{\star}$
- various nice guarantees:
  - constant-time code
  - memory safe
  - functionally correct
- "cryptographically verified" - proof in code?

How are they doing that?

Our Contributions

Make it easier to do:

Possible applications:

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- modular composed proofs

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- TLS

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Our Contributions

Make it easier to do:

- modular composed proofs
- key composition
- hybrid arguments
- (partially) machine-checkable proofs

Possible applications:

- TLS
- Messaging
- Multi-Instance
- $\mathrm{F}^\star$, other proof assistants

Related Work

- Universal Composability ([C01])
- Abstract- and Constructive Crypto ([MR11],[M11])
- “The Joy of Cryptography” (Rosulek)
- EasyCrypt ([BGHB11])

Packages - Security Games

$\texttt{IND-CPA}_e^b$

GEN()

assert $k = \perp$

$k \leftarrow e.\mathsf{KGen}(1^n)$

return ()

ENC(m)

assert $k \neq \perp$

if $b = 0$ **then**

  $c \leftarrow e.\mathsf{Enc}(k, m)$

else

  $c \leftarrow e.\mathsf{Enc}(k, 0^{|m|})$

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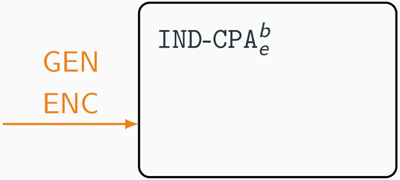
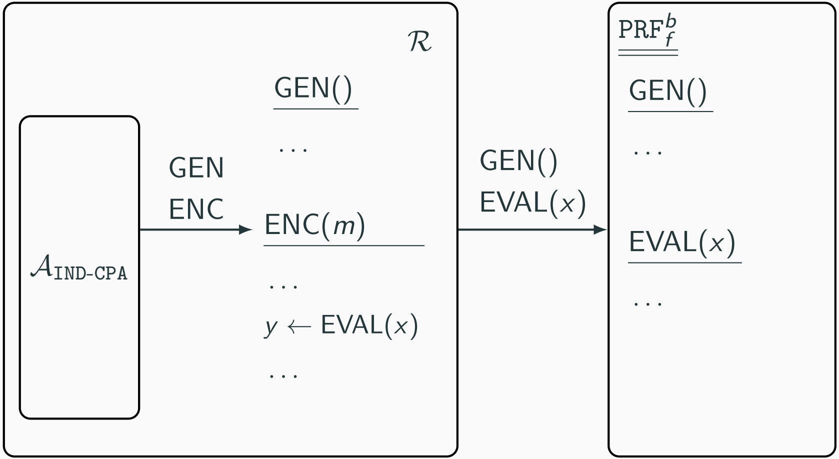


- a package contains oracle descriptions and their state
- it provides these oracles for other algorithms to use
- packages are composable.

Packages - Reductions



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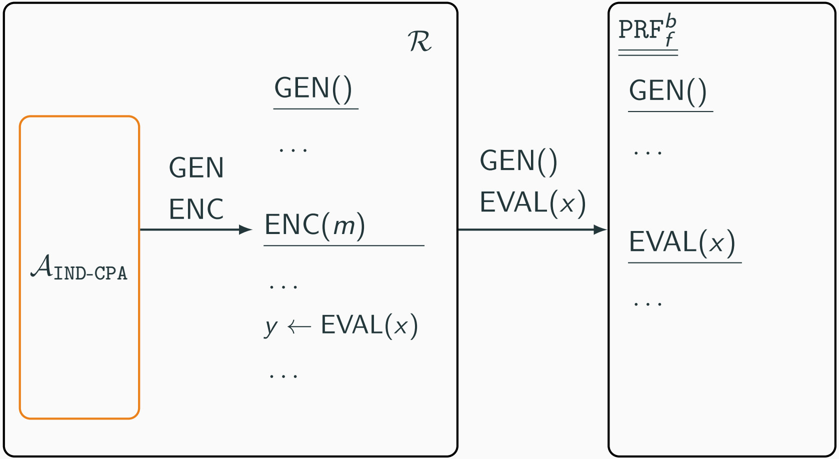

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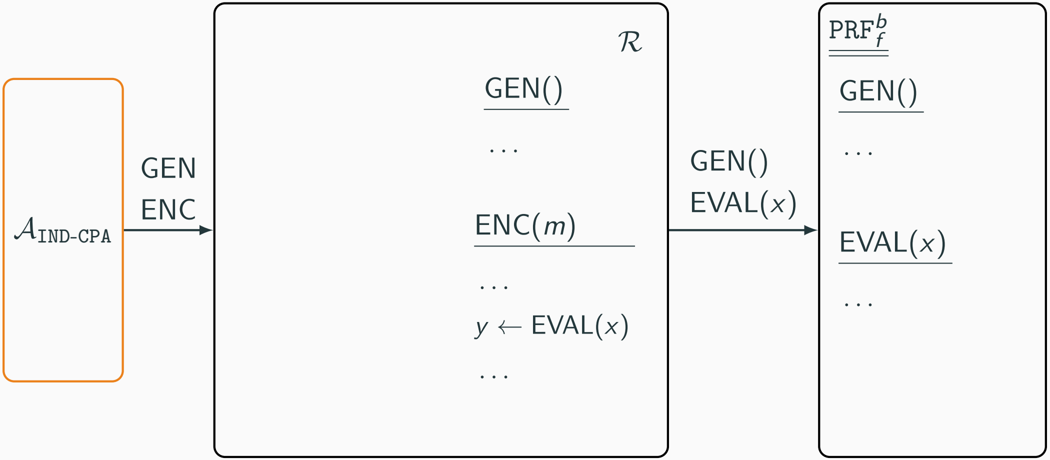

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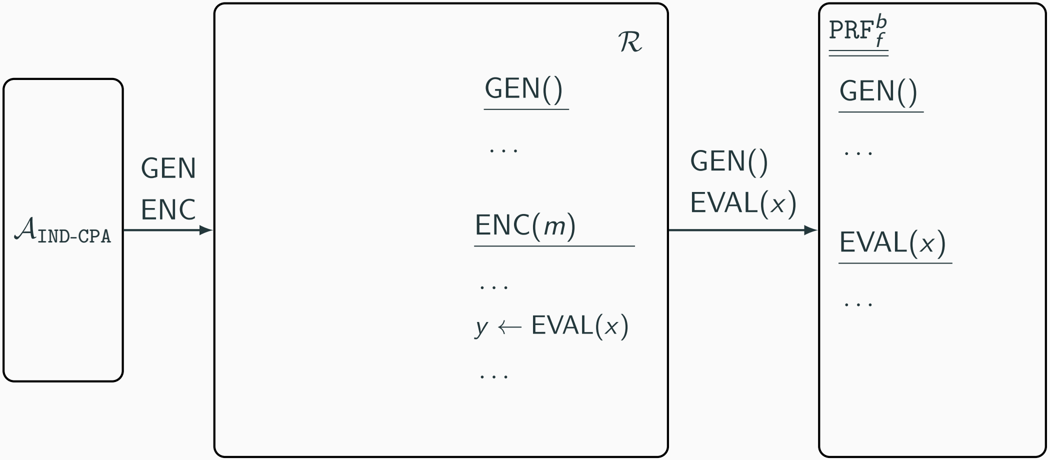
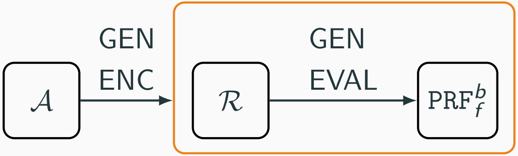


Packages - Reductions

$$\mathcal{A} \xrightarrow{\substack{\text{GEN} \\ \text{ENC}}} \mathcal{R} \xrightarrow{\substack{\text{GEN} \\ \text{EVAL}}} \mathrm{PRF}_f^b$$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GEN
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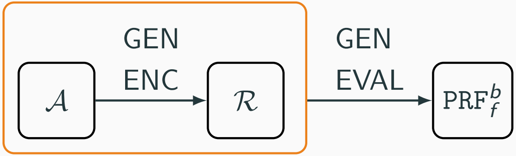

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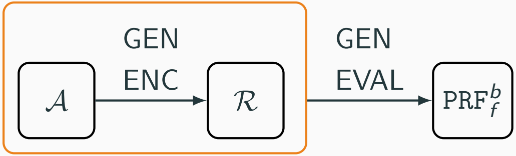

Packages . . .

- contain oracle descriptions and state,
- can provide oracles to other packages,
- and can call oracles provided by other packages.

Example I

Reducing $\texttt{IND-CPA}_e$ to $\texttt{PRF}_f$

Overview

Proof Goal:

$$\boxed{\texttt{IND-CPA}_e^0} \overset{\epsilon_1(\mathcal{A}_{\texttt{IND-CPA}})}{\approx} \boxed{\texttt{IND-CPA}_e^1}$$

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Assumption:

$$\boxed{\mathtt{PRF}_f^0} \overset{\epsilon_2(\mathcal{A}_{\mathtt{PRF}})}{\approx} \boxed{\mathtt{PRF}_f^1}$$

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Concrete Security

Relate $\epsilon_1(\cdot)$ to $\epsilon_2(\cdot)$ in two steps:

1. Simulation correctness
2. Applying assumptions

Step 1: Simulation Correctness

$\boxed{\mathcal{R}} \rightarrow \boxed{\mathrm{PRF}_f^0}$

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To prove: Perfect Indistinguishability

$\boxed{\texttt{IND-CPA}_e^0} \equiv \boxed{\mathcal{R}} \to \boxed{\texttt{PRF}_f^0}$

and

$\boxed{\texttt{IND-CPA}_e^1} \equiv \boxed{\mathcal{R}} \to \boxed{\texttt{PRF}_f^1}$

Step 2: Applying Assumptions

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and

$$\boxed{\texttt{IND-CPA}_e^1} \equiv \boxed{\mathcal{R}} \to \boxed{\texttt{PRF}_f^1}$$

$$\boxed{\texttt{PRF}_f^0} \overset{\epsilon_2(\mathcal{A}_{\text{PRF}})}{\approx} \boxed{\texttt{PRF}_f^1}$$

Step 2: Applying Assumptions

$$\boxed{\texttt{IND-CPA}_e^0} \equiv \boxed{\mathcal{R}} \to \boxed{\texttt{PRF}_f^0}$$

$$\wr\!\wr\ \epsilon_2\Big(\underbrace{\boxed{\mathcal{A}_{\texttt{IND-CPA}}} \to \boxed{\mathcal{R}}}_{\mathcal{A}_{\text{PRF}}}\Big)$$

$$\boxed{\texttt{IND-CPA}_e^1} \equiv \boxed{\mathcal{R}} \to \boxed{\texttt{PRF}_f^1}$$

$$\boxed{\texttt{PRF}_f^0} \overset{\epsilon_2(\mathcal{A}_{\text{PRF}})}{\approx} \boxed{\texttt{PRF}_f^1}$$

General Proof Pattern

$$
\begin{aligned}
&\boxed{\texttt{IND-CPA}^0_e} \equiv \boxed{\mathcal{R}} \to \boxed{\texttt{PRF}^0_f} \\
&\qquad\qquad\qquad\quad \wr\ \epsilon_2\Big(\underbrace{\boxed{\mathcal{A}_{\texttt{IND-CPA}}} \to \boxed{\mathcal{R}}}_{\mathcal{A}_{\mathrm{PRF}}}\Big) \\
&\boxed{\texttt{IND-CPA}^1_e} \equiv \boxed{\mathcal{R}} \to \boxed{\texttt{PRF}^1_f}
\end{aligned}
$$

Some notes:

- graphs have precise meaning
- an inline notation exists

Example II

Key Composition

Overview

Proof Goal:

$$\boxed{\mathtt{SN}_p^0} \overset{\epsilon_1(\mathcal{A}_{\mathtt{SN}})}{\approx} \boxed{\mathtt{SN}_p^1},$$ where SN (security notion) could be PKE-CCA

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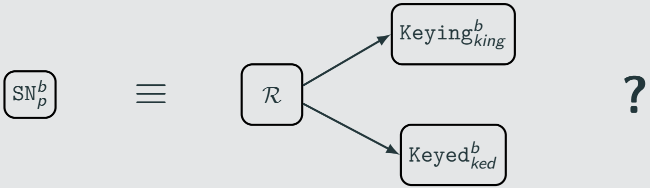
Common Pattern



where Keying could be KEM-CCA and Keyed could be DEM-CCA.

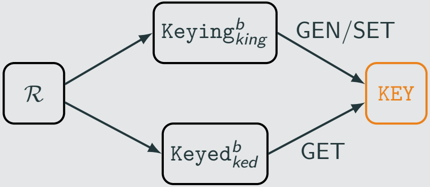
Overview

Proof Goal:

$$\boxed{\mathtt{SN}_p^0} \overset{\epsilon_1(\mathcal{A}_{\mathtt{SN}})}{\approx} \boxed{\mathtt{SN}_p^1},$$ where $\mathtt{SN}$ (security notion) could be $\mathtt{PKE\text{-}CCA}$

Common Pattern

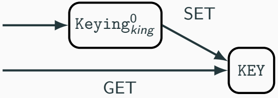
$$\boxed{\mathtt{SN}_p^b} \quad \equiv$$



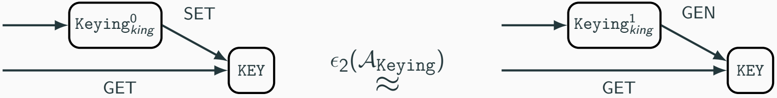
where $\mathtt{Keying}$ could be $\mathtt{KEM\text{-}CCA}$ and $\mathtt{Keyed}$ could be $\mathtt{DEM\text{-}CCA}$.

Assumptions

`Keying` Assumption



`Keying` **Assumption**



Assumptions

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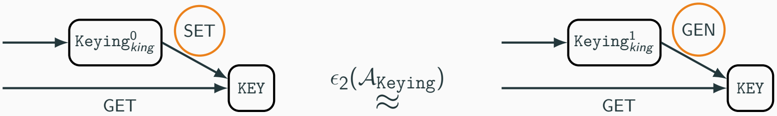

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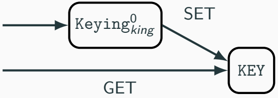
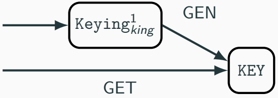
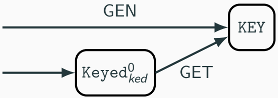


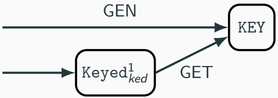
$$\epsilon_2(\mathcal{A}_{\texttt{Keying}})$$
$$\approx$$



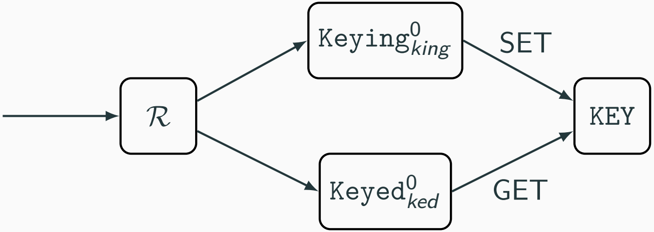
`Keyed` Assumption



$$\epsilon_3(\mathcal{A}_{\texttt{Keyed}})$$
$$\approx$$



Applying the Assumptions



$$\epsilon_1(\mathcal{A}_{\texttt{SN}}) =$$

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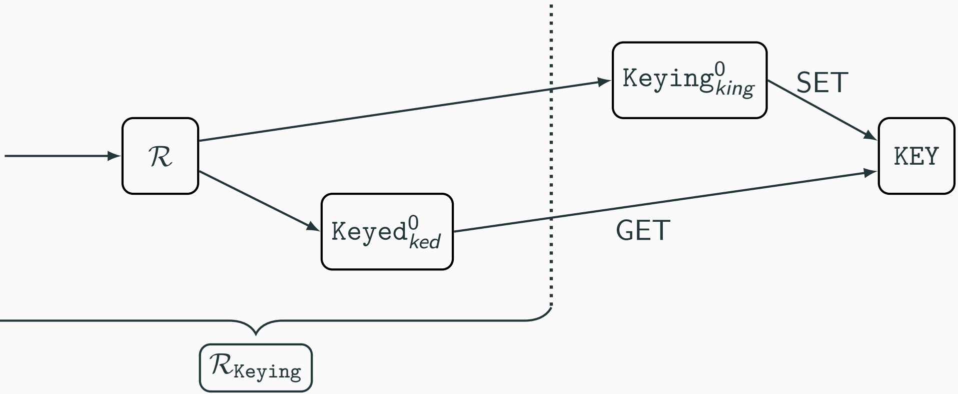

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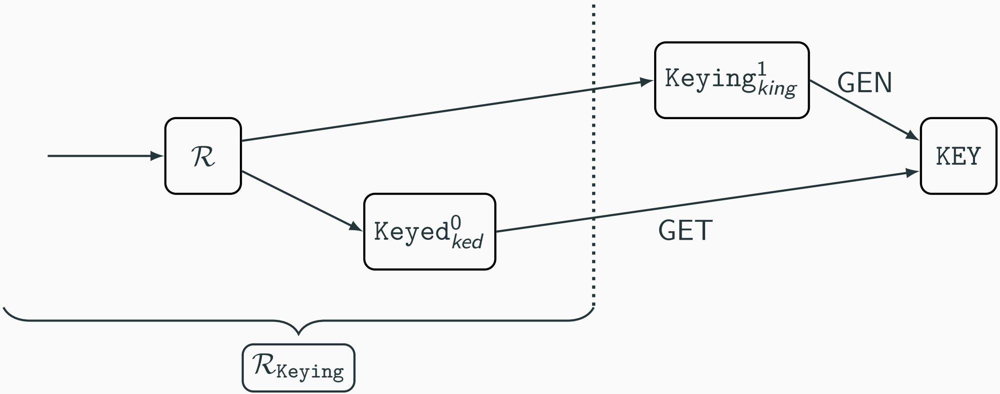

$$\epsilon_1(\mathcal{A}_{\mathtt{SN}}) = \epsilon_2(\boxed{\mathcal{A}_{\mathtt{SN}}} \rightarrow \boxed{\mathcal{R}_{\mathtt{Keying}}})$$

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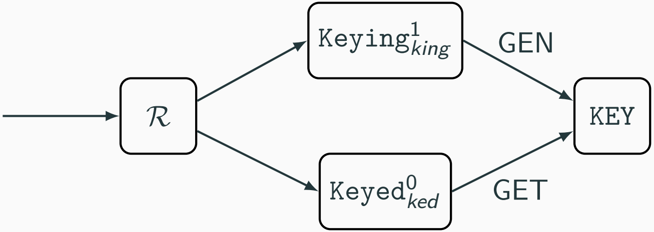

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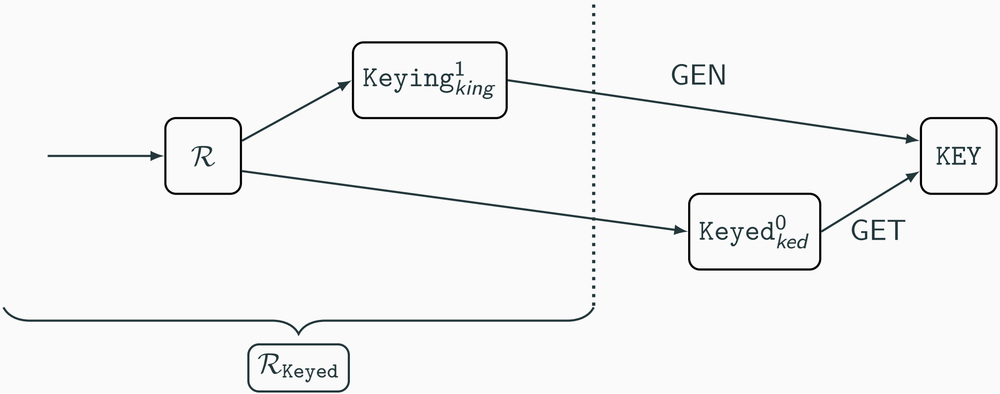

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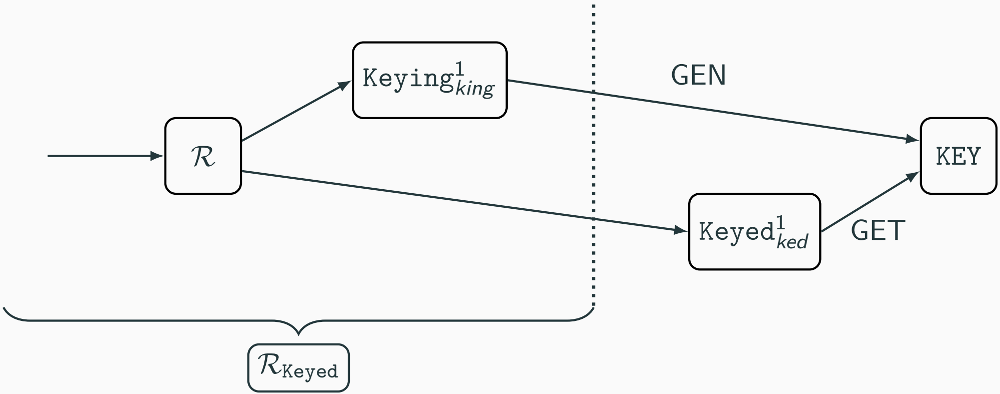

$$\epsilon_1(\mathcal{A}_{\mathtt{SN}}) = \epsilon_2(\boxed{\mathcal{A}_{\mathtt{SN}}} \rightarrow \boxed{\mathcal{R}_{\mathtt{Keying}}}) + \epsilon_3(\boxed{\mathcal{A}_{\mathtt{SN}}} \rightarrow \boxed{\mathcal{R}_{\mathtt{Keyed}}})$$

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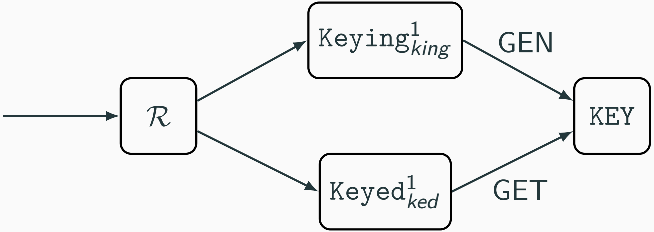

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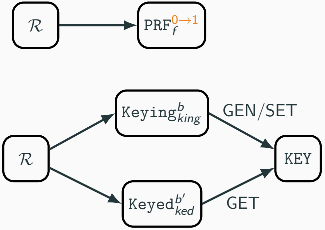

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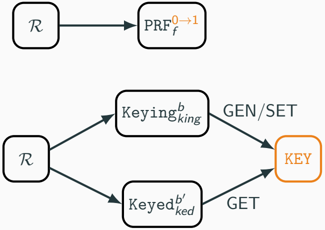

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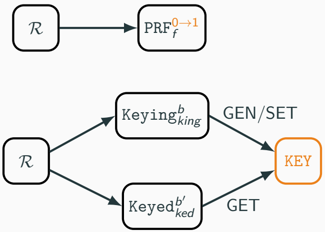
  - packaging of code and state.
- useful for composed protocols (TLS)
- enables key composition (Messaging)
- less useful for . . .
  - implications ($\texttt{AE} \implies \texttt{IND-CCA}$)
  - smaller proofs



Work in Progress

- TLS 1.3 Key Schedule (for miTLS)
- Multi-Party Computation (Yao's Garbled Circuits)
- Protocol Design (Secret Handshake 2)