

ARE YOU EXPLOITING YOUR ASSUMPTIONS?

TOWARDS EXPRESSIVE PRIORS FOR BIOMARKER DISCOVERY AND FUNCTIONAL PREDICTION

Melanie F. Pradier

Harvard University



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Research in collaboration
with...



ML SYSTEMS ARE TRANSFORMING OUR SOCIETY





machine learning

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Unveiling Biology with Deep Microscopy

An AI-inspired Revolution in the Life Sciences

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Powerful antibiotic discovered using machine learning for first time

Thu 20 Feb 2020 11:28 EST

Team at MIT says halicin kills some of the world's most dangerous strains

MAGAZINE • ARTIFICIAL INTELLIGENCE

A.I. breakthroughs in natural-language processing are big for business

BY JEREMY KAHN

January 20, 2020 4:30 AM EST



The American Journal of Medicine

Volume 132, Issue 7, July 2019, Pages 795-801



Review

Artificial Intelligence Transforms the Future of Health Care

Nariman Noorbakhsh-Sabet MD ^{1,2}, Ramin Zand MD, MPH ^{1,2,3}, Yanfei Zhang PhD ⁴, Vida Abedi PhD ^{5,6,7,8}Article | [Open Access](#) | Published: 05 February 2020

Inferring structural variant cancer cell fraction

[Nature Communications](#) 11, Article number: 730 (2020) | [Cite this article](#)

Research

26 February 2020 | [Open Access](#)

[Impact of a deep learning assistant on the histopathologic classification of liver cancer](#)

Amirhossein Kiani, Bora Uyumazturk [1] · Jeanne Shen

[npj Digital Medicine](#) 3, 23

 **nature** [Subscribe](#)

NEWS FEATURE • 09 OCTOBER 2019

Why deep-learning AIs are so easy to fool

Artificial-intelligence researchers are trying to fix the flaws of neural networks.

HDSR Published: Jan 31, 2020

Should We Trust Algorithms?

 **nature**
machine intelligence

Perspective | Published: 13 May 2019

Stop explaining black box machine learning models for high stakes decisions and use interpretable models instead

Cynthia Rudin 

Artificial Intelligence, Machine Learning, and Bias in Finance: Toward Responsible Innovation

K Johnson, F Pasquale, J Chapman - Fordham L. Rev., 2019 - HeinOnline

Over the last decade, a growing number of digital startups launched bids to lure business

fr
an

RESEARCH ARTICLE

Improving Fairness in Machine Learning Systems: What Do Industry Practitioners Need?



Publication: CHI '19: Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems • May 2019

TECHNOLOGY AND IIOT > DIGITAL TOOLS

Making AI Work with Small Data

It's a true manufacturing dilemma: 'teaching' AI when defects are a rarity.

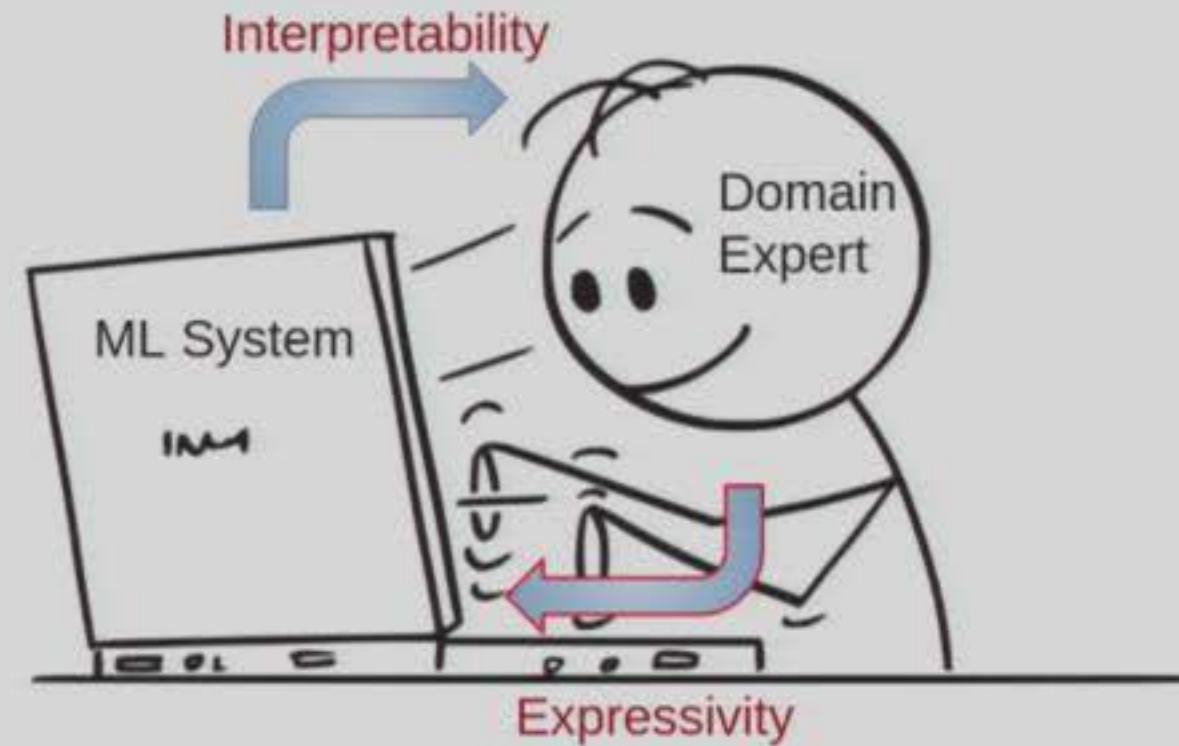
Alejandro Betancourt

FEB 12, 2020

The real test of an AI machine is when it can admit to not knowing something

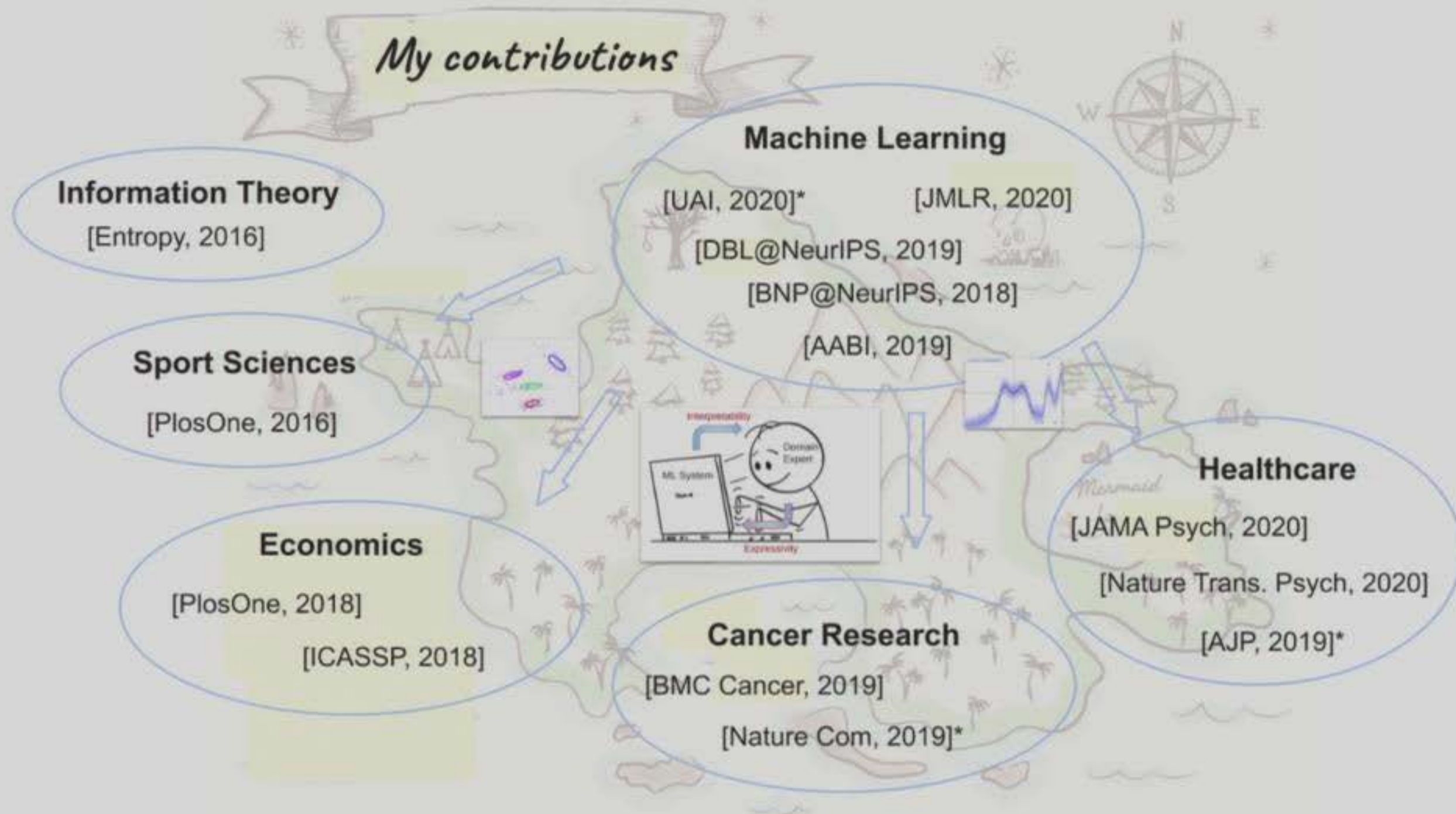
John Naughton

INTERPRETABILITY-EXPRESSIVITY LOOP



INTERPRETABILITY

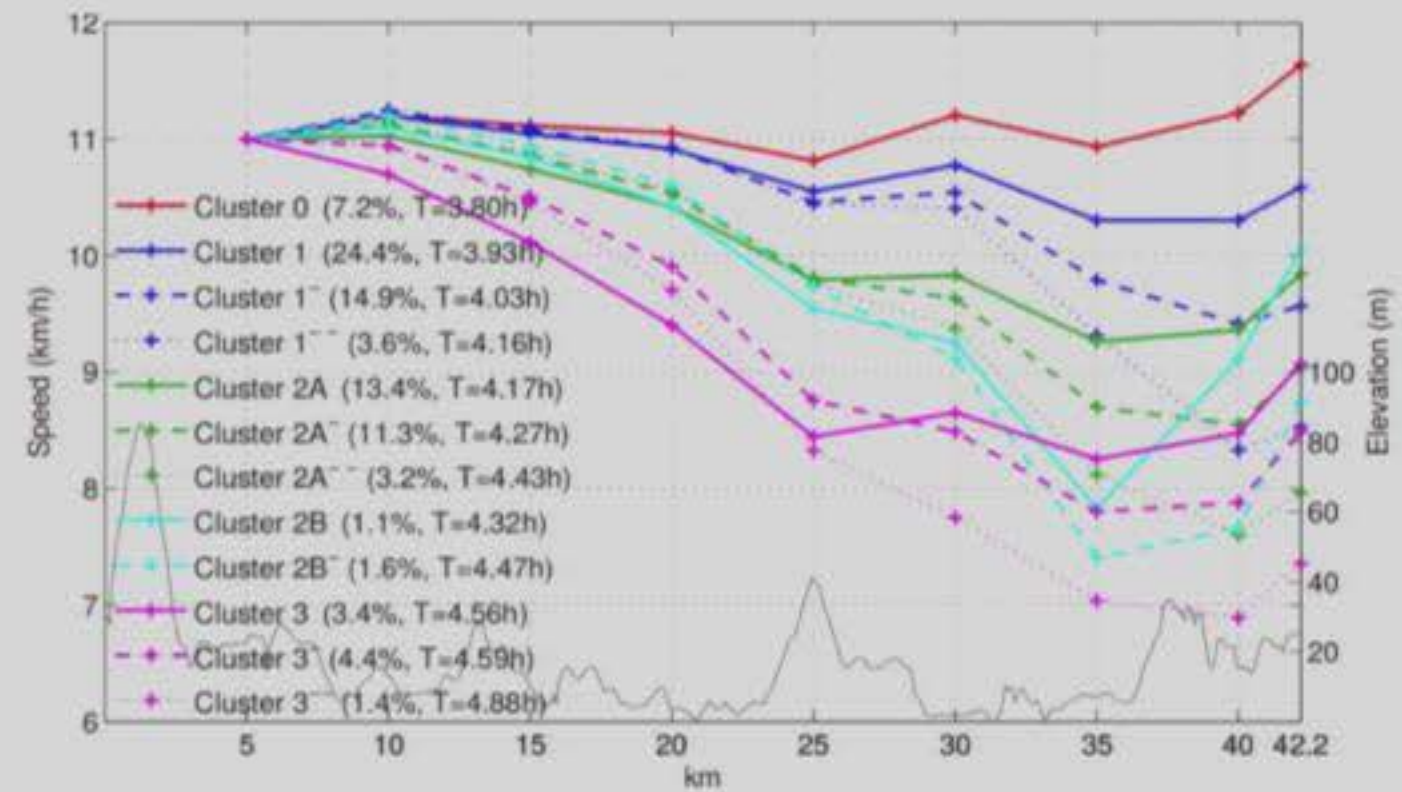
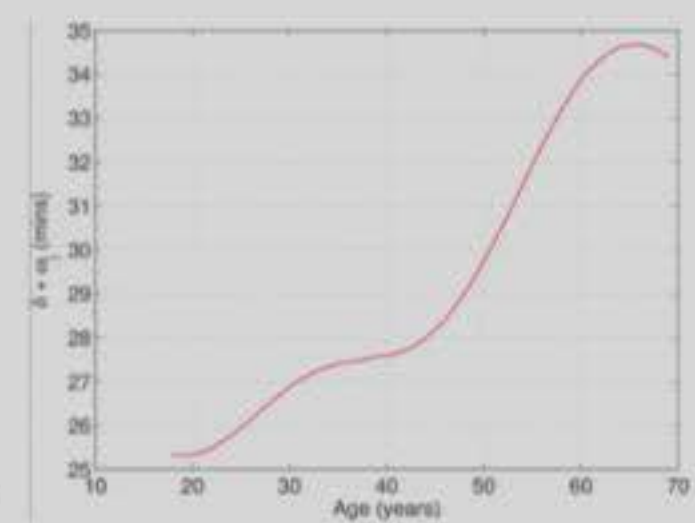
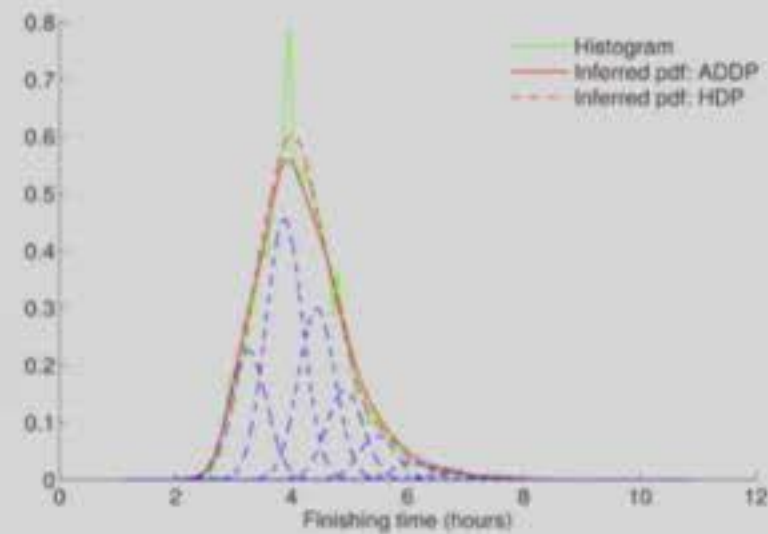
- “ability to present in understandable terms to a human” (Doshi-Velez et.al, 2017; 2018 EU GDPR)



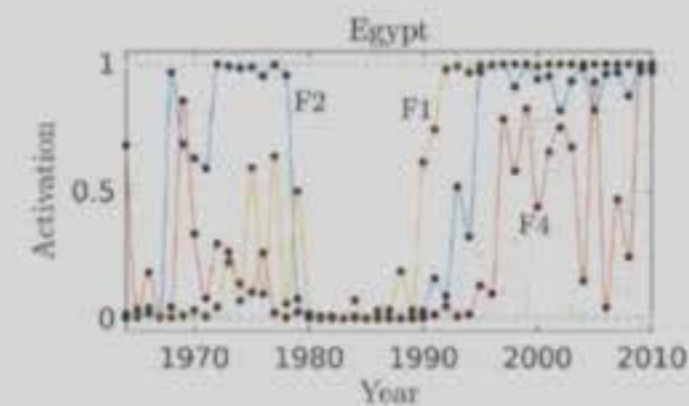
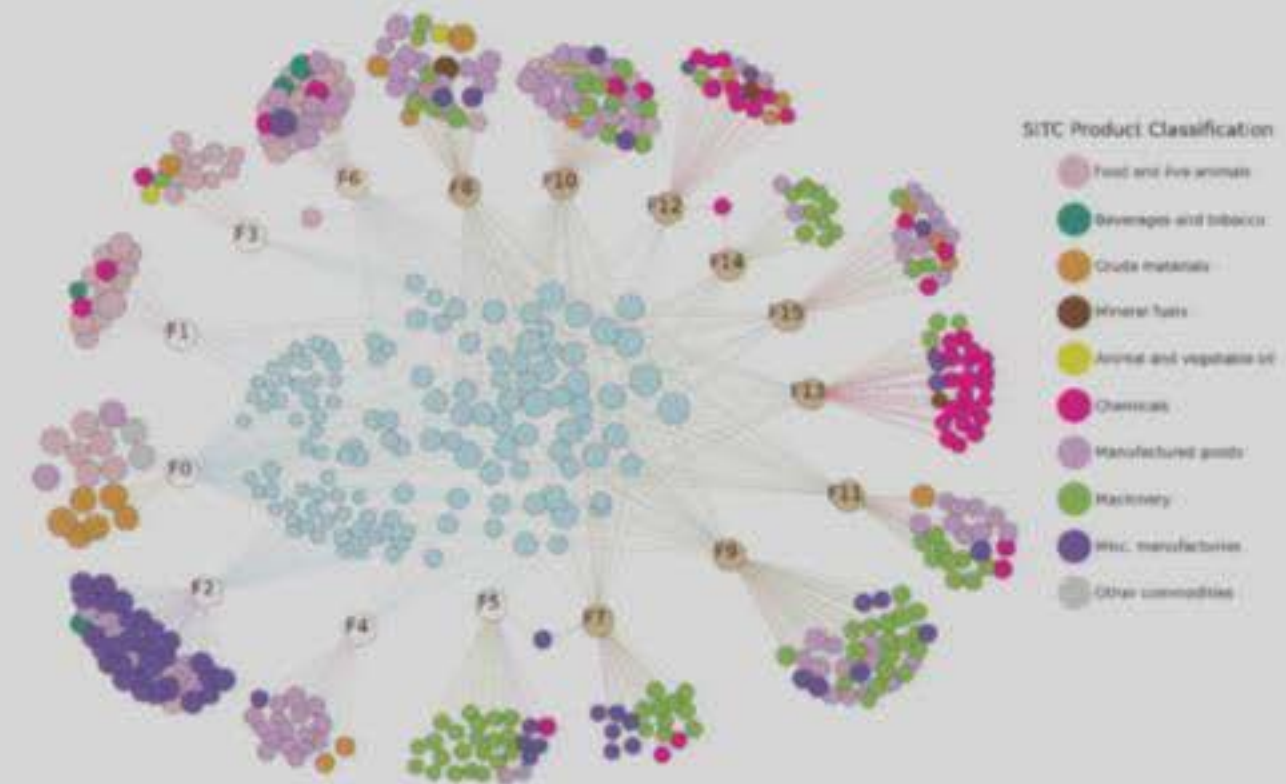
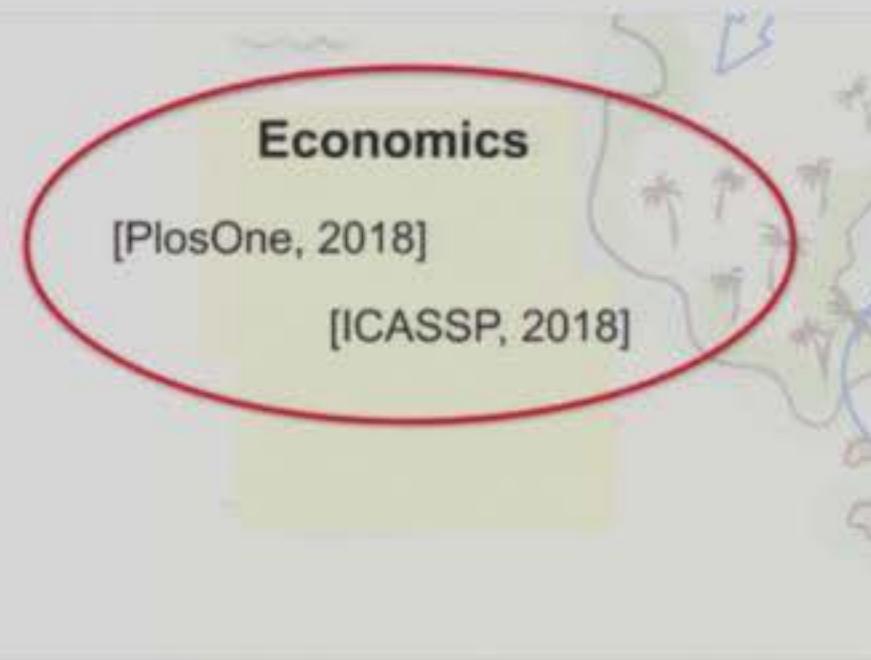
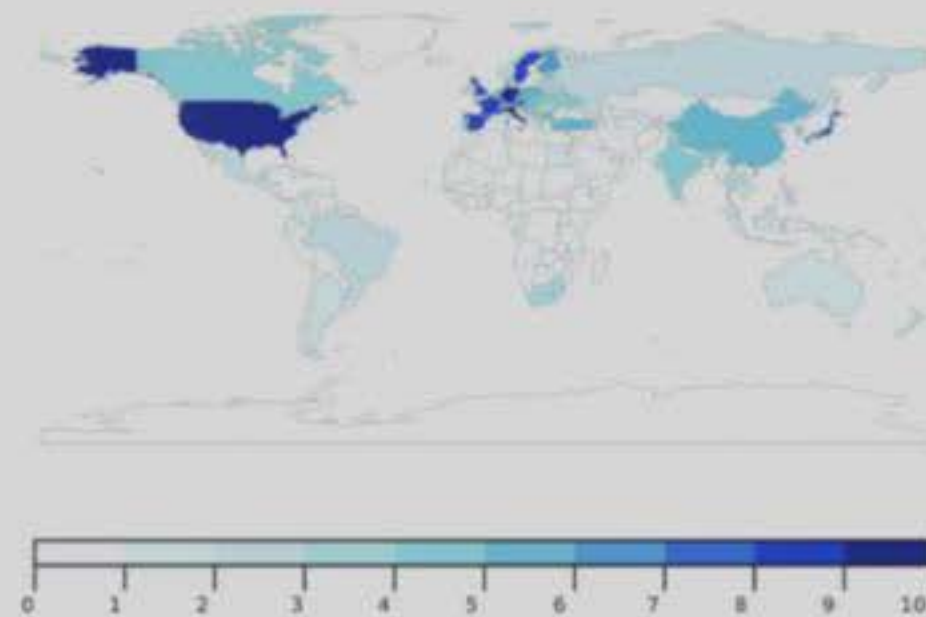


How does aging impact our athletic performance?

M. F. Pradier, F. J. R. Ruiz, and F. Perez-Cruz. **Prior Design for Dependent Dirichlet Processes: An Application to Marathon Modeling.** *PlosONE*. 2016.

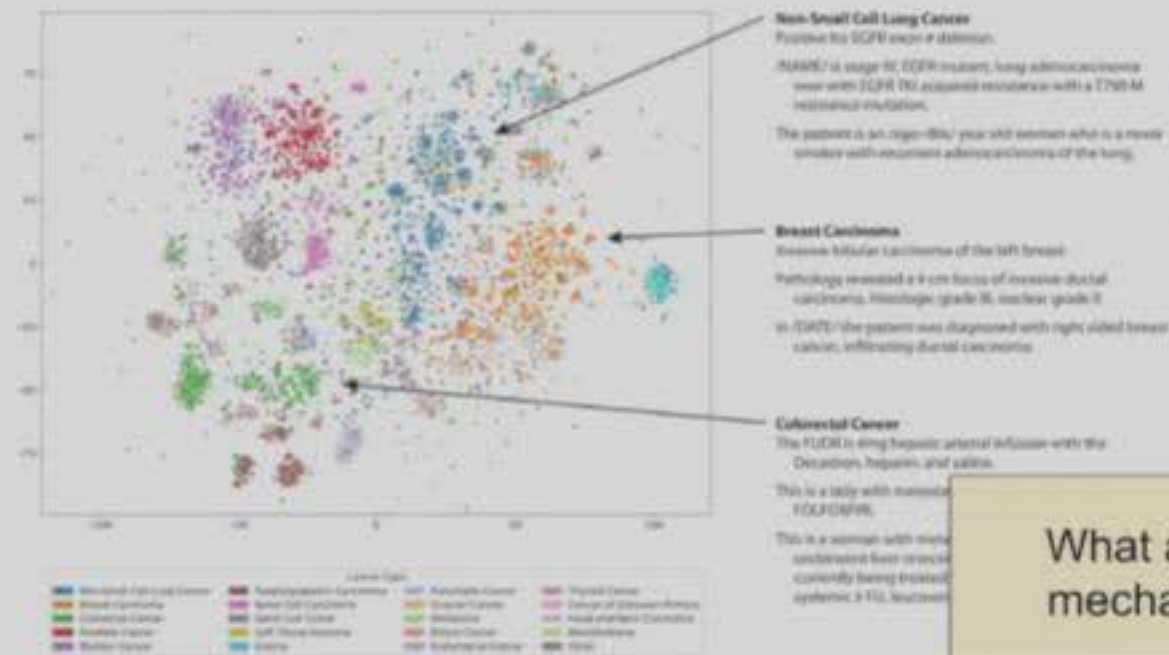


Which factors make countries wealthier than others?



Which factors make countries wealthier than others?

M. F. Pradier*, Z. Utkovski*, V. Stojkoski, L. Kocarev and F. Perez-Cruz. **Economic Complexity Unfolded: An Interpretable Model for the Productive Structure of Economies.** *PlosONE*. 2018.



What are the underlying mechanisms of cancer?

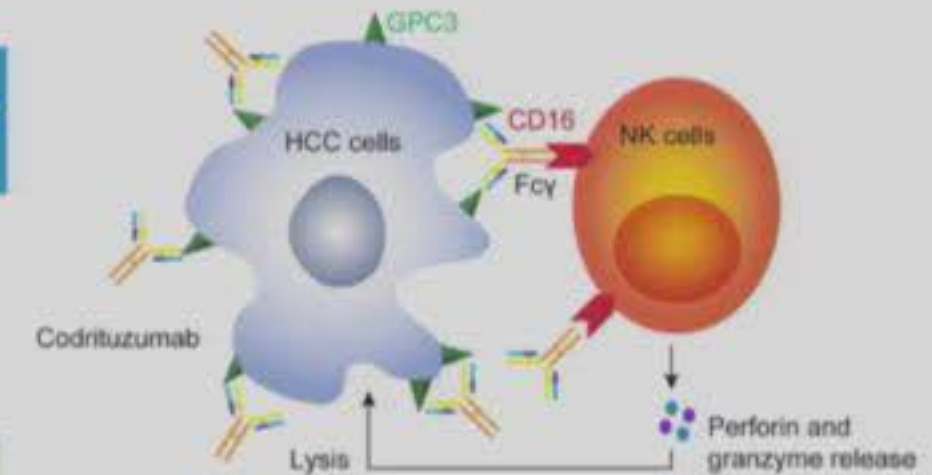
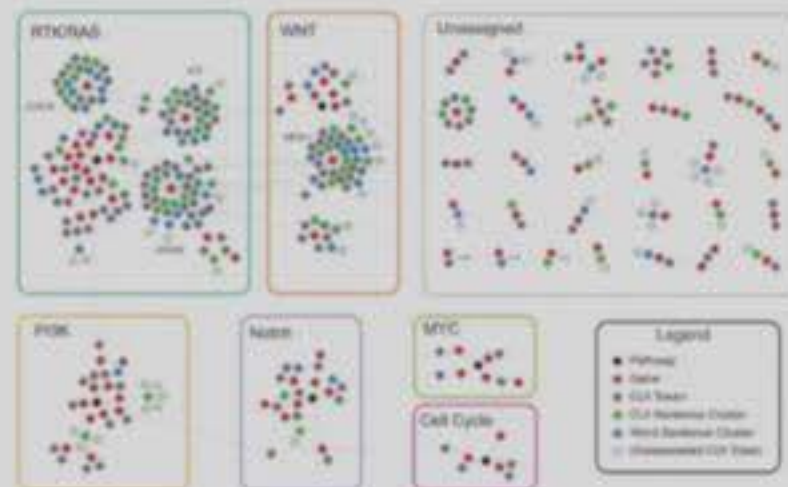
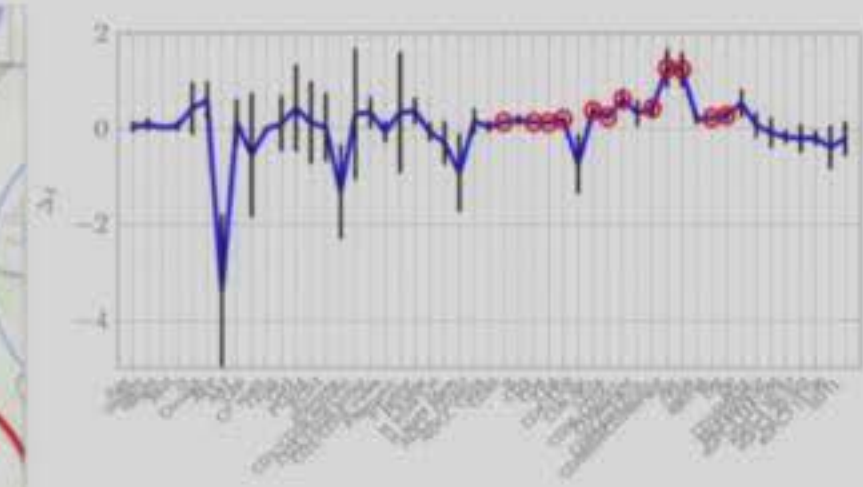
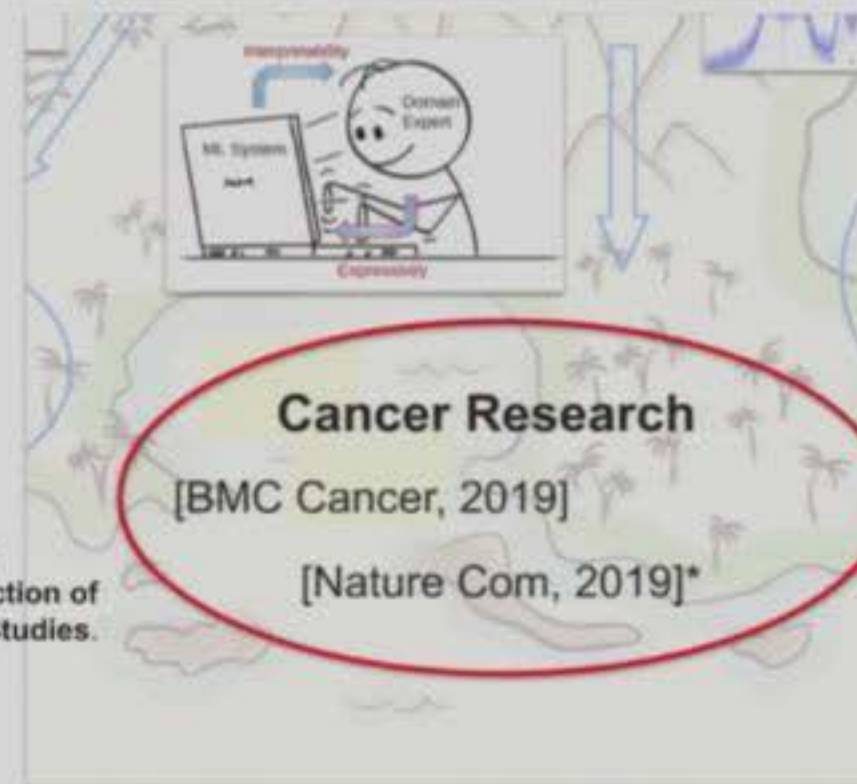


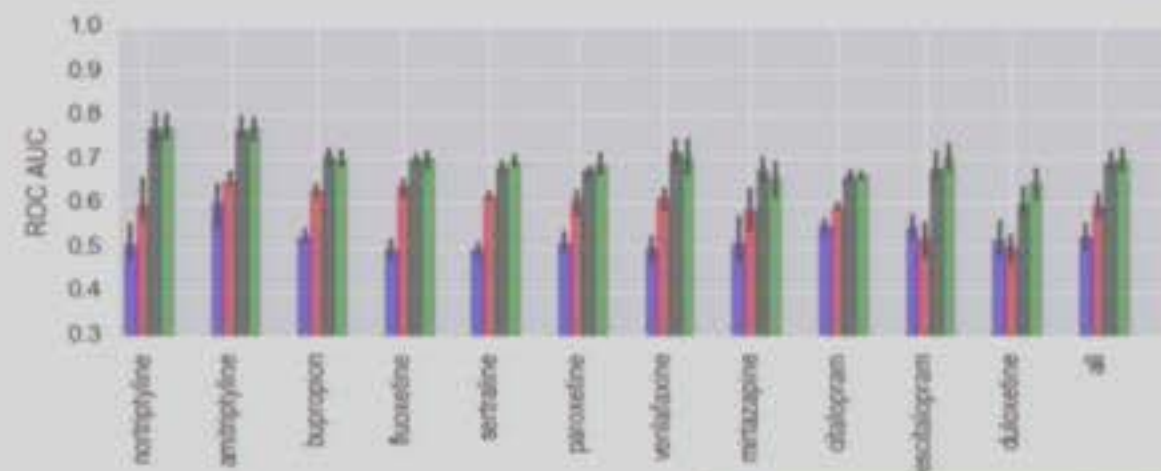
Diagram: Mechanism of codrituzumab-induced antibody-dependent cytotoxicity through the interaction of Fc CD16 in NK cells



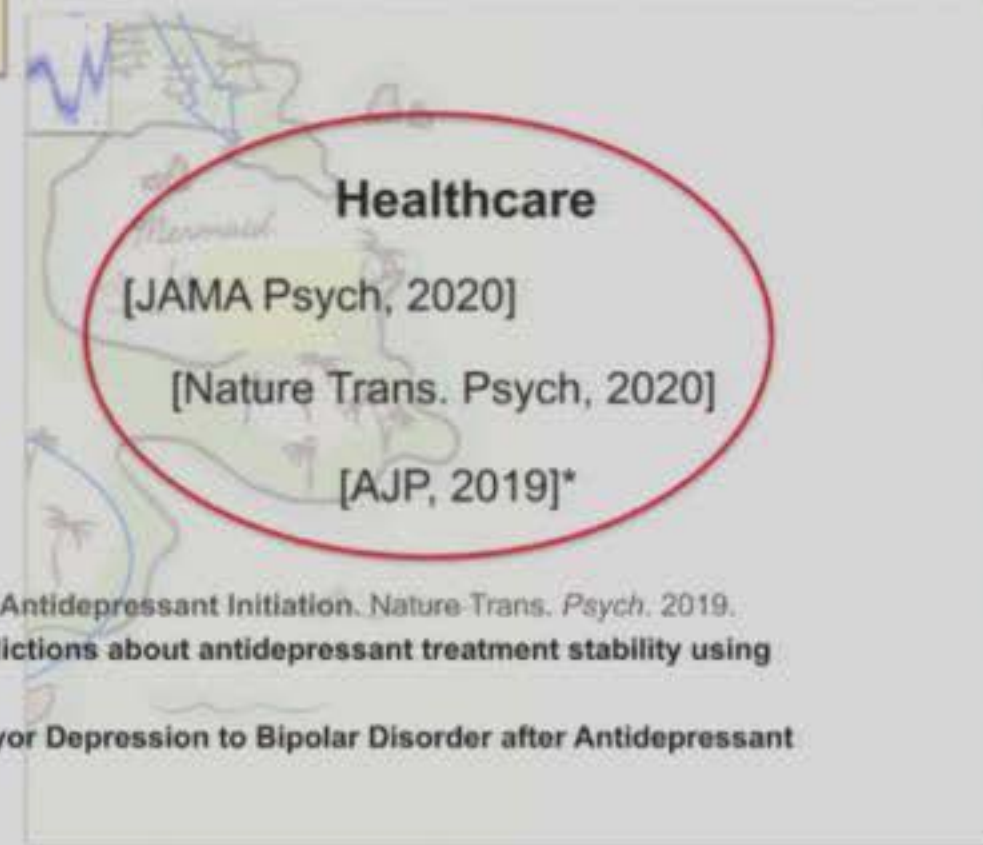
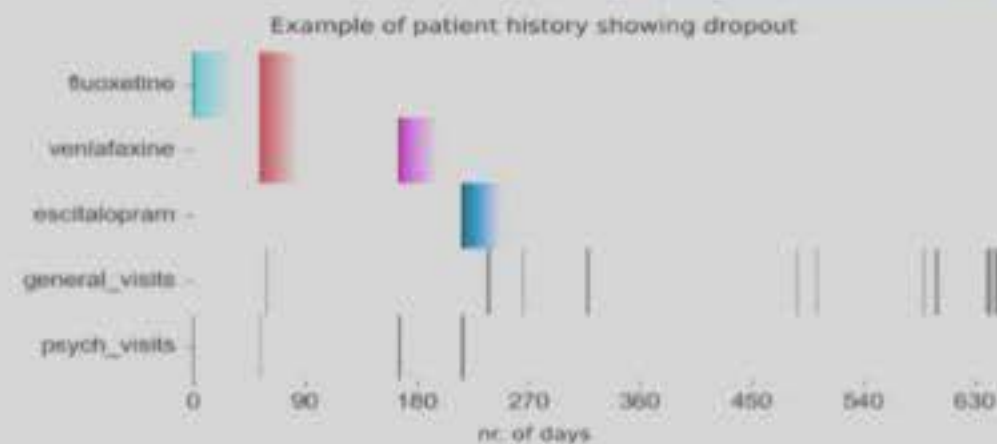
S. Stark, S. L. Hyland, M. F. Pradier, K. Lehmann, A. Wicki, F. Perez-Cruz, J. E. Vogt, and G. Ratsch. **Unsupervised Extraction of Phenotypes from Cancer Clinical Notes for Association Studies.** *In submission to Nature Communications*. 2019



M. F. Pradier, B. Reis, L. Jukofsky, F. Milletti, T. Ohtomo, F. Perez-Cruz, and O. Puig. **Case-control Indian Buffet Process identifies biomarkers of response to Codrituzumab.** *BMC Cancer*. 2019.



Can we personalize prescription of antidepressants?



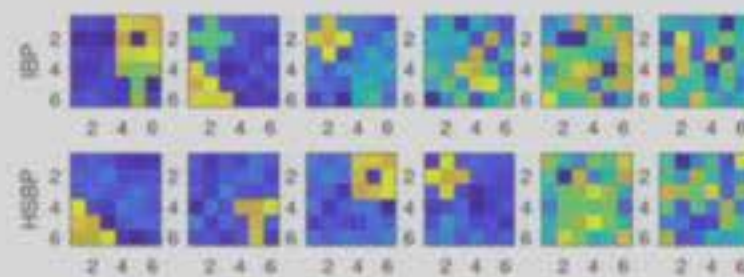
M. F. Pradier, T. H. McCoy, M. Hughes, R. H. Perlis and F. Doshi-Velez. Predicting Treatment Discontinuation after Antidepressant Initiation. *Nature-Trans. Psych.* 2019.

M. C. Hughes, M. F. Pradier, A. S. Ross, T. H. McCoy, R. H. Perlis and F. Doshi-Velez. Generating interpretable predictions about antidepressant treatment stability using supervised topic models. *Accepted JAMA Psychiatry.* 2019.

M. F. Pradier, M. Hughes, T. H. McCoy, S. Barroilhet, F. Doshi-Velez and R. H. Perlis. Predicting Transition from Major Depression to Bipolar Disorder after Antidepressant Initiation. *Submitted to American Journal of Psychiatry.* 2019.

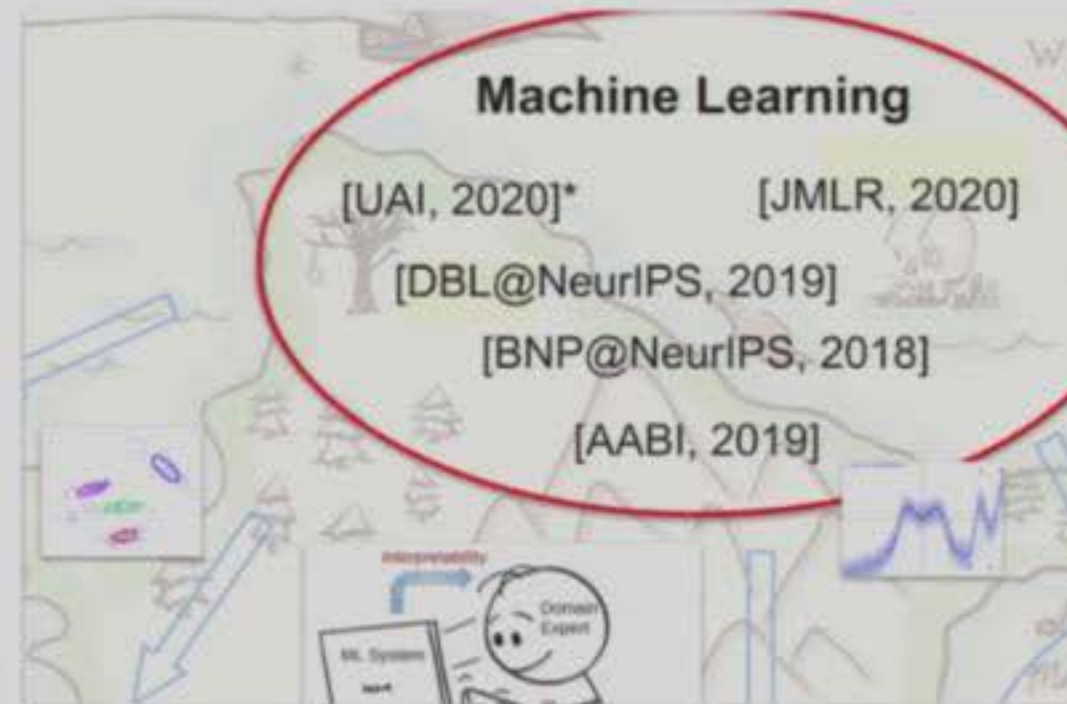
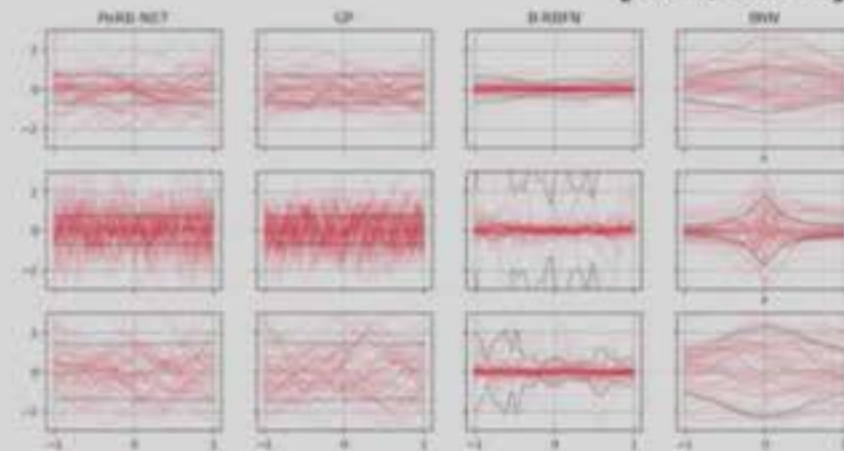
0											1
π_1					π_0						
π_{11}					π_{01}						
π_{111}						π_{010}					

[BNP@NeurIPS, 2018]

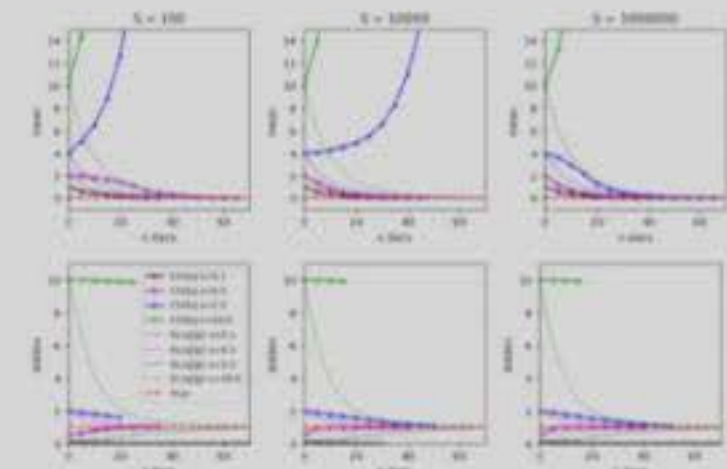


Can we design expressive priors?

[UAI, 2020]*

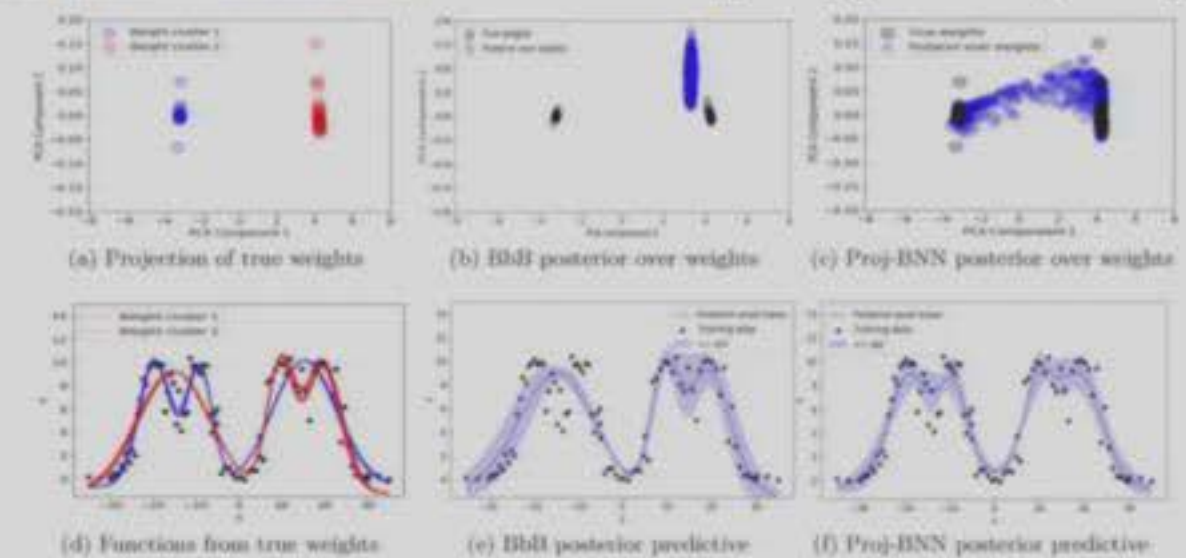


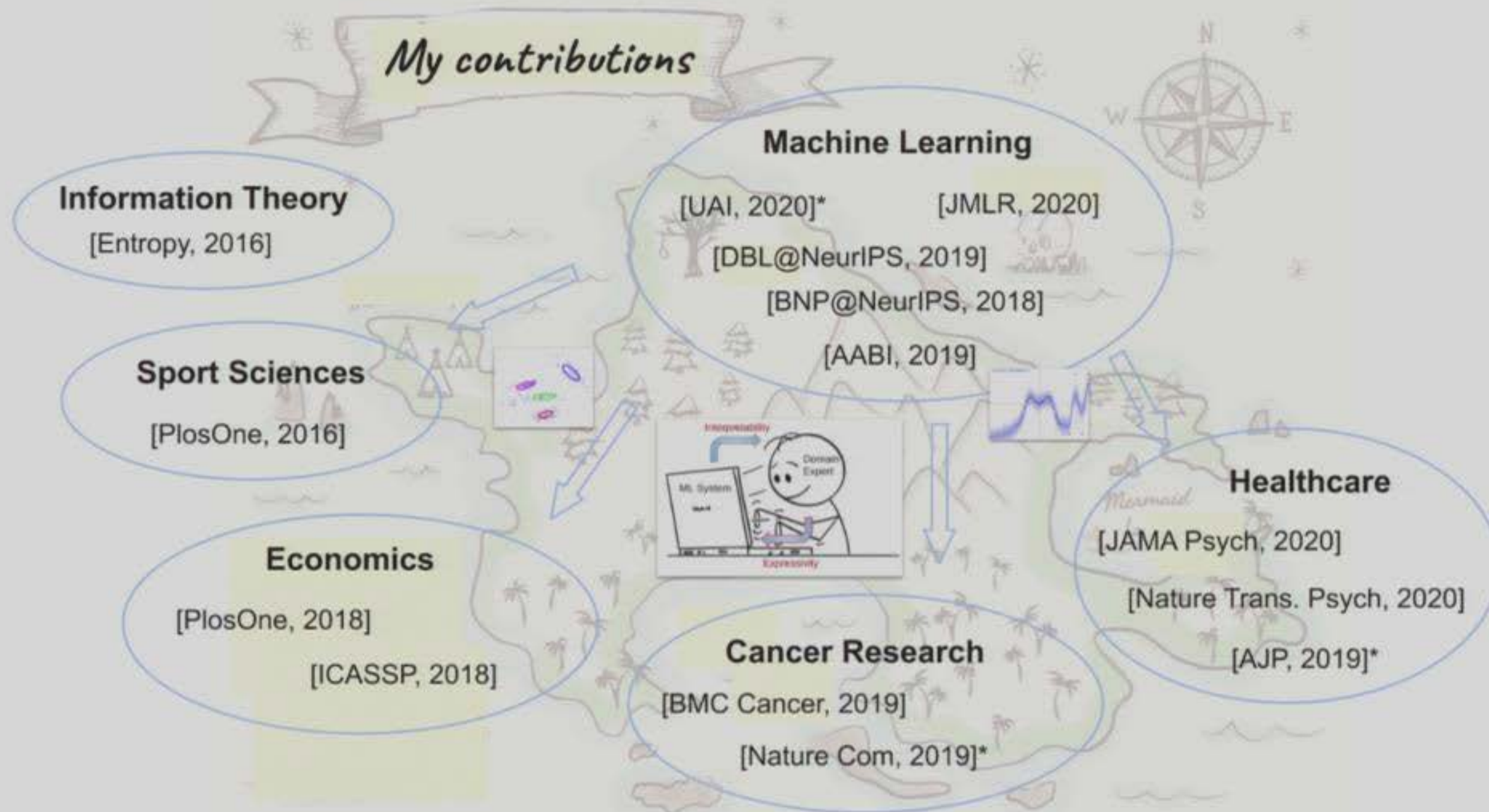
[AABI, 2019]

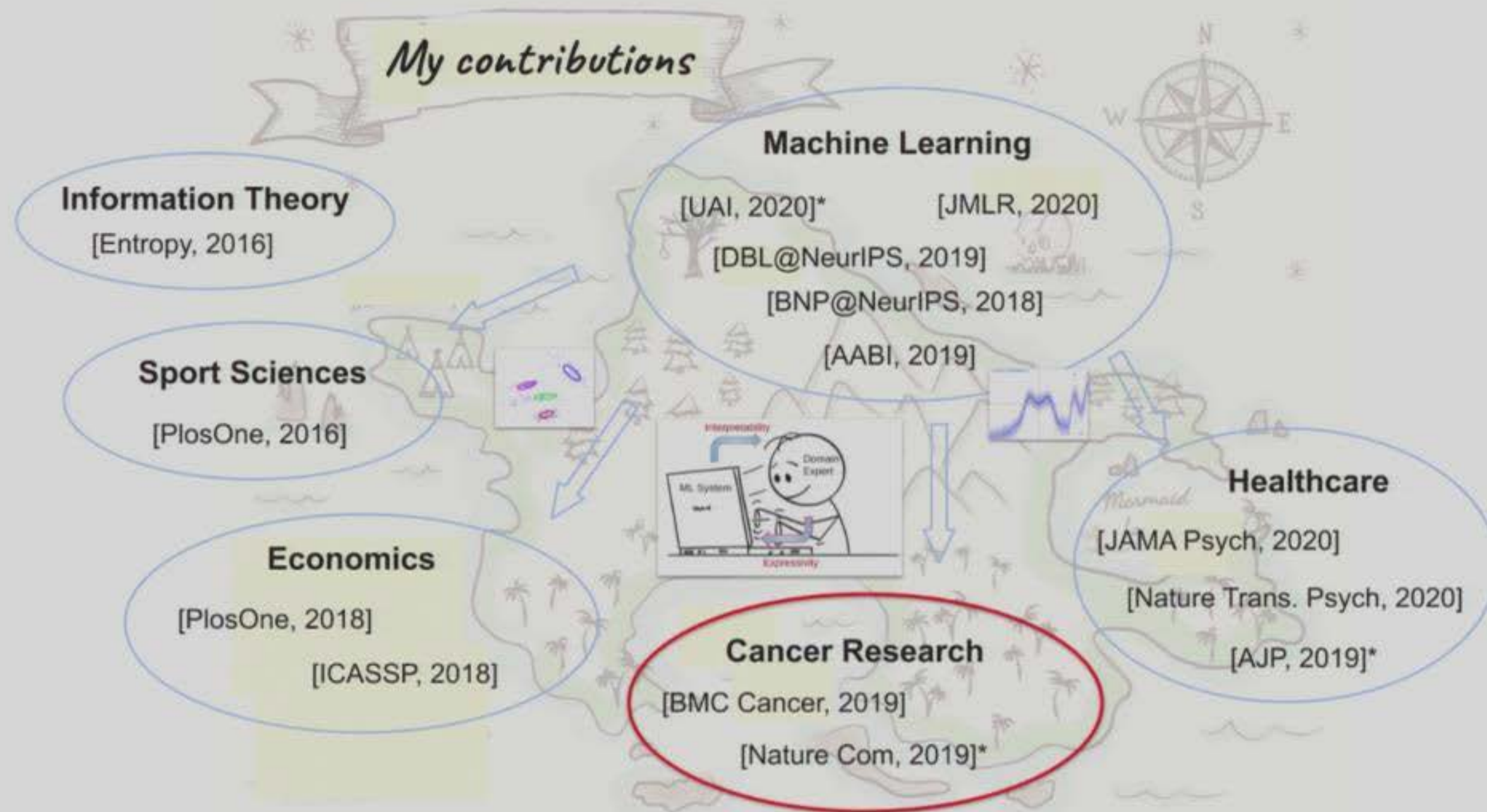


Can we improve inference?

[DBL@NeurIPS, 2019]







OUR FOCUS: BIOMARKER DISCOVERY

DEF: "ANY VARIABLE THAT CAN BE USED AS AN INDICATOR OF A PARTICULAR DISEASE STATE".

Application: Phase II clinical trial for immunotherapy for liver cancer

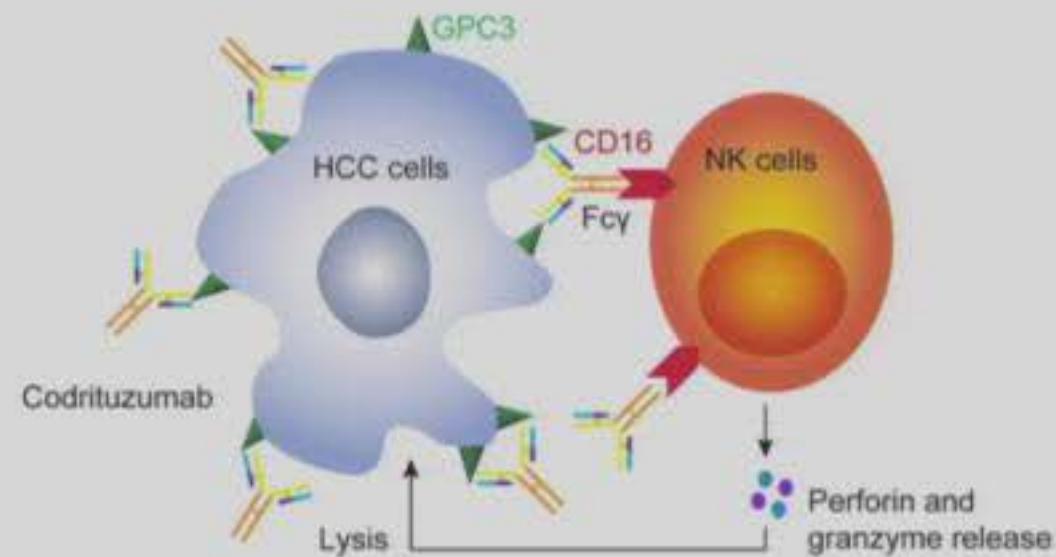


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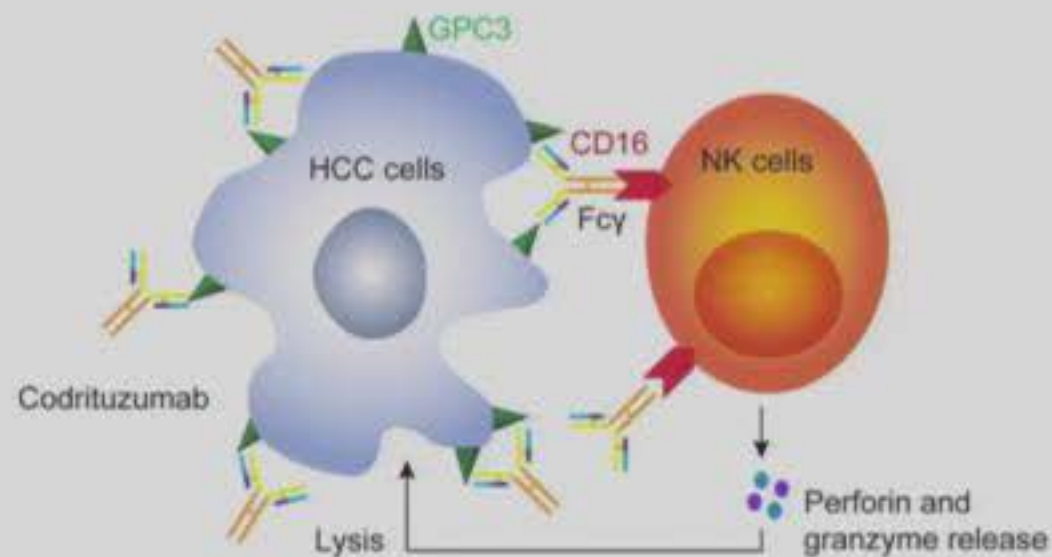
[ABOU-ALFA ET.AL, 2016]

- **Method:** classical statistical tests
- **Conclusion:** no evidence for treatment effectiveness

OUR FOCUS: BIOMARKER DISCOVERY

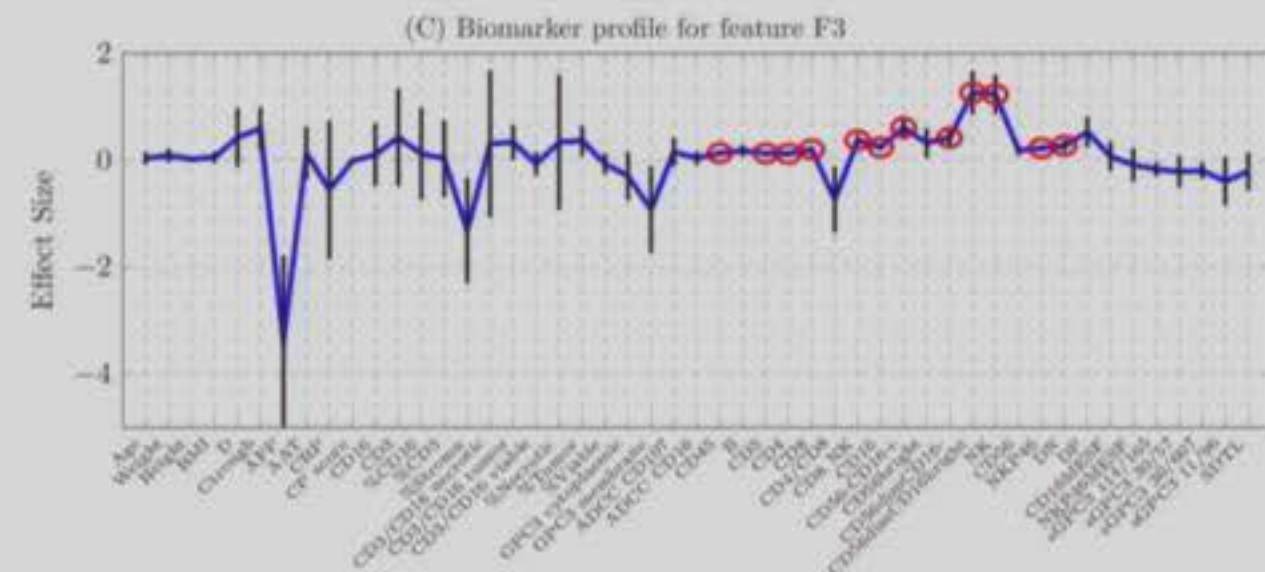
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Application: Phase II clinical trial for immunotherapy for liver cancer



[ABOU-ALFA ET.AL, 2016]

- **Method:** classical statistical tests
- **Conclusion:** no evidence for treatment effectiveness



[BMC CANCER, 2019]

- **Method:** novel Bayesian approach
- **Conclusion:** treatment effective for subgroup

WHY IS THIS PROBLEM HARD?

CHALLENGES

WHY IS THIS PROBLEM HARD?

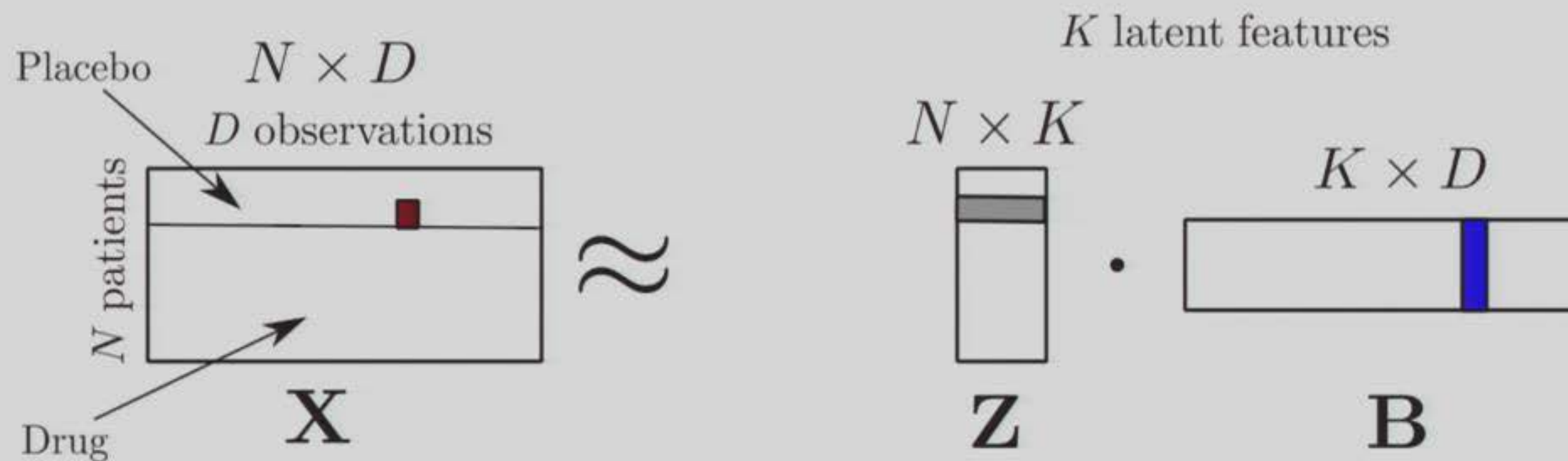
CHALLENGES

1. complex correlations
2. patient heterogeneity
3. few data available
4. very different data types
5. drug effect vs natural response



CHALLENGE 1: HOW TO DEAL WITH COMPLEX CORRELATIONS?

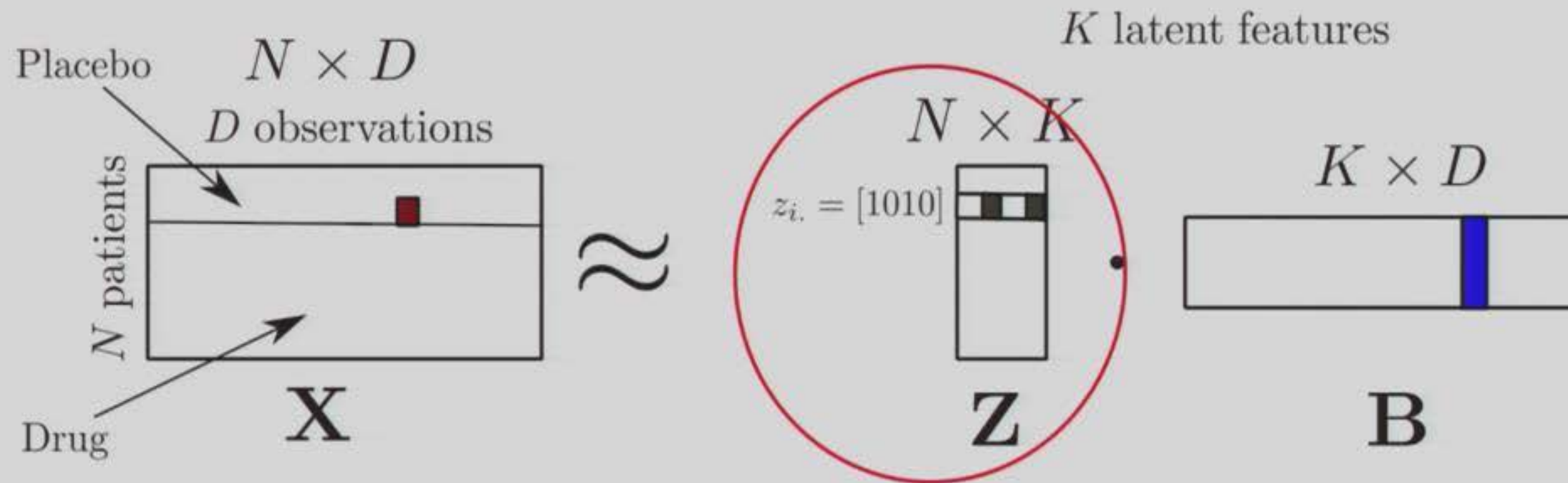
OUR APPROACH: LATENT FEATURE MODEL



$$\blacksquare x_{id} = 173 \text{ ml/dL} = 73 + 0 + 100 \text{ ml/dL}$$

CHALLENGE 2: HOW TO DEAL WITH PATIENT HETEROGENEITY?

OUR APPROACH: BINARY LATENT FEATURE MODEL



- Binary latent feature active: correlation pattern applies to patient

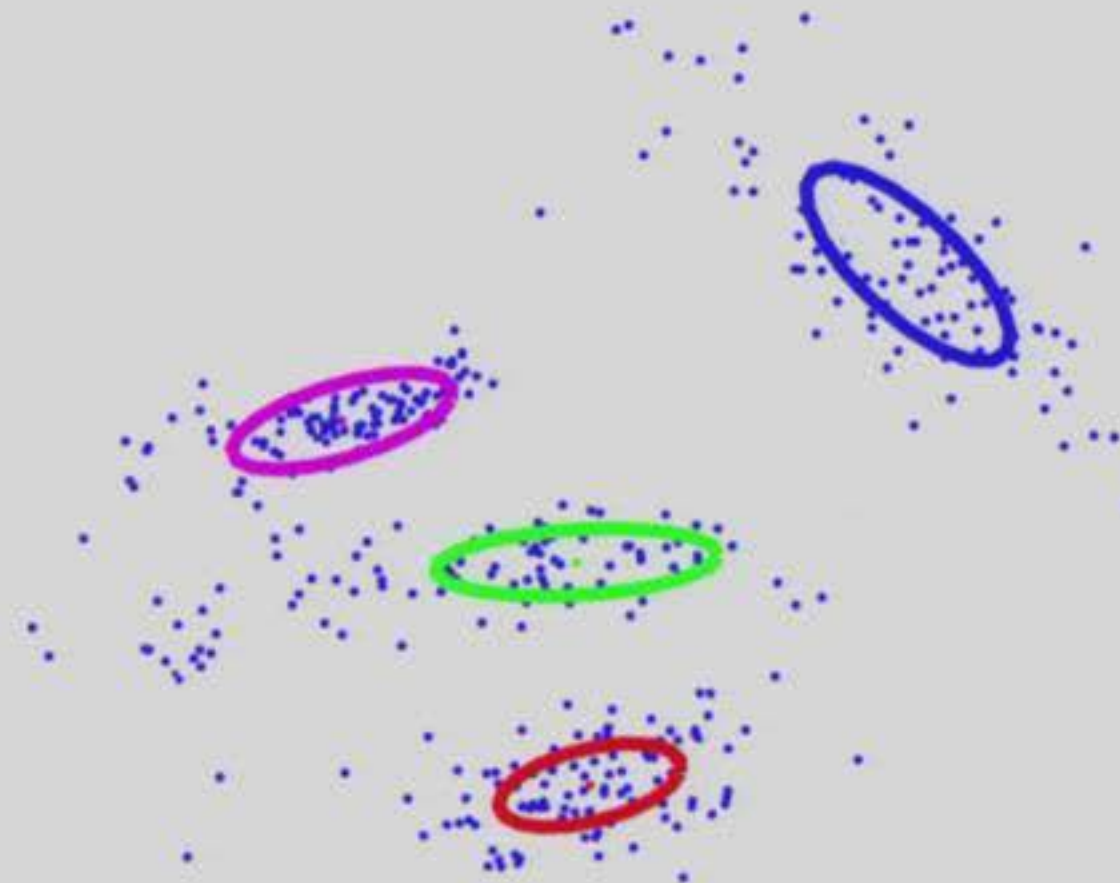
CHALLENGE 3: HOW TO DEAL WITH **LITTLE DATA**?

OUR APPROACH: **BAYESIAN NONPARAMETRICS**

- **Bayesian:** to handle uncertainty

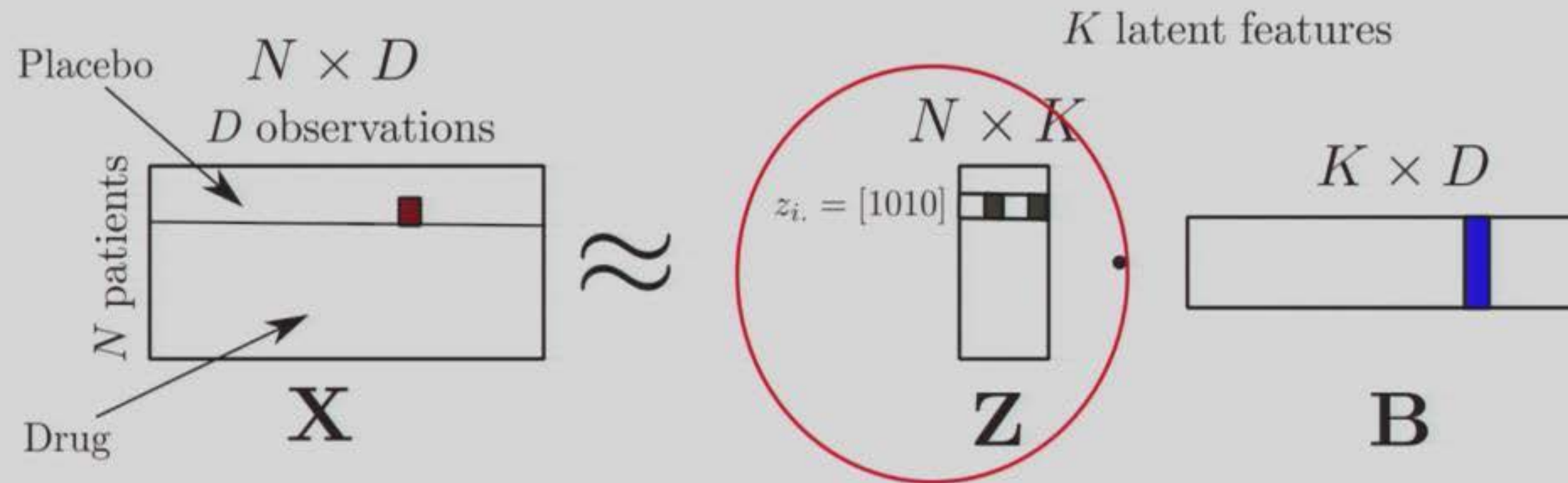
$$p(\text{parameters}|\text{data}) \propto p(\text{data}|\text{parameters})p(\text{parameters})$$

- **Nonparametric:** to adapt model complexity (hypothesis generation)



CHALLENGE 2: HOW TO DEAL WITH PATIENT HETEROGENEITY?

OUR APPROACH: BINARY LATENT FEATURE MODEL



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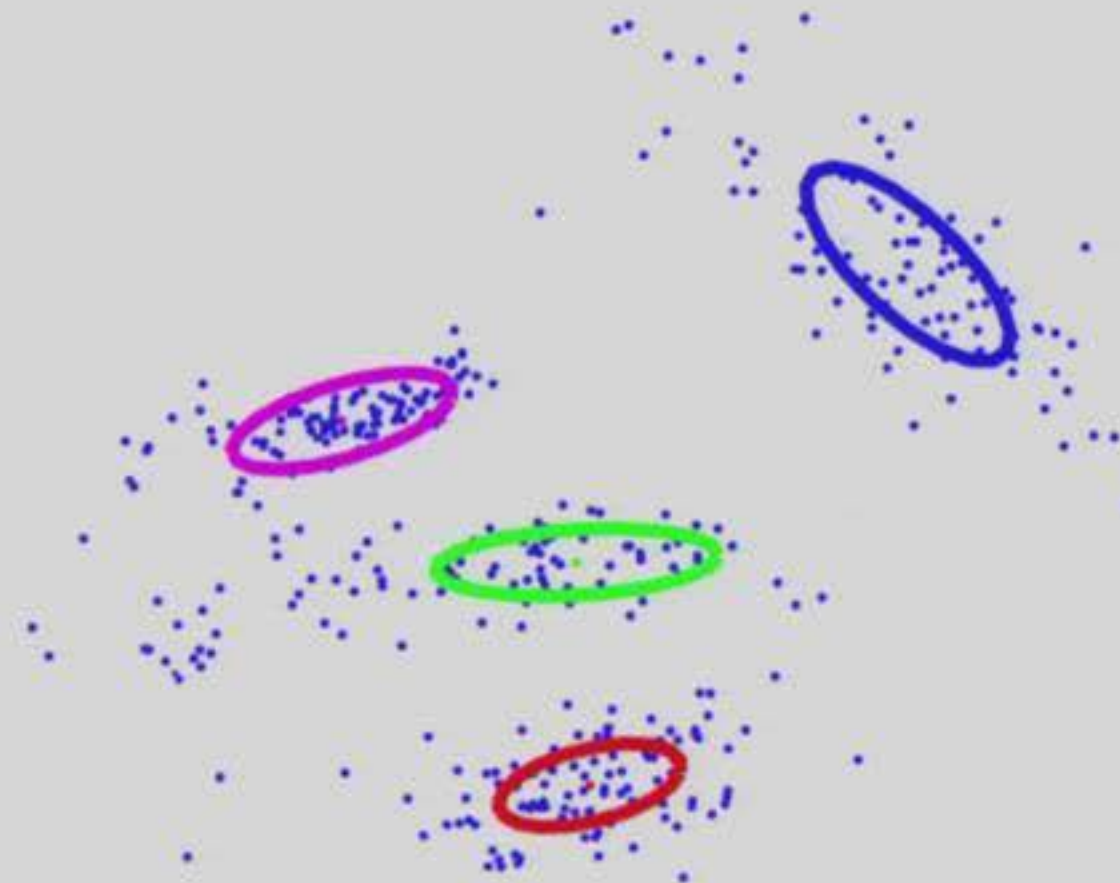
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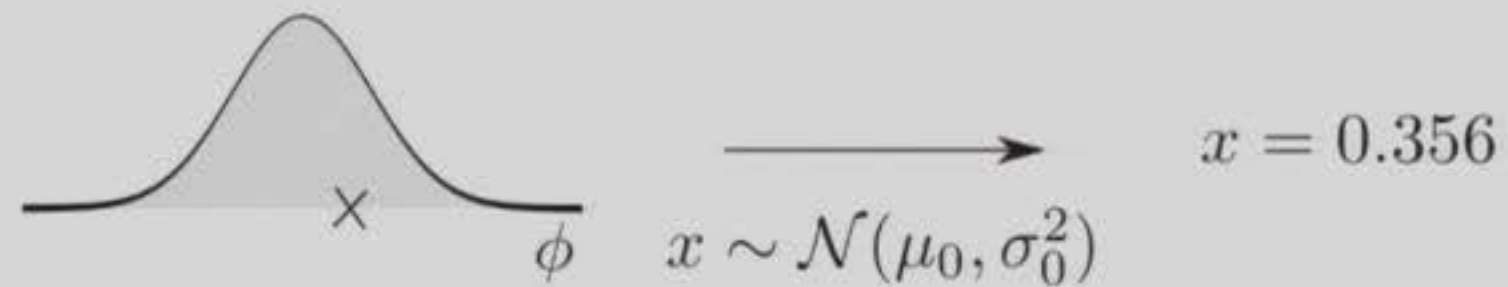
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CHALLENGE 3: HOW TO DEAL WITH LITTLE DATA?

OUR APPROACH: BAYESIAN NONPARAMETRICS

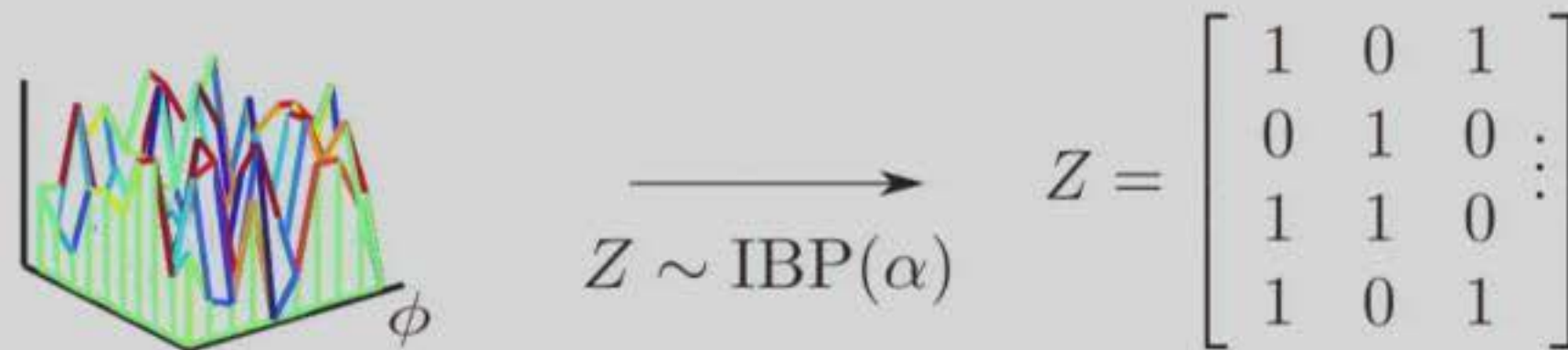
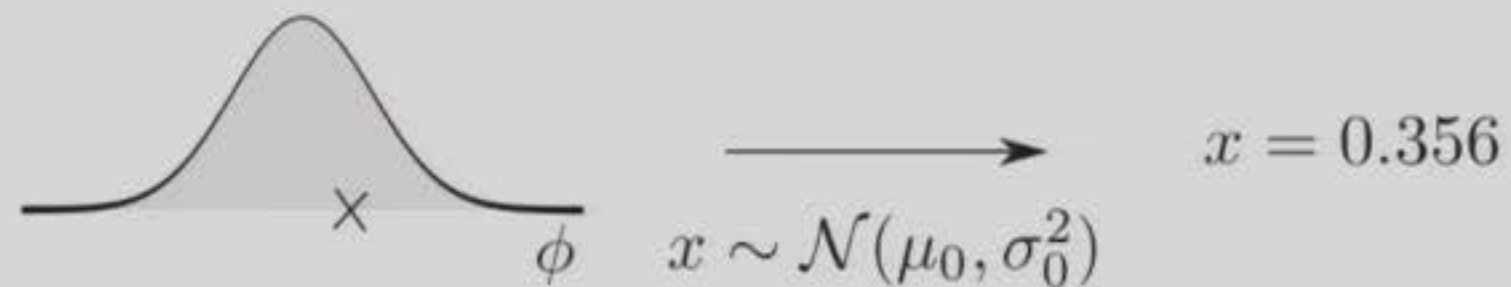
- Indian Buffet Process [Ghahramani et.al, 2006]



CHALLENGE 3: HOW TO DEAL WITH **LITTLE DATA**?

OUR APPROACH: **BAYESIAN NONPARAMETRICS**

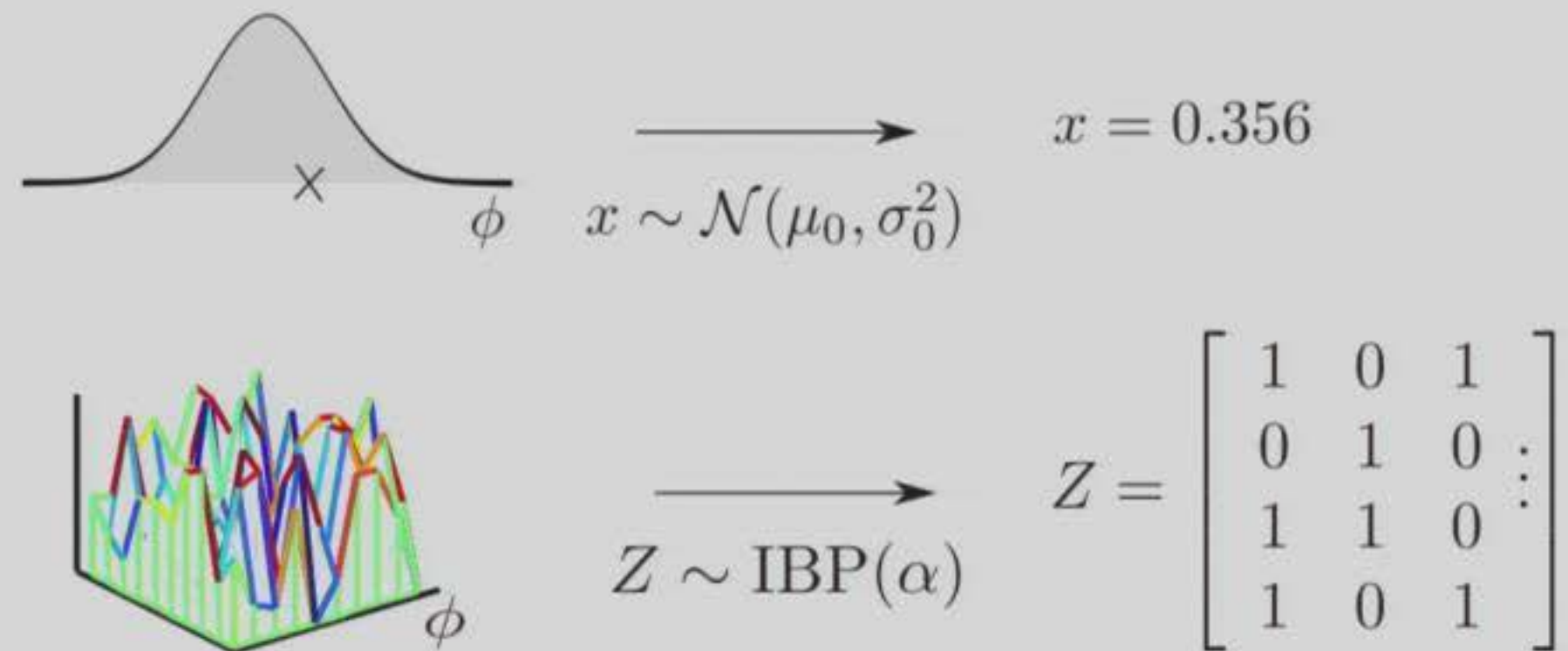
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CHALLENGE 3: HOW TO DEAL WITH **LITTLE DATA**?

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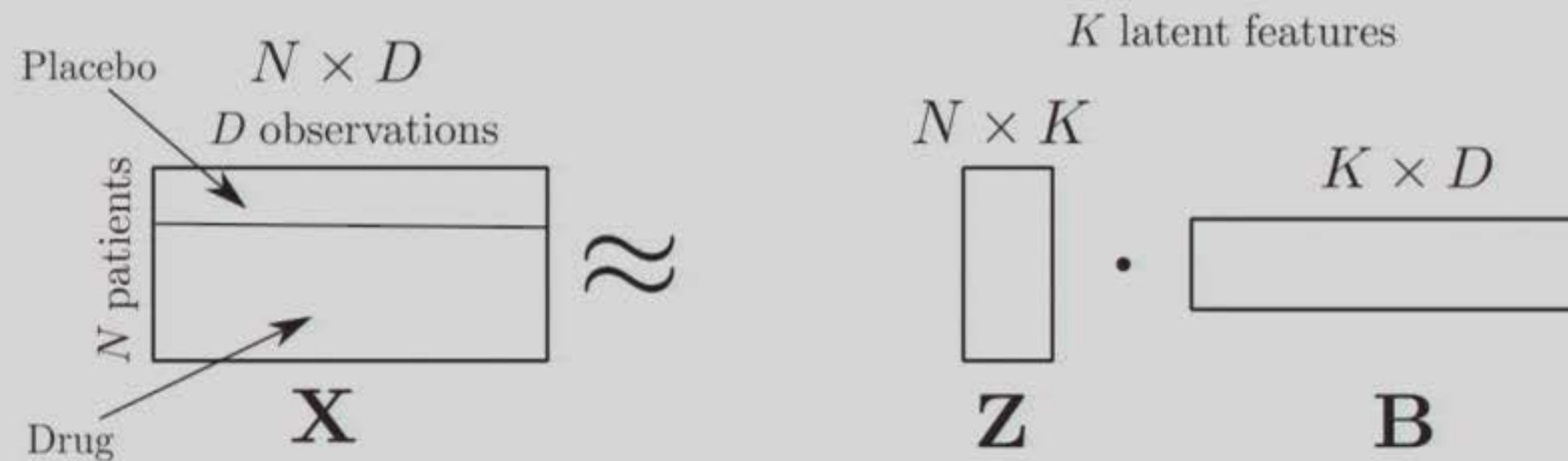
- Indian Buffet Process [Ghahramani et.al, 2006]



- Prior over binary matrices with infinite number of columns
- $\mathbf{Z} \sim \text{IBP}(\alpha)$, where α : concentration parameter

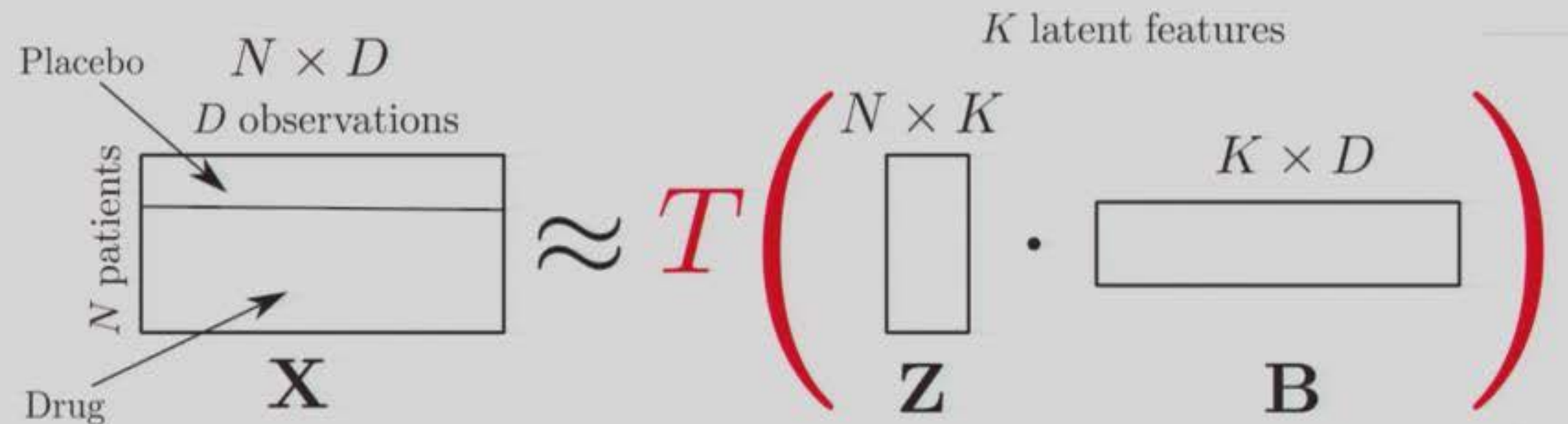
CHALLENGE 4: HOW TO DEAL WITH DIFFERENT DATA TYPES?

OUR APPROACH: USE LINK FUNCTIONS



CHALLENGE 4: HOW TO DEAL WITH DIFFERENT DATA TYPES?

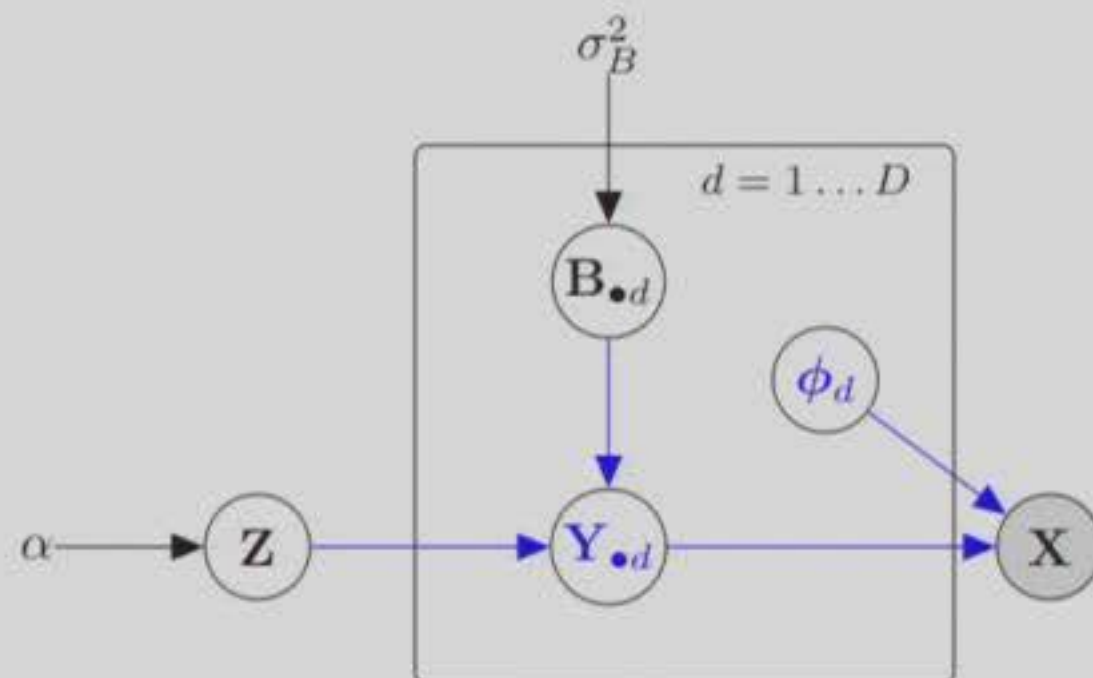
OUR APPROACH: USE LINK FUNCTIONS



- ▶ Link functions T_d for each observed feature d
- ▶ Each T_d depends on type of data

SOLVING ALL 4 CHALLENGES TOGETHER

GENERAL LATENT FEATURE MODEL (GLFM) [JMLR, 2020]



$$\begin{aligned}
 x_{nd} &= T_d(y_{nd}; \phi_d) \\
 y_{nd} | \mathbf{Z}, \mathbf{B} &\sim \mathcal{N}(\mathbf{Z}_{n\bullet} \mathbf{B}_{\bullet d}, \sigma_y^2) \\
 B_{kd} &\sim \mathcal{N}(0, \sigma_B^2) \\
 \mathbf{Z} &\sim \text{IBP}(\alpha)
 \end{aligned}$$

Infinite latent feature model for heterogeneous datasets

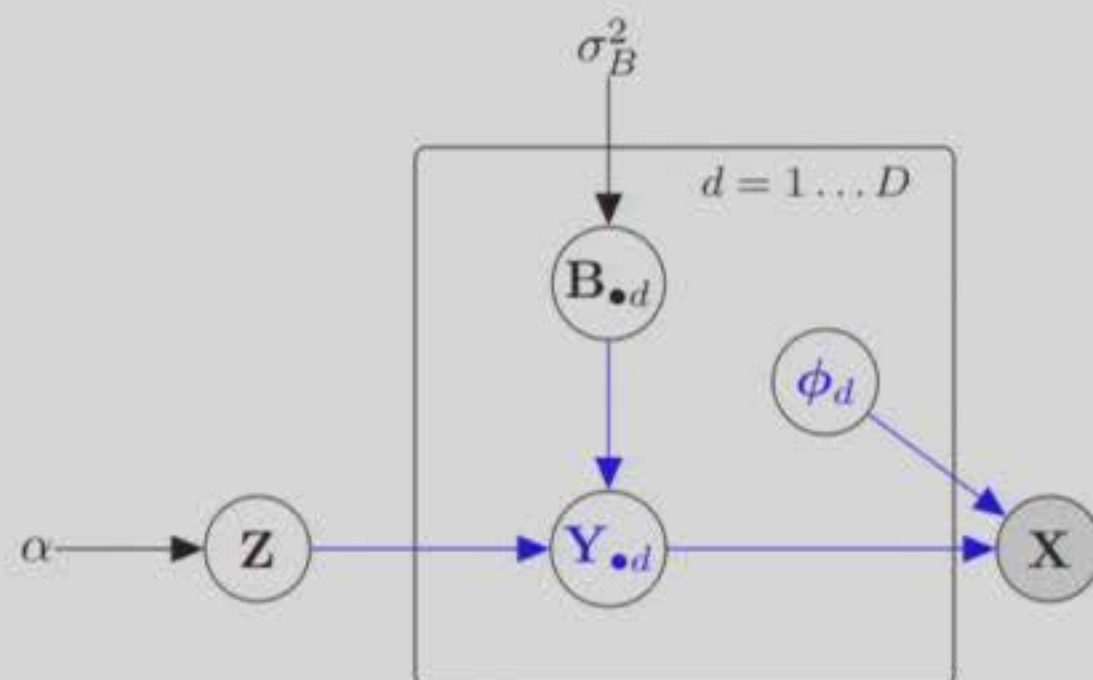
GLFM PACKAGE

- ▶ Open-source python/matlab/R code
- ▶ Discovers complex correlations
- ▶ Is able to deal with data heterogeneity, noisy observations, missing data, different data types
- ▶ Adjusts model complexity

<https://github.com/ivaleraM/GLFM>

SOLVING ALL 4 CHALLENGES TOGETHER

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REVISITING LIST OF CHALLENGES

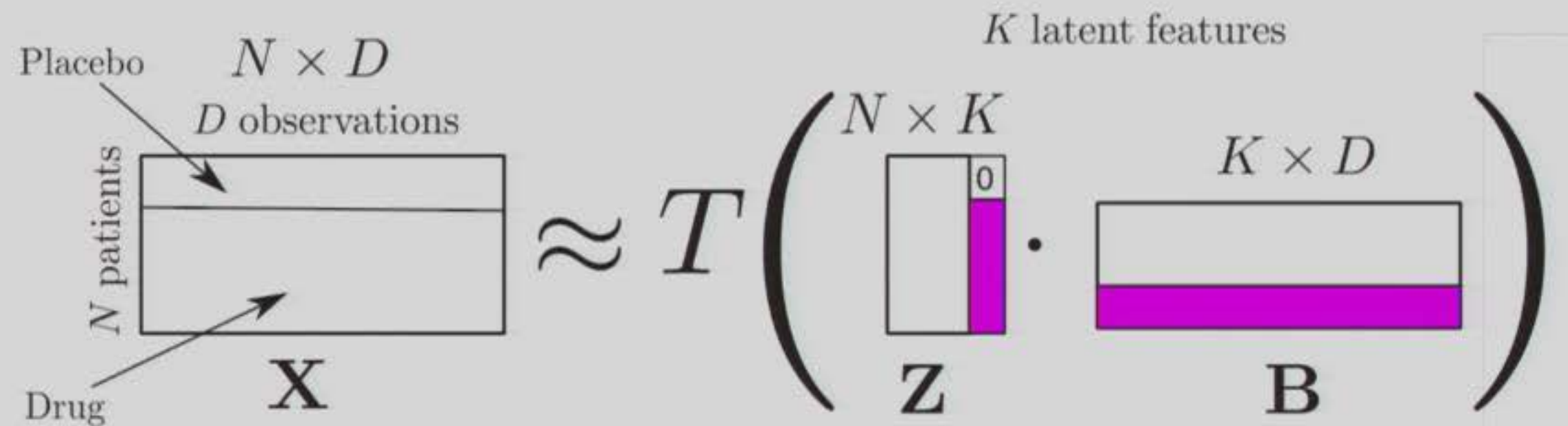
CHALLENGES

1. complex correlations
2. patient heterogeneity
3. few data available
4. very different data types
5. drug effect vs natural response



CHALLENGE 5: DRUG EFFECT VS NATURAL RESPONSE?

OUR APPROACH: STRUCTURED PRIOR

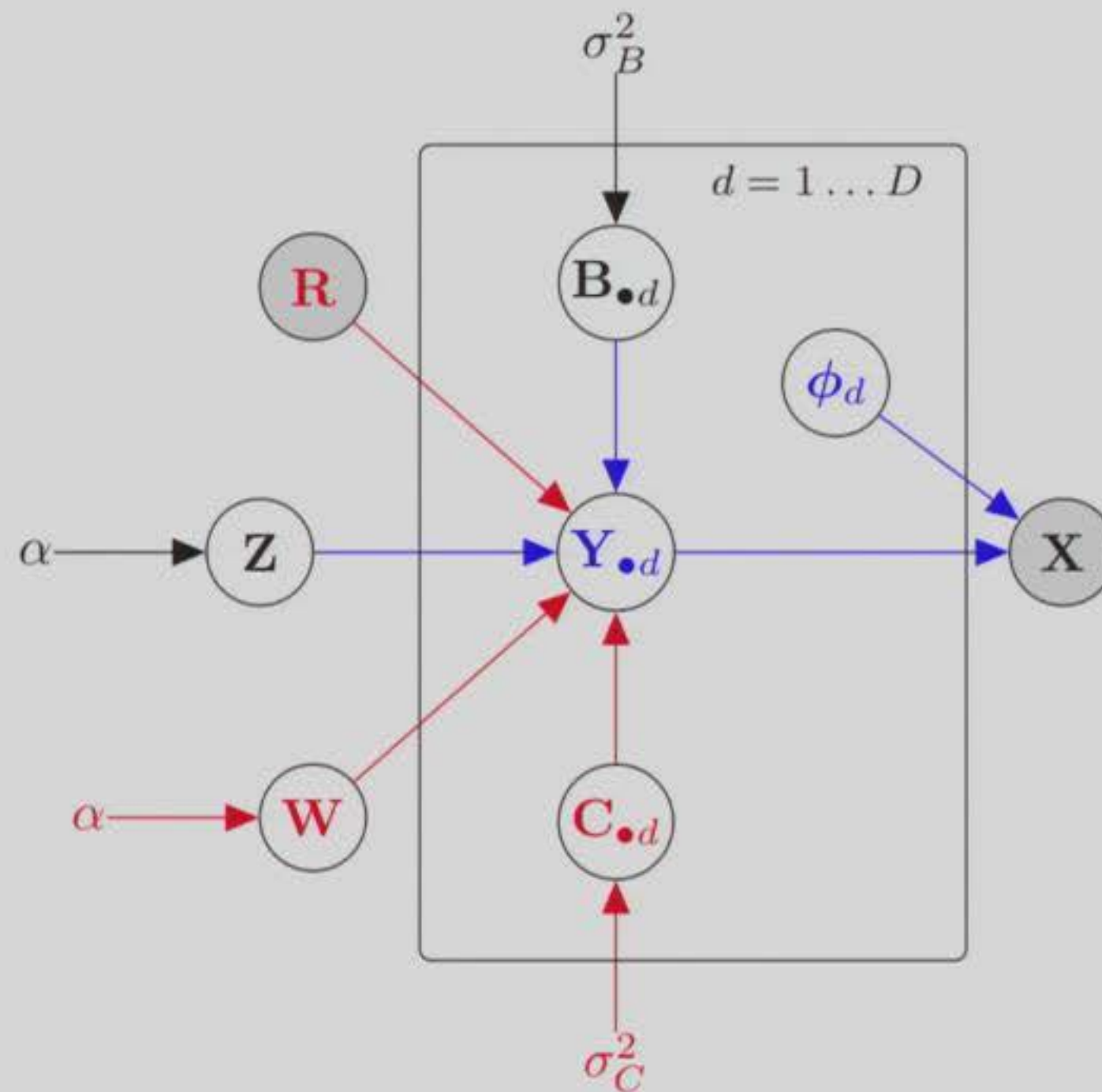


TREATMENT-SPECIFIC LATENT FEATURES

- ▶ can only activate for patients in treatment arm
- ▶ number learned automatically

HOW TO DISTINGUISH DRUG EFFECT VS NATURAL RESPONSE?

CASE-CONTROL INDIAN BUFFET PROCESS (C-IBP) [BMC CANCER, 2019]



R_n : drug indicator for patient n

$$x_{nd} = T_d(y_{nd}; \phi_d)$$

$$y_{nd} | \mathbf{Z}, \mathbf{W}, \mathbf{B}, \mathbf{C}, \mathbf{R} \sim$$

$$\mathcal{N}(\mathbf{Z}_n \bullet \mathbf{B}_{\bullet d} + \mathbb{1}[\mathbf{R}_n = 1] \mathbf{W}_n \bullet \mathbf{C}_{\bullet d}, \sigma_y^2)$$

$$B_{kd} \sim \mathcal{N}(0, \sigma_B^2)$$

$$\mathbf{Z} \sim \text{IBP}(\alpha)$$

$$C_{kd} \sim \mathcal{N}(0, \sigma_C^2)$$

$$\mathbf{W} \sim \text{IBP}(\alpha)$$

- **Inference:** MCMC approach with accelerated Gibbs sampling
- **Biomarker discovery:** statistical multiple hypothesis testing

RESULTS: SUBPOPULATIONS

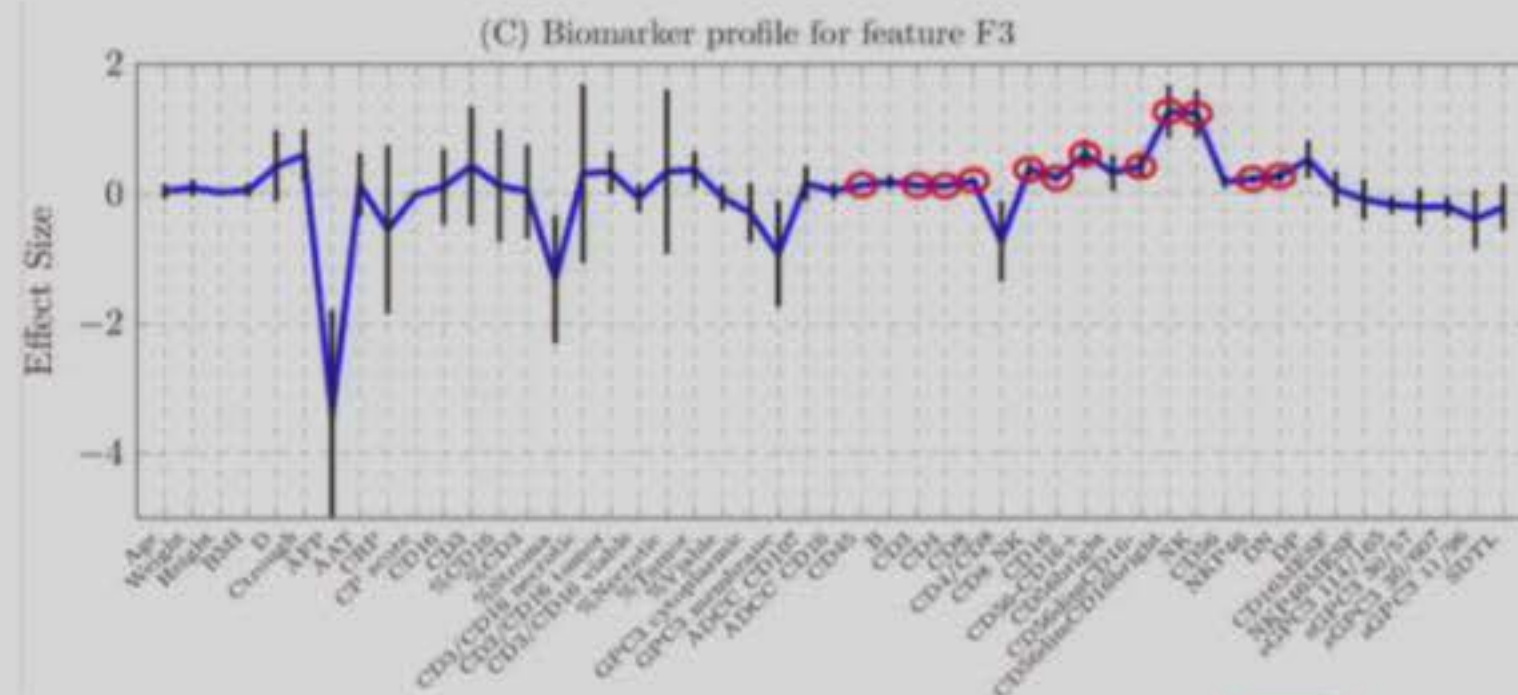
GPC3 Antibody Treatment against Liver Cancer (J. Hepatology. 2016 Apr, Abou-Alfa et.al.)

- ▶ 180 patients: 60 took a placebo, 120 took the drug
- ▶ PFS: Progression Free Survival

Sub-population	Drug Identifier	F1	F2	F3	Size (number of patients)	Mean PFS (months)	Median PFS (months)
1.	0	0	0	0	33.37	3.06	1.65
2.	0	0	1	0	4.07	2.29	2.24
3.	0	1	0	0	17.84	2.72	1.81
4.	0	1	1	0	4.72	7.05	7.18
5.	1	0	0	0	51.52	3.22	2.55
6.	1	0	0	1	16.77	4.17	3.65
7.	1	0	1	0	8.38	1.74	1.33
8.	1	0	1	1	2.07	2.69	2.65
9.	1	1	0	0	29.88	3.36	2.03
10.	1	1	0	1	4.90	4.44	4.34
11.	1	1	1	0	4.53	6.31	5.31
12.	1	1	1	1	1.94	10.04	10.01

RESULTS: BIOMARKER DISCOVERY

TREATMENT-SPECIFIC FEATURE F3



M. F. Pradier, B. Reis, L. Jukofsky, F. Milletti, T. Ohtomo, F. Perez-Cruz, and O. Puig. **Case-control Indian Buffet Process identifies biomarkers of response to Codrituzumab.** *BMC Cancer*. 2019.

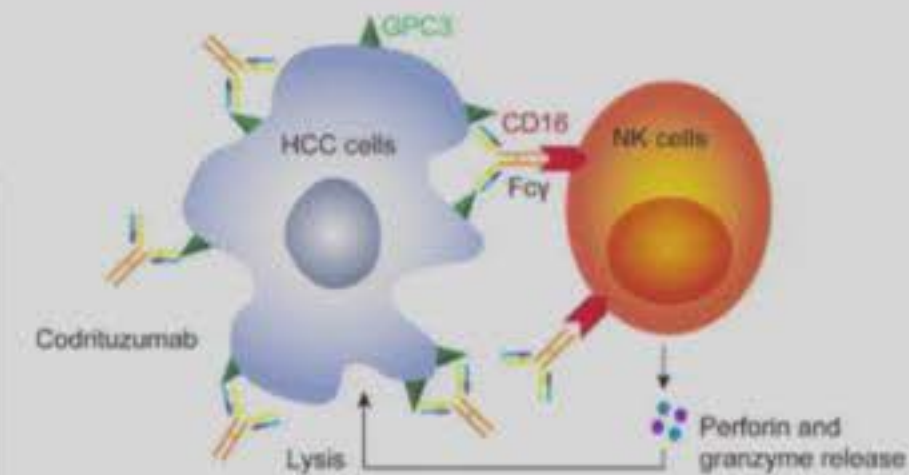


Diagram: Mechanism of codrituzumab-induced antibody-dependent cytotoxicity through the interaction of Fc CD16 in NK cells

What did we find?

- Subgroup for which treatment is especially effective
- Relevant biomarkers (drug acting as expected)



RESULTS: SUBPOPULATIONS

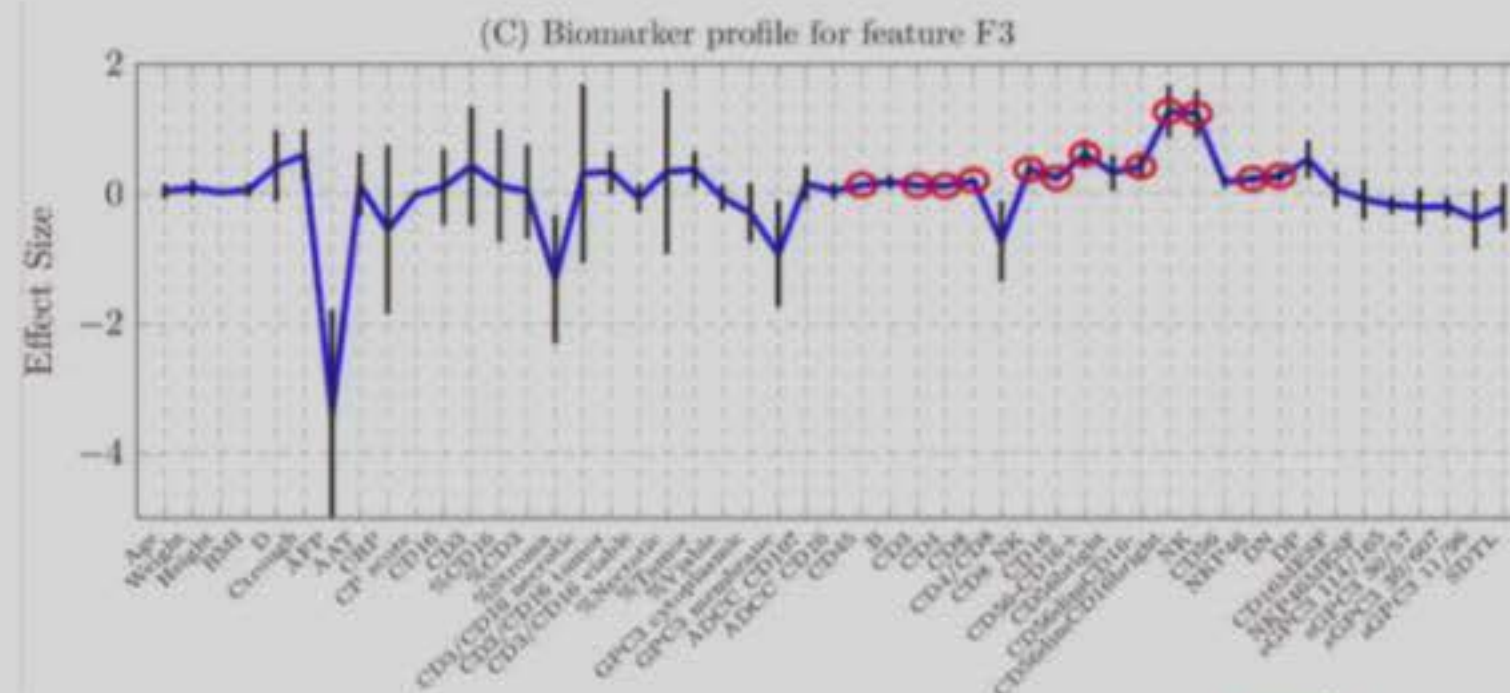
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8.	1	0	1	1	2.07	2.69	2.65
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RESULTS: BIOMARKER DISCOVERY

TREATMENT-SPECIFIC FEATURE F3



M. F. Pradier, B. Reis, L. Jukofsky, F. Milletti, T. Ohtomo, F. Perez-Cruz, and O. Puig. **Case-control Indian Buffet Process identifies biomarkers of response to Codrituzumab.** *BMC Cancer*. 2019.

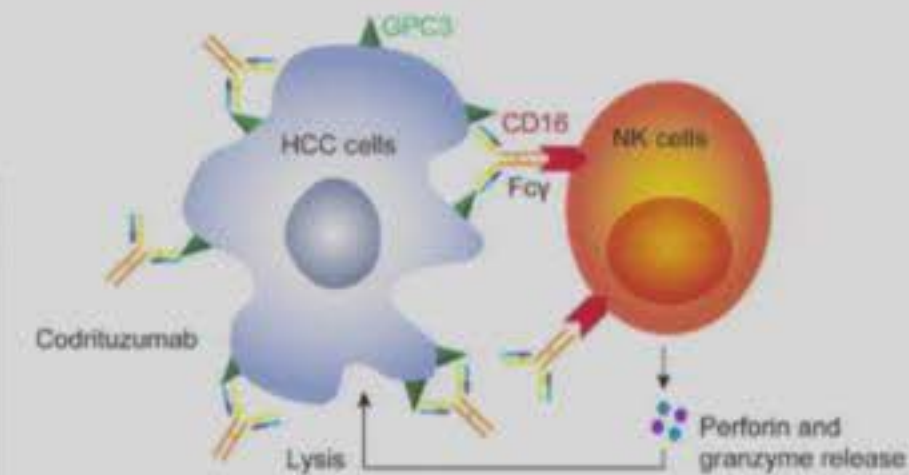


Diagram: Mechanism of codrituzumab-induced antibody-dependent cytotoxicity through the interaction of Fc CD16 in NK cells

What did we find?

- Subgroup for which treatment is especially effective
- Relevant biomarkers (drug acting as expected)



RESULTS: SUBPOPULATIONS

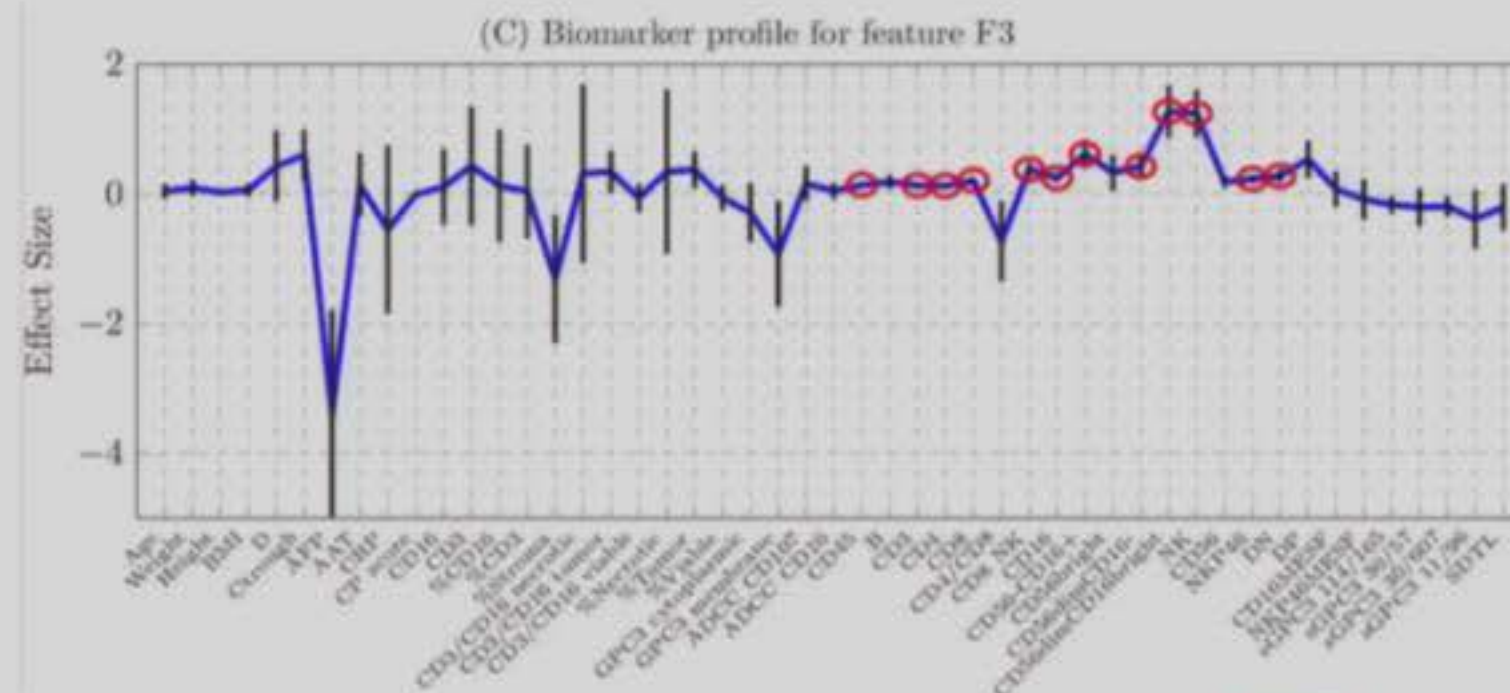
GPC3 Antibody Treatment against Liver Cancer (J. Hepatology. 2016 Apr, Abou-Alfa et.al.)

- ▶ 180 patients: 60 took a placebo, 120 took the drug
- ▶ PFS: Progression Free Survival

Sub-population	Drug Identifier	F1	F2	F3	Size (number of patients)	Mean PFS (months)	Median PFS (months)
1.	0	0	0	0	33.37	3.06	1.65
2.	0	0	1	0	4.07	2.29	2.24
3.	0	1	0	0	17.84	2.72	1.81
4.	0	1	1	0	4.72	7.05	7.18
5.	1	0	0	0	51.52	3.22	2.55
6.	1	0	0	1	16.77	4.17	3.65
7.	1	0	1	0	8.38	1.74	1.33
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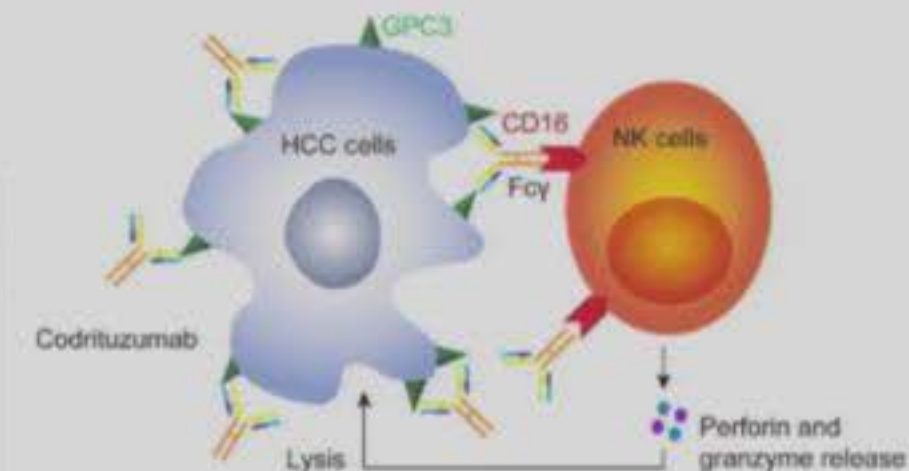


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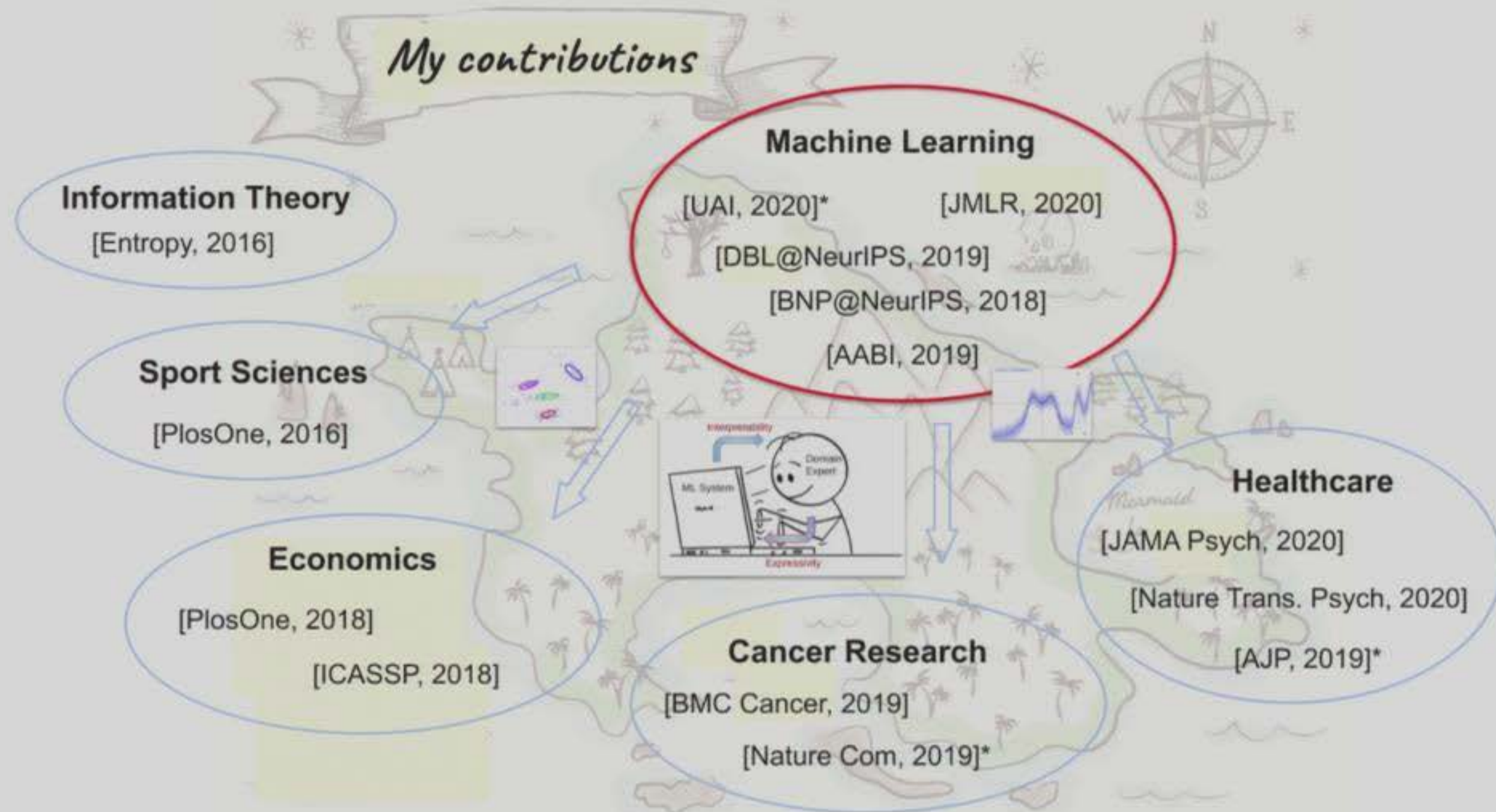


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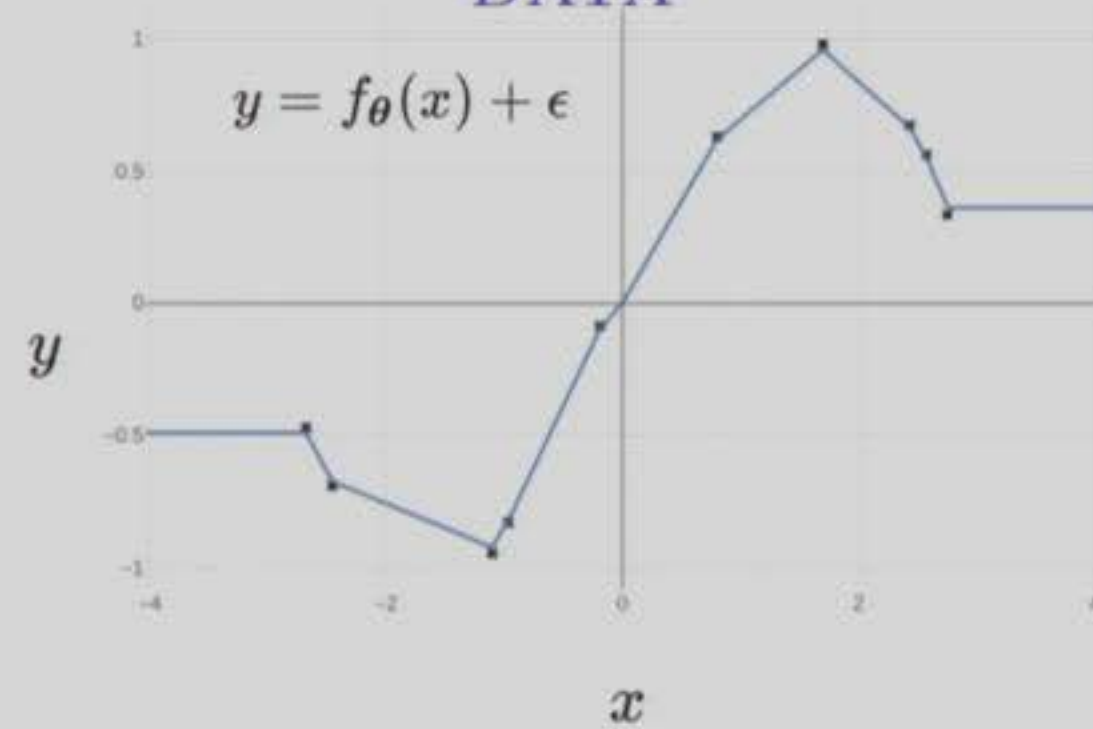
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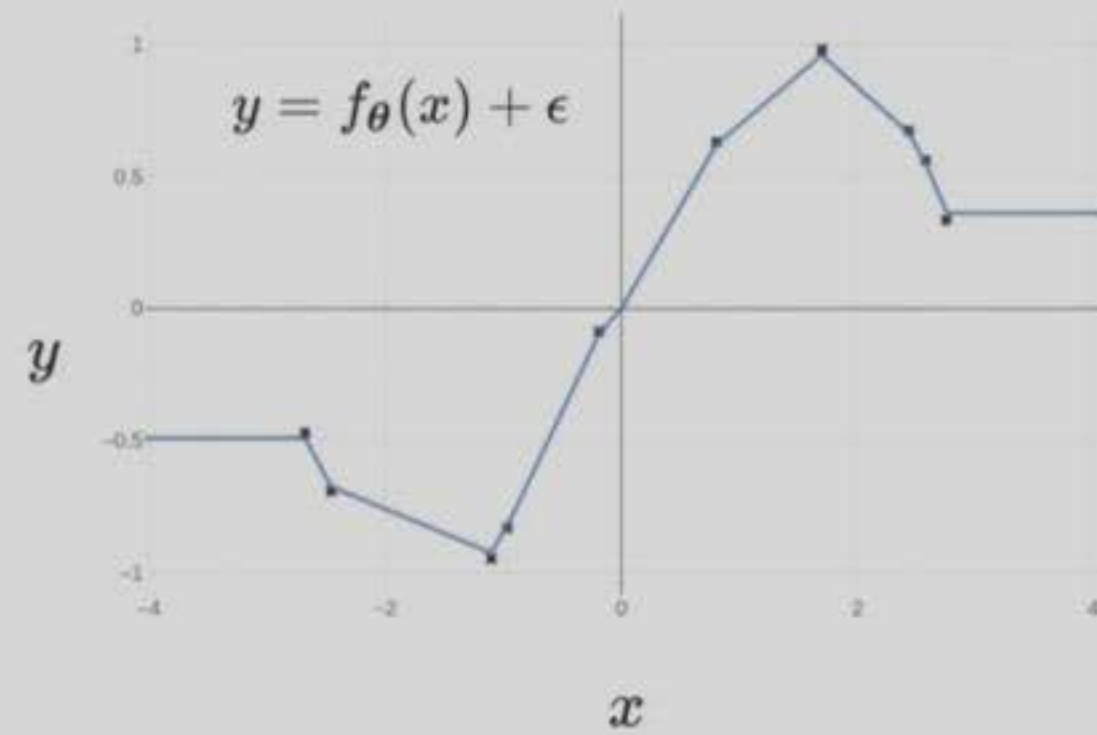
NEURAL NETWORKS (NNs) CAN FIT COMPLICATED TRENDS IN DATA



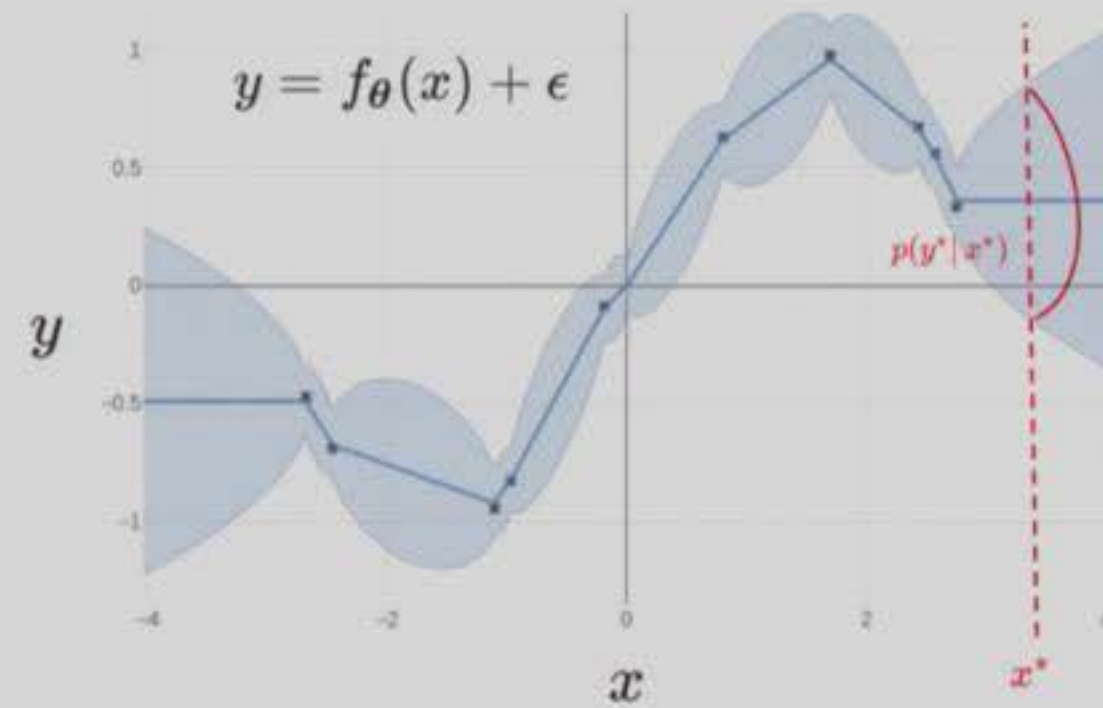
Several success stories...



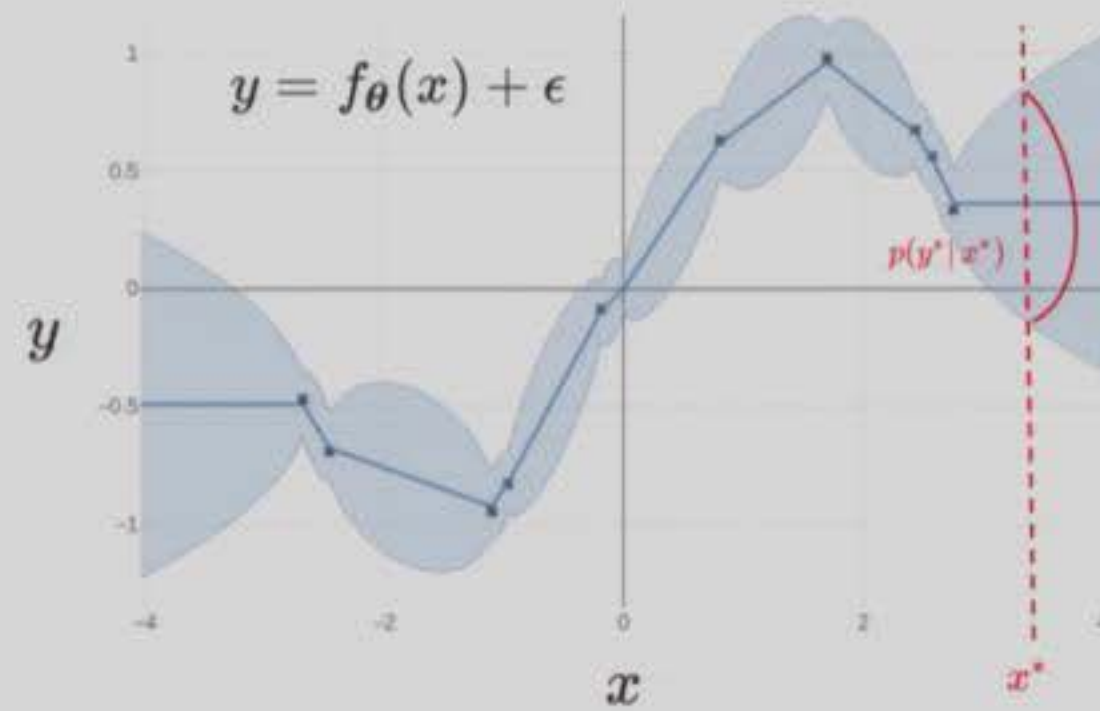
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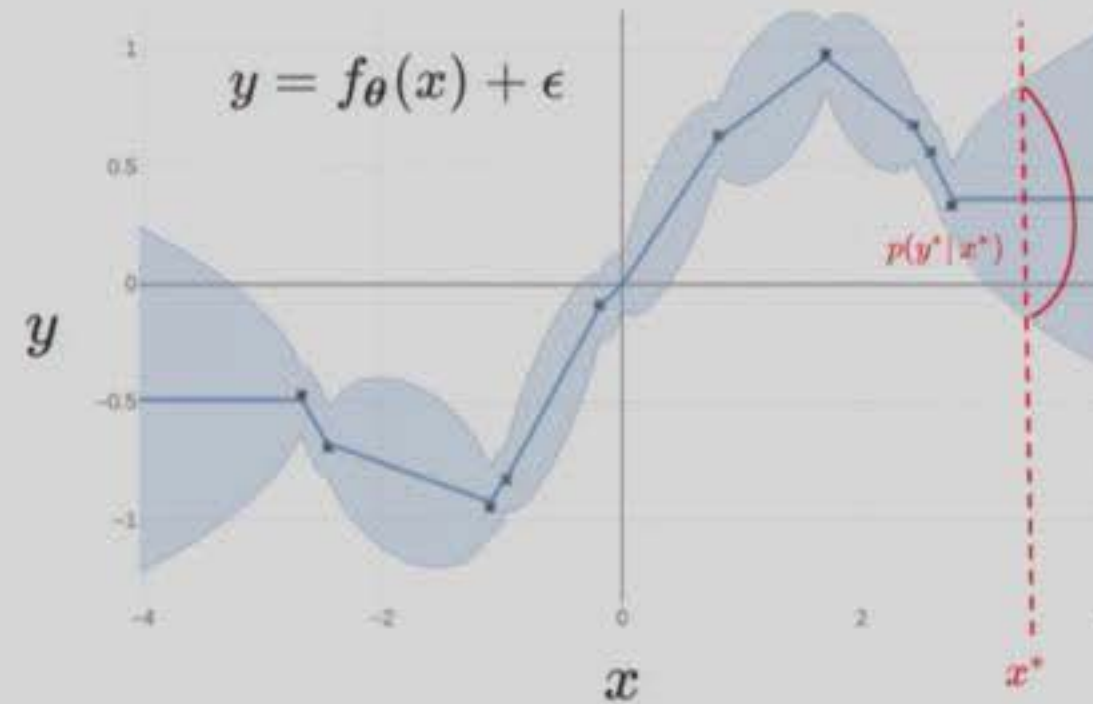
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Uncertainty estimation becomes crucial!



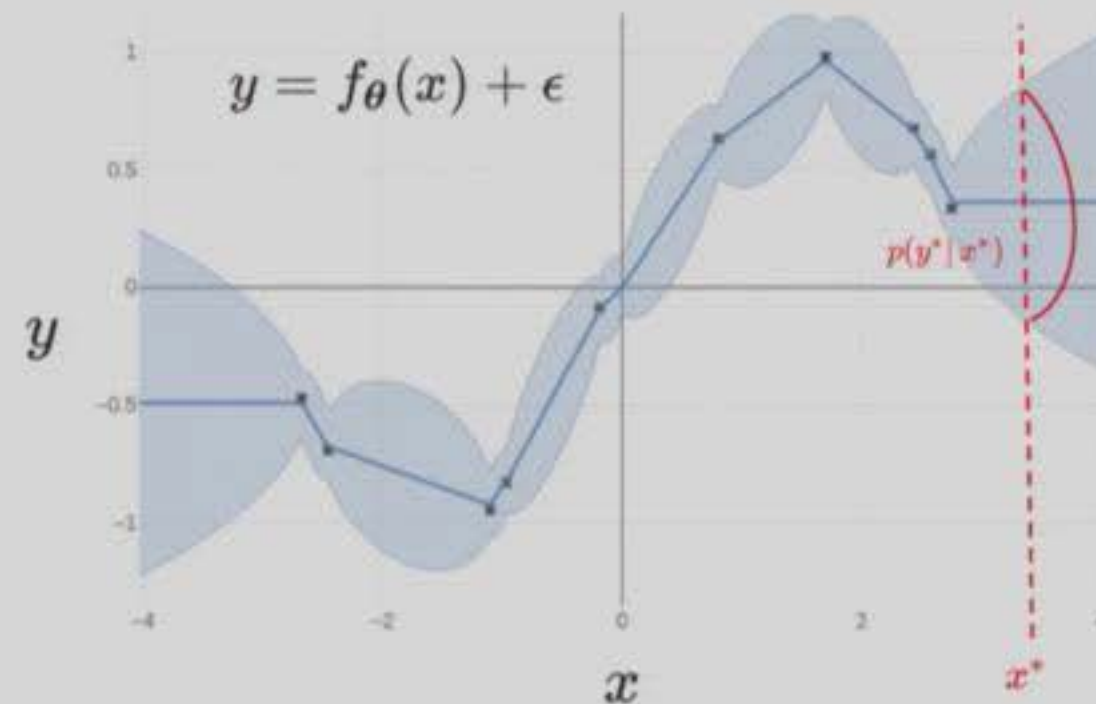
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Some examples of assumptions:

- ▶ Range of heart rate at rest between 60-100 bpm.
- ▶ Slow/fast variation of air pollutant; volatility of stock market

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Key point: All of these translate into restrictions on learned functions.

Question: How can we incorporate such desiderata into the model?

AN EASY WAY TO SPECIFY FUNCTIONAL DESIDERATA: GAUSSIAN PROCESSES (GPs)

Definition: a Gaussian process is a collection of random variables, any finite number of which have (consistent) Gaussian distributions.

$$f \sim \text{GP}(\mu(\cdot), k(\cdot, \cdot))$$

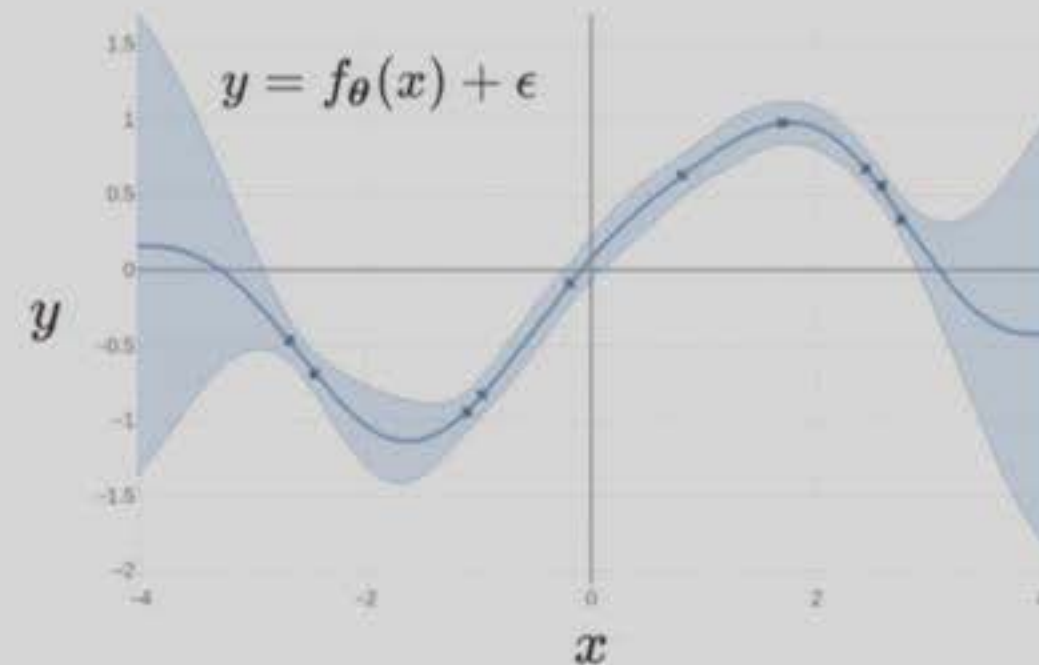
Example: RBF kernel as covariance function:

$$k(x, x') = \sigma^2 \exp\left(-\frac{(x - x')^2}{2\gamma^2}\right)$$

► Stationarity

► Lengthscale

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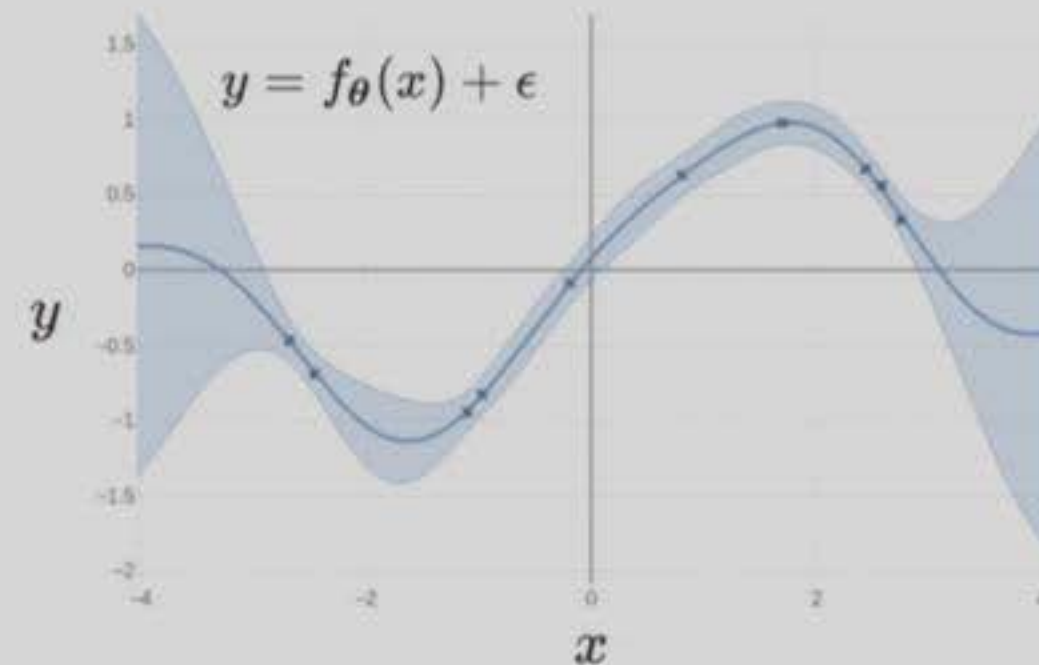
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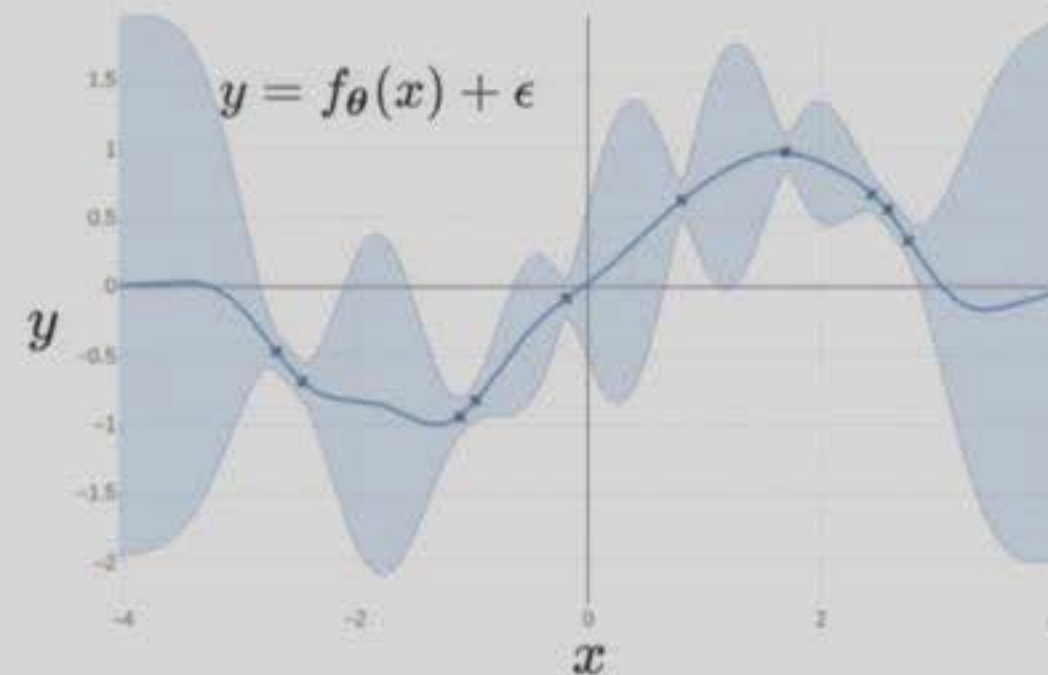
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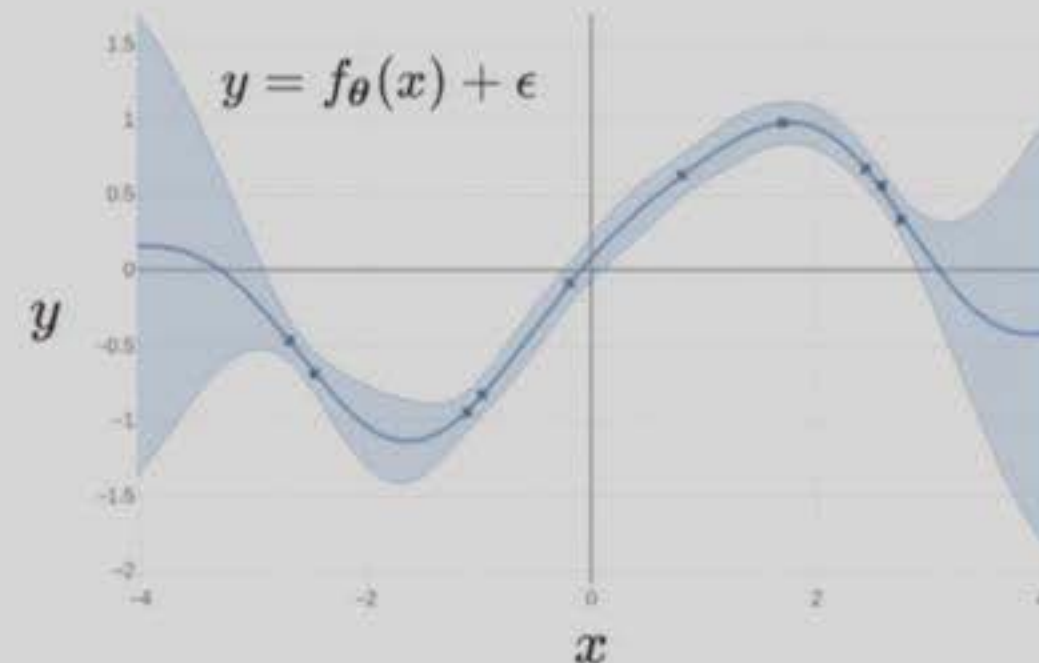
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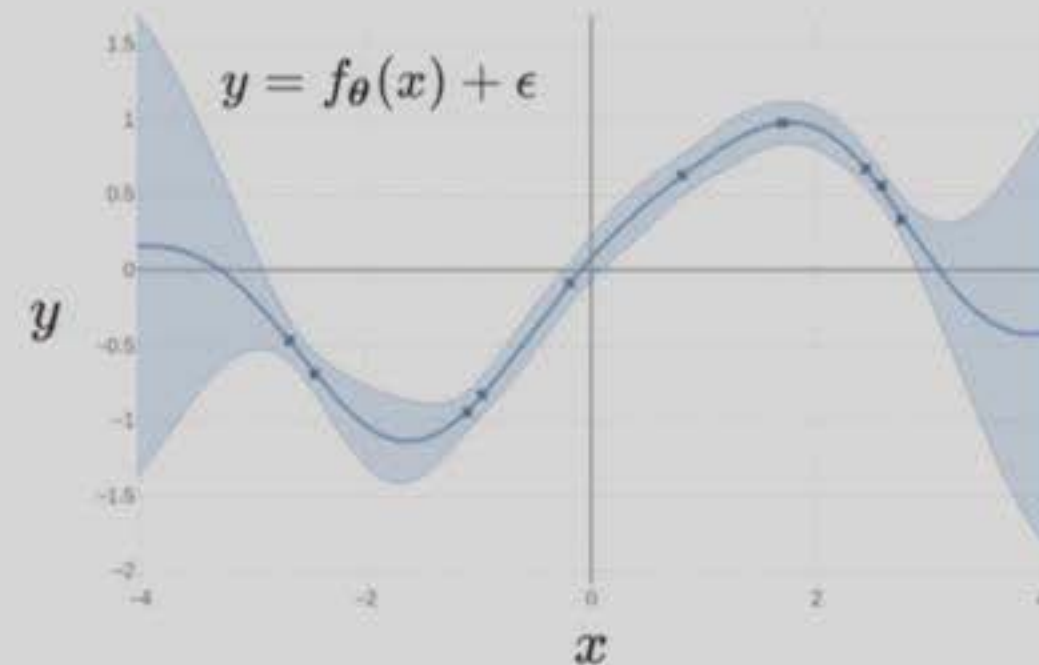
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GPs ARE GREAT, BUT WHAT IF I STILL WANT A NN?

Benefits of NN approaches:

- ▶ widely used (many tools available)
- ▶ parametric expression
- ▶ fast at train and evaluation time

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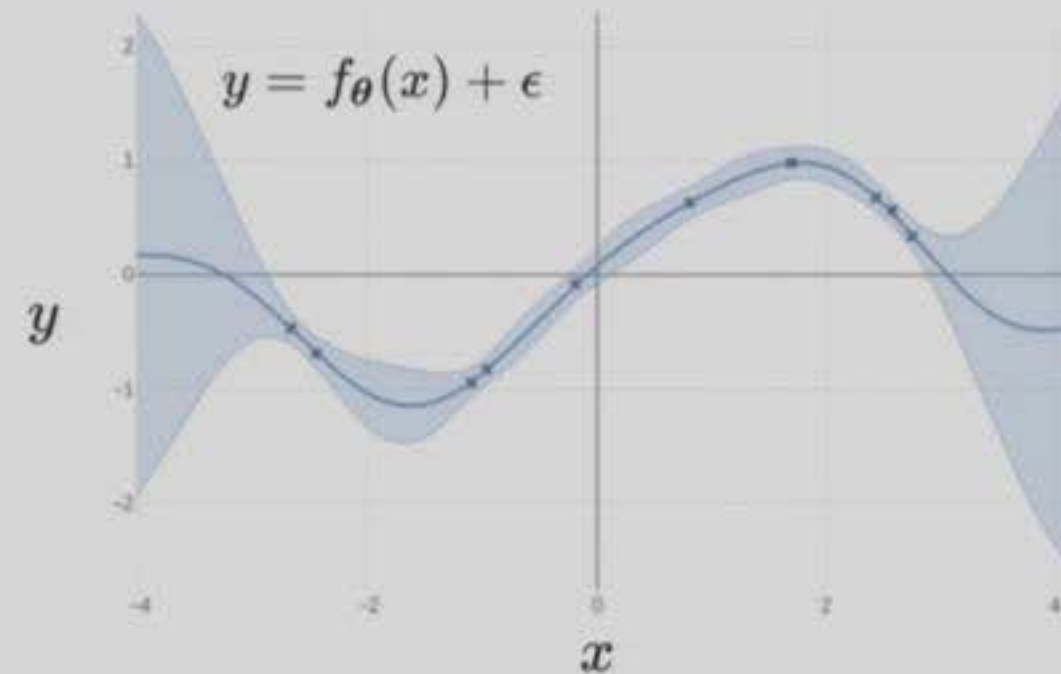
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KEY RESEARCH QUESTIONS:

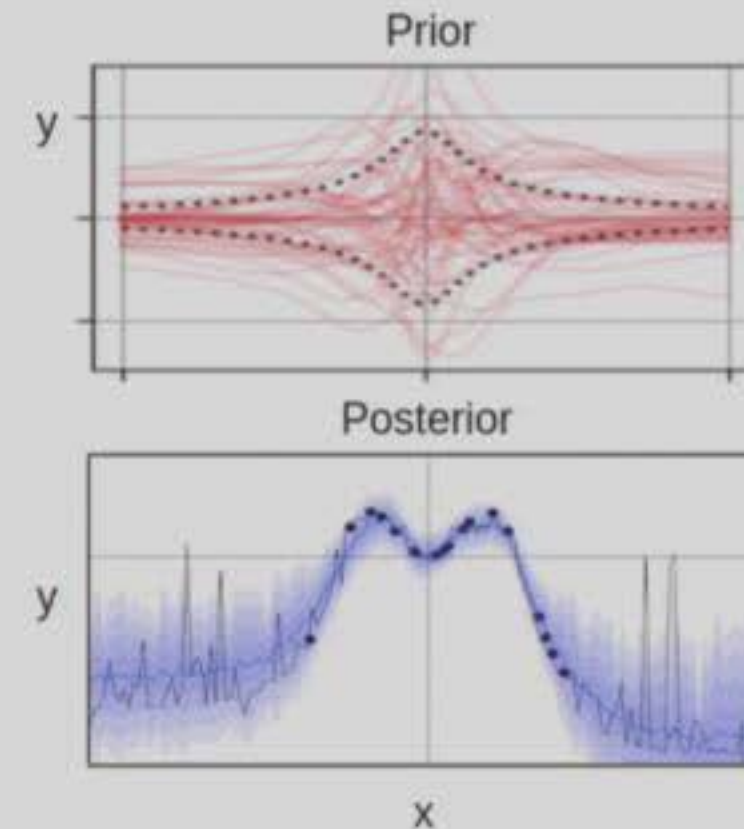
1. Can we design Bayesian NN priors that encode **stationarity properties** like a GP while retaining the benefits of neural networks?
2. Can we easily specify lengthscale and amplitude variance in a **decoupled** fashion?

BACKGROUND: BAYESIAN NEURAL NETWORKS

- ▶ Assume prior on network parameters
- ▶ Most common, i.i.d Gaussians

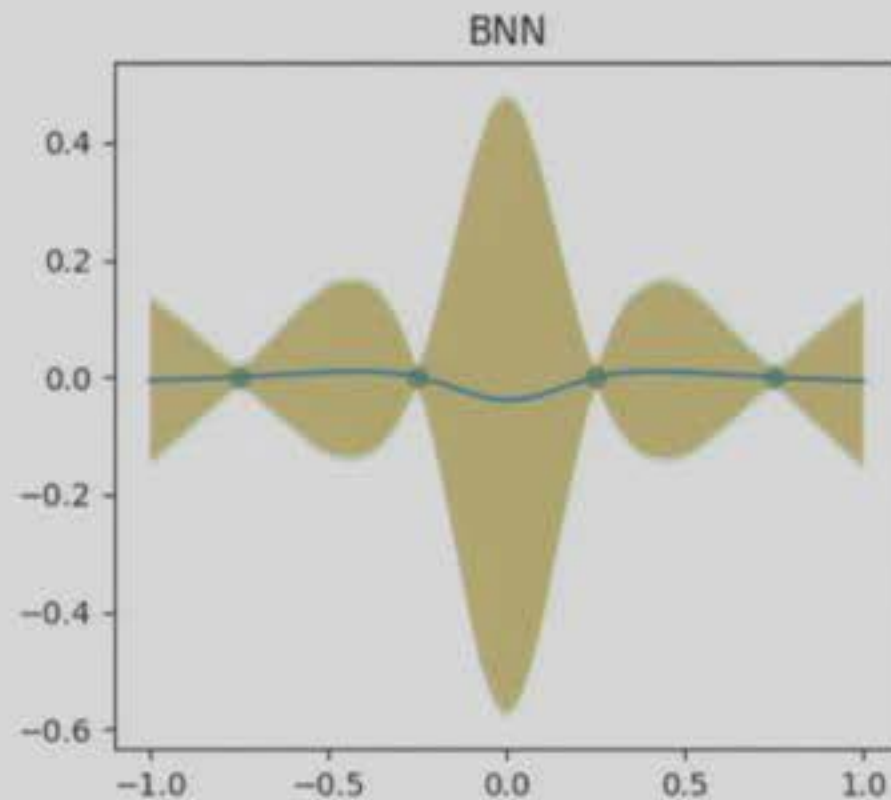
$$\begin{aligned} \mathbf{y} &= f_{\boldsymbol{\theta}}(\mathbf{x}) + \boldsymbol{\epsilon} \\ \boldsymbol{\epsilon} &\sim \mathcal{N}(0, \sigma_y^2 I) \\ \theta_i &\sim \mathcal{N}(0, \sigma_{\theta}^2 I) \quad \forall i \end{aligned}$$

- ▶ $p(\boldsymbol{\theta}) \implies p(f)$



NOT ONLY HARD TO ENCODE FUNCTIONAL PROPERTIES WITH BNNs; SOME PROPERTIES ARE IMPOSSIBLE TO GET

- For example, a standard BNN (with RBF activations) is nonstationary in amplitude variance (Williams, 1997)

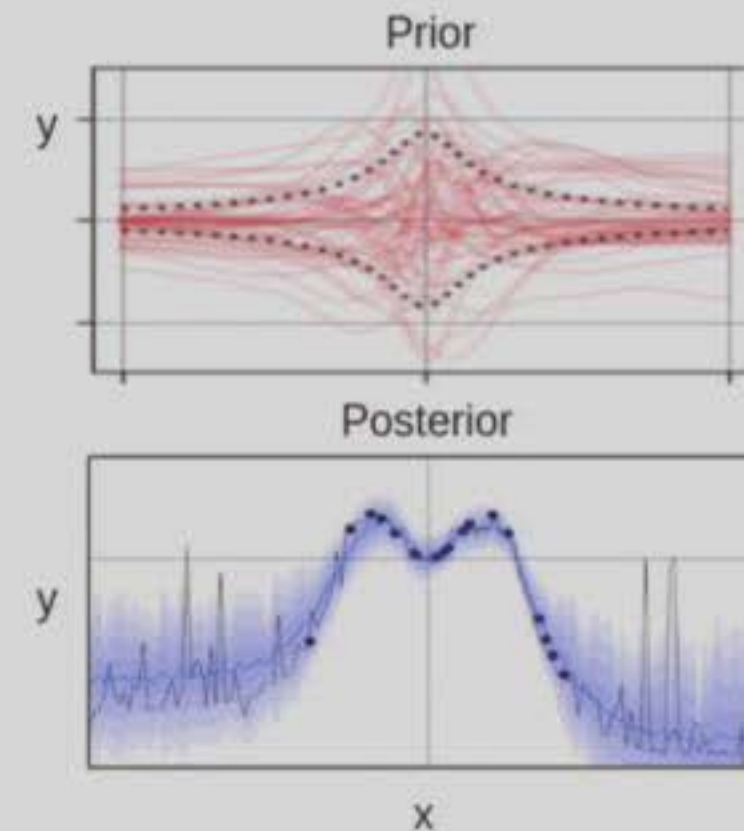


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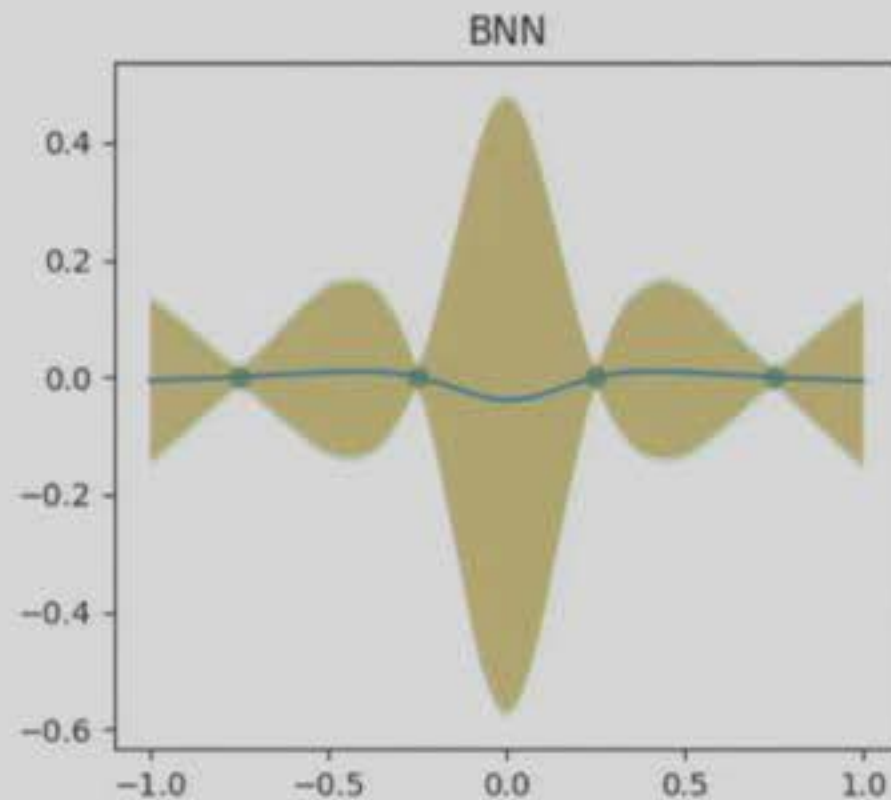
- ▶ $p(\boldsymbol{\theta}) \implies p(f)$



- ▶ But what does a prior over weights mean in function space?
 - ▶ Hard to know!

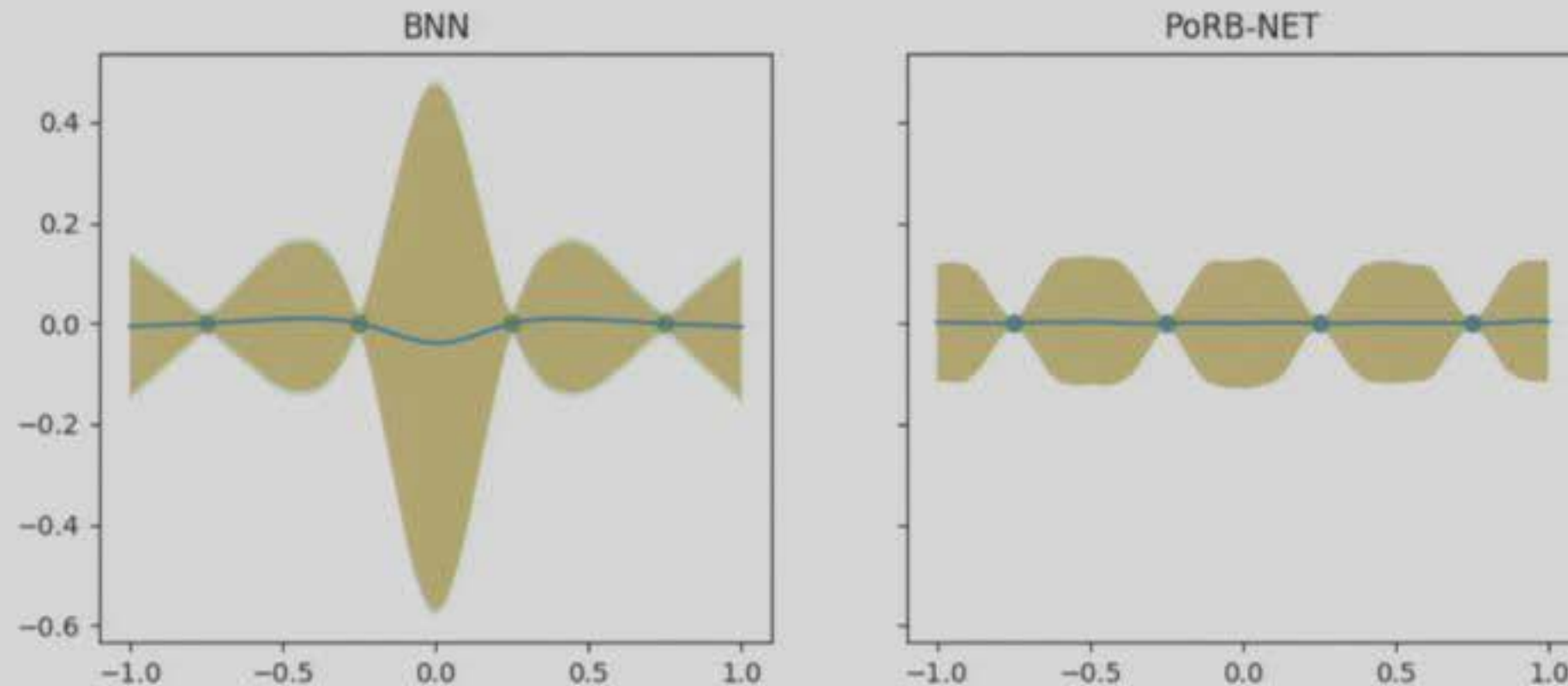
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Question: can we design a Bayesian NN that exhibits stationarity? **Yes!**

RADIAL BASIS FUNCTION NETWORKS (RBFNs)

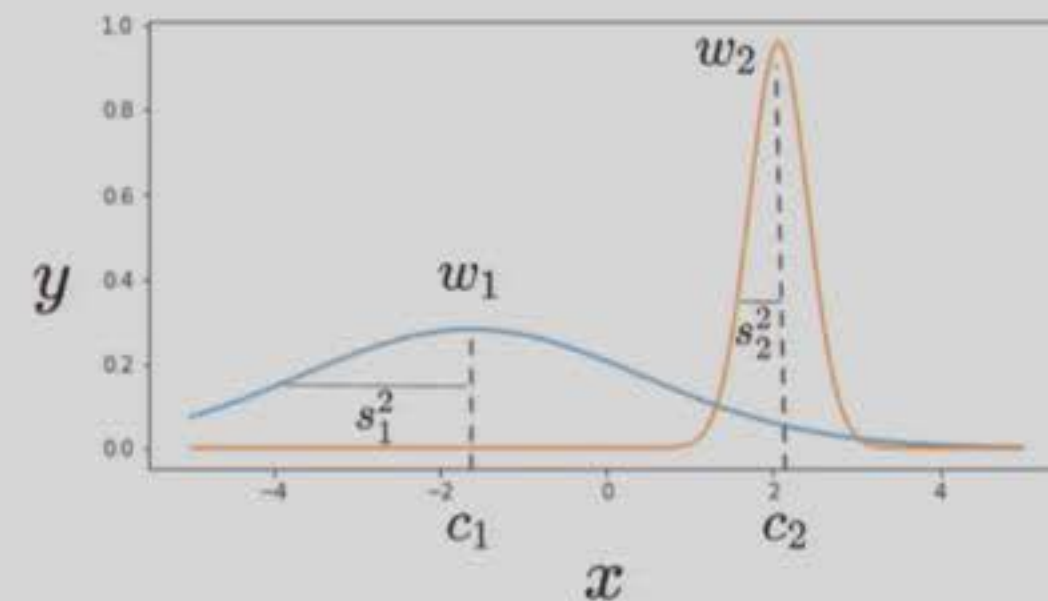
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- ▶ NN based on radial basis $\phi(\cdot)$, e.g., $\phi(x) = \exp(-x^2)$

$$f_{\theta}(x) = b + \sum_{k=1}^K w_k \phi(s_k(x - c_k)),$$

- ▶ s_k : scale
- ▶ c_k : center
- ▶ w_k : output weight
- ▶ b : output bias

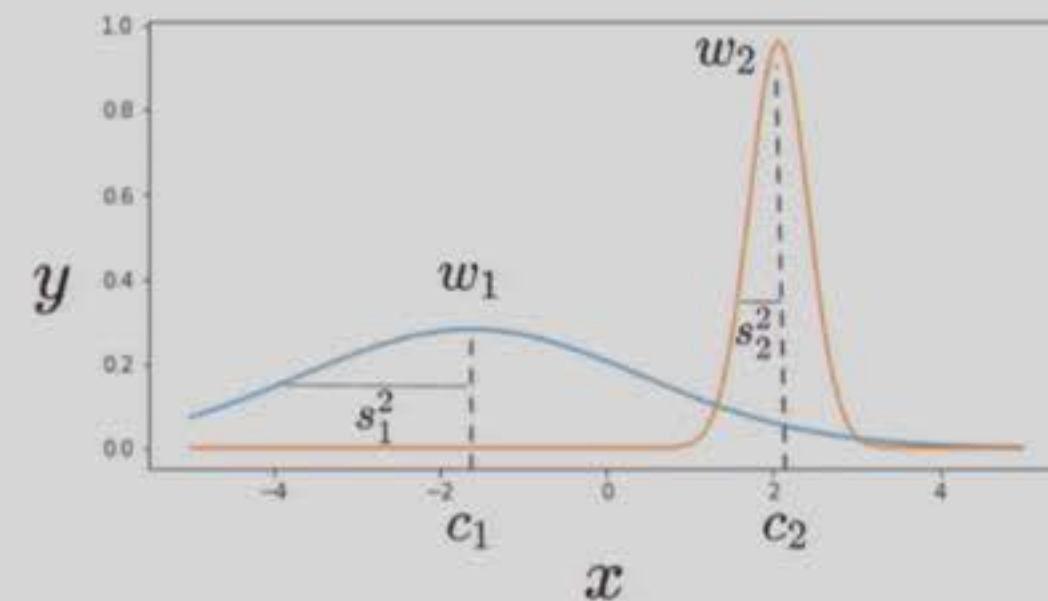


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- ▶ Equivalence to standard BNN exists, but not interpretable!

OUR CONTRIBUTION: POISSON PROCESS RADIAL BASIS FUNCTION NETWORK (PoRB-NET) [UAI, 2020]*

1. We propose an expressive prior for NNs
2. We show desirable properties:
 - 2.1 Stationarity
 - 2.2 Decoupling of lengthscale and amplitude variance
 - 2.3 Consistency
3. We demonstrate successful behavior empirically

(*) submitted

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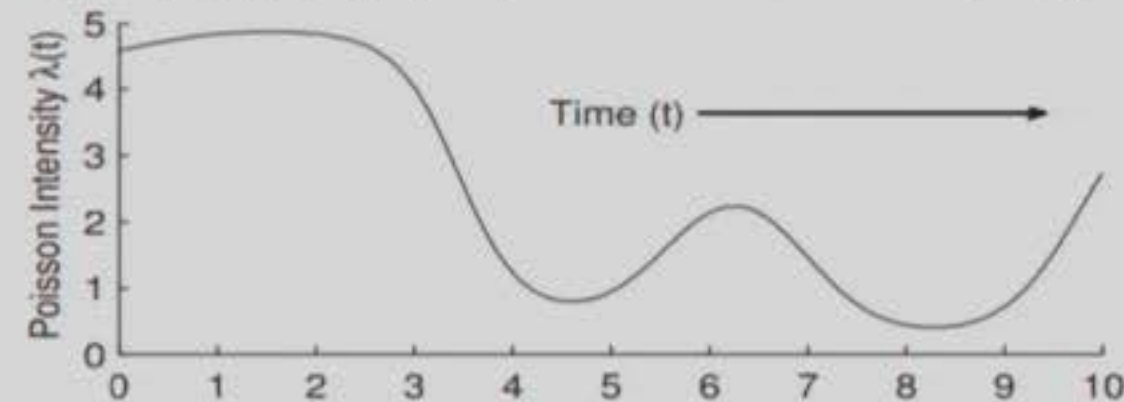
- **Key point:** Expressive prior over centers $\mathbf{c}$ and scales $\{s_k\}$.

- Poisson process priors over $\mathbf{c}$.

$$\mathbf{c} | \lambda \sim \text{Poisson Process}(\lambda)$$

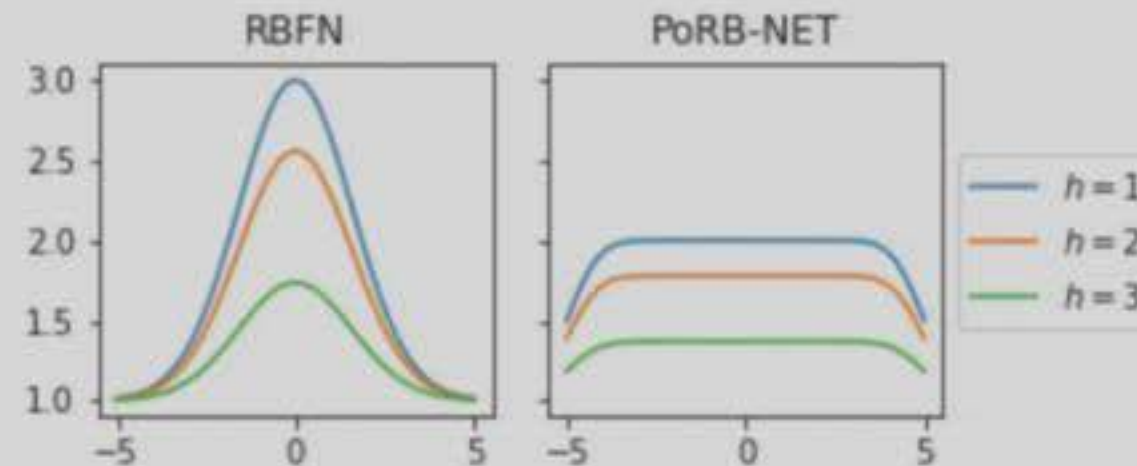
- Deterministic relationship between $\mathbf{c}$ and $\{s_k\}$.

$$s_k^2 = \lambda^2(c_k)$$



PROPERTIES OF PoRB-NET

1. Stationarity



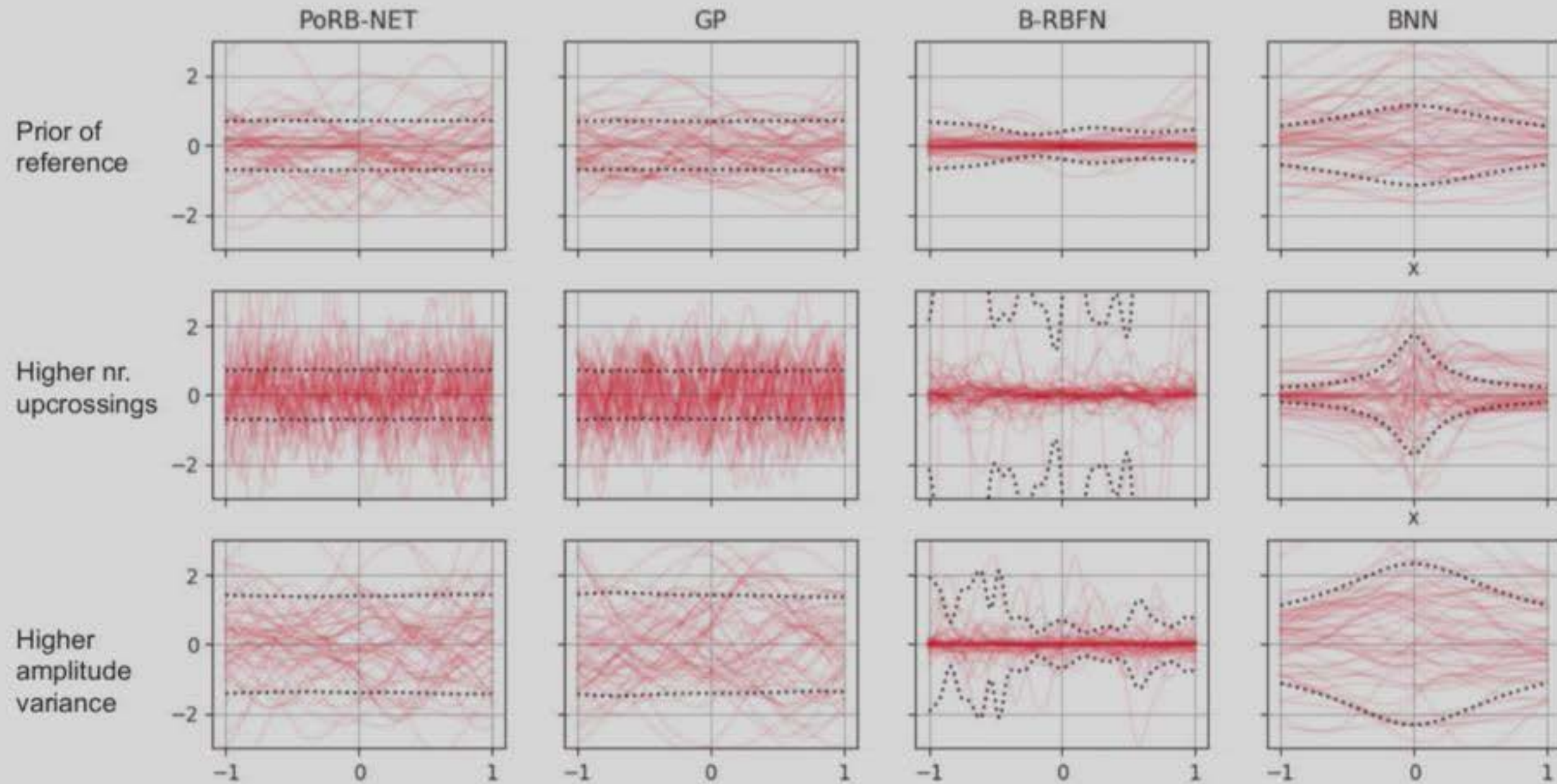
2. Decoupled lengthscale and amplitude variance

$$\text{Cov}(f(x_1), f(x_2)) = \sigma_b^2 + \tilde{\sigma}_w^2 \exp \left\{ -\lambda^2 \left(\frac{x_1 - x_2}{2} \right)^2 \right\}$$

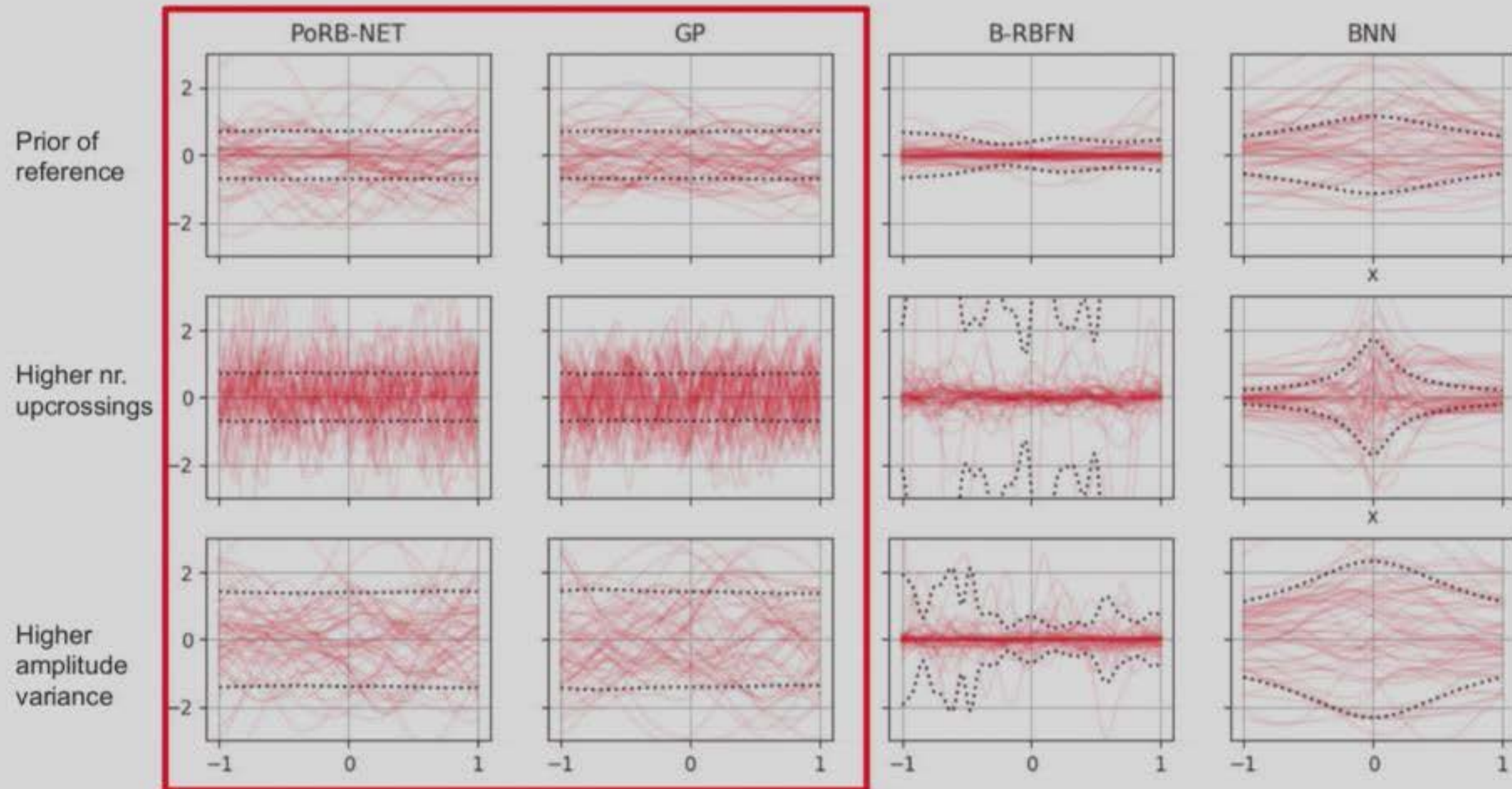
3. Consistency Theorem:

A PoRB-NET with uniform intensity function is Hellinger consistent as the number of observations goes to infinity.

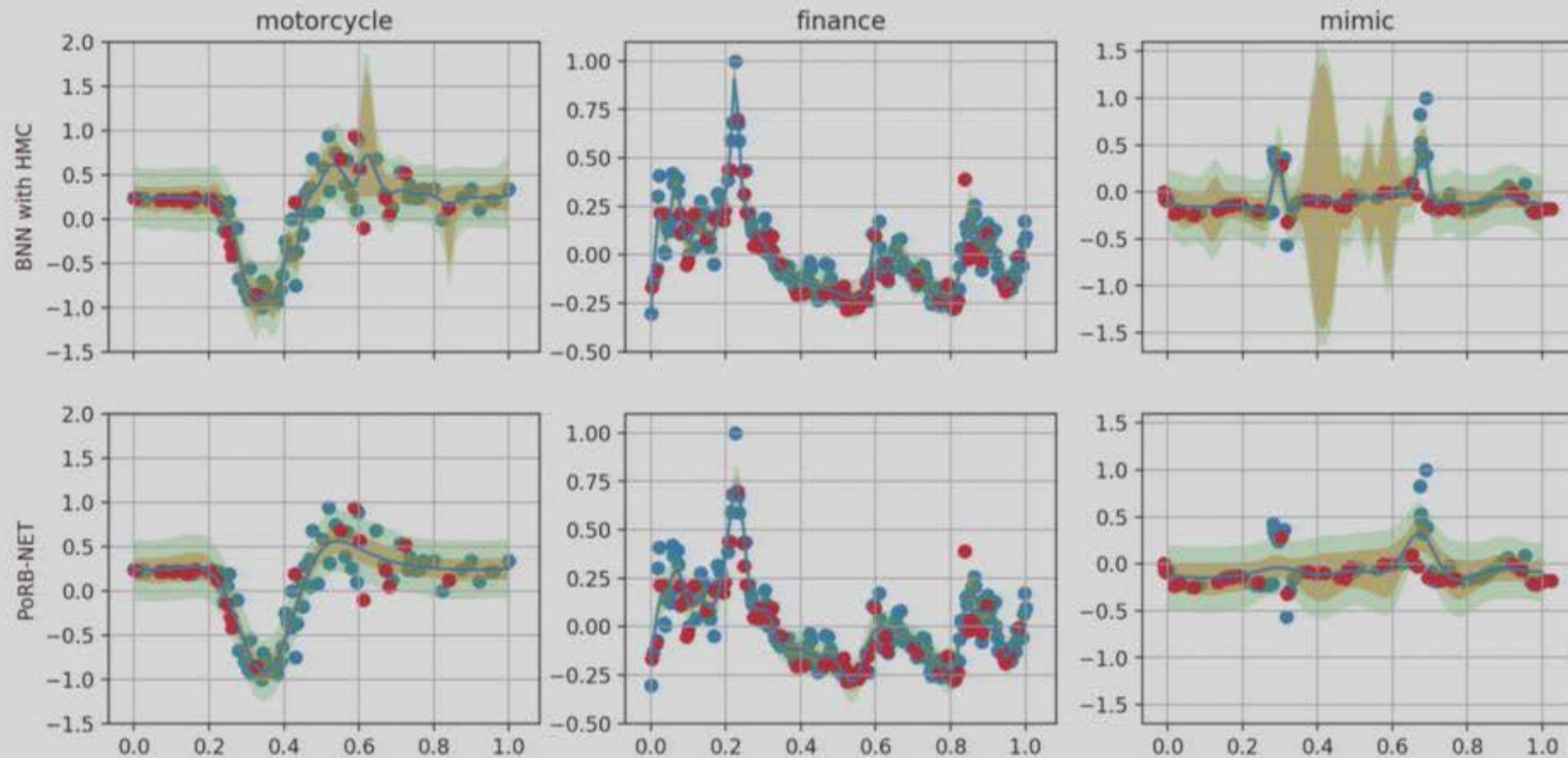
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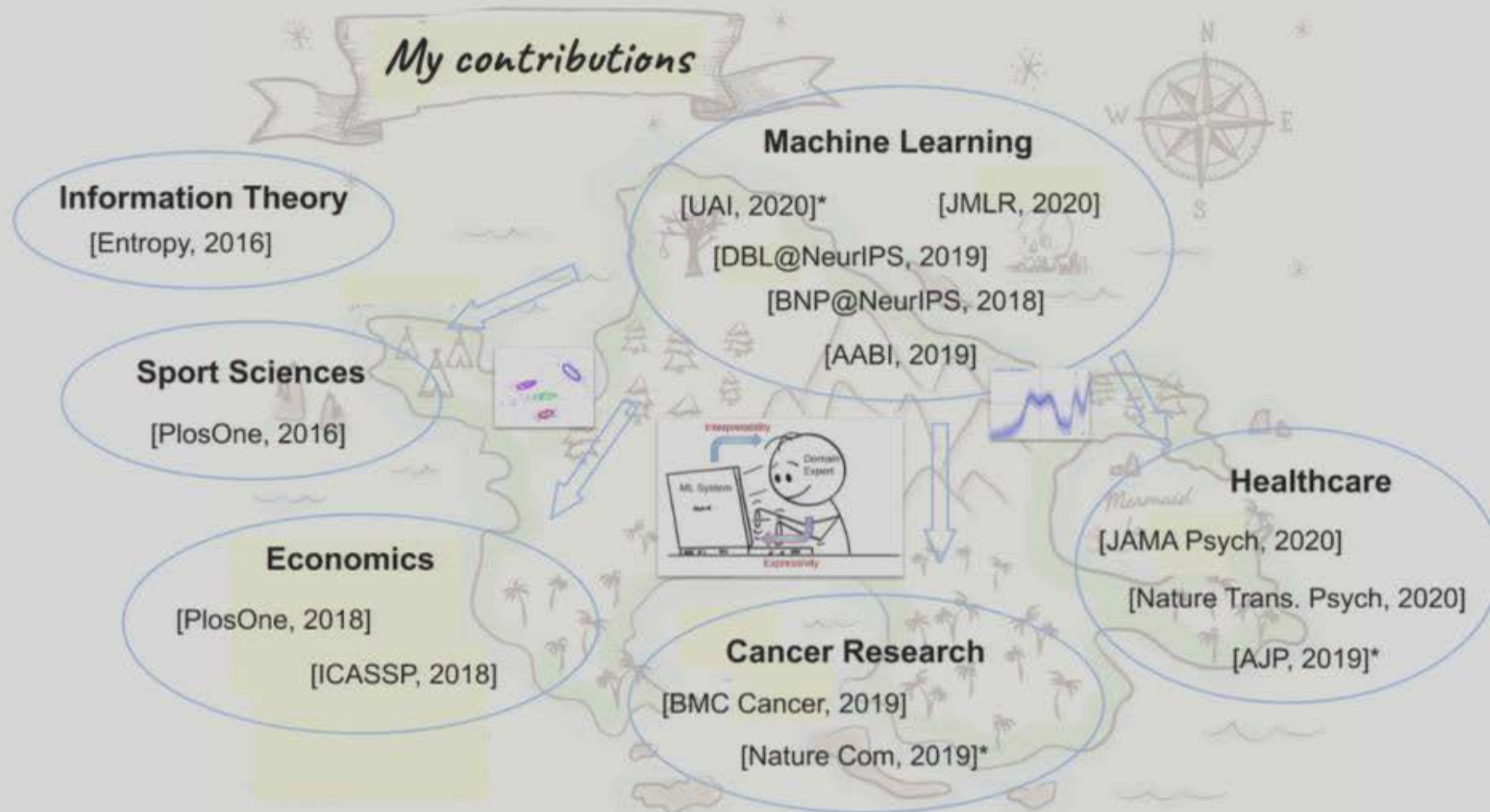


PORB-NET ALLOWS FOR EASY SPECIFICATION OF LENGTHSCALE AND SIGNAL VARIANCE LIKE A GP



PORB-NET IS ABLE TO CAPTURE NON-STATIONARY PATTERNS IN REAL SCENARIOS, ADAPTING THE LENGTHSCALE LOCALLY





From the lab to the clinic

- Ongoing user study at MGH, Boston
 - Impact of explanations
 - Usefulness, trust...

Why are these therapies being recommended?

The following **patient features** had the highest contributions to system 13's predictions:



Which antidepressant medication would you be most likely to prescribe in this situation?



Patient Details:

Jessica is a 37 year old woman who is married and works part time. She presents with 9 months of depressed mood and lack of appetite. She has a seizure disorder, and current medications include Omeprazole and Celecoxib. Prior treatment with Citalopram had no effect on depressed mood.

System 15 Recommendation: FLUOXETINE

Top 5 therapies with highest probability for stability:

Therapy	Predicted Stability*	Predicted Dropout Risk**
Fluoxetine	.76	.05
Sertraline	.67	.05
Paroxetine	.64	.10
Venlafaxine	.60	.14
Vortioxetine	.55	.15

*Stability: continued use of the same medication for at least 3 months

**Dropout: early treatment discontinuation following prescription

Why are these therapies being recommended?

The following **rules** had the highest contributions to system 15's predictions:

1. If underweight or lack of appetite, favor weight gain, favor Mirtazapine
2. If underweight or lack of appetite, avoid appetite suppressants, avoid nausea-inducing, avoid SNRI's, avoid Sertraline
3. If lack of response to Paroxetine, avoid SSRI's

Current and future research agenda



Contact: melanie@seas.harvard.edu

<https://melaniefp.github.io/>

Impactful real-world problems:

- Personalize prescription of antidepressants
- Prognosticate outcomes for in-vitro fertilization
- ...

Useful machine learning methodology:

- How to better quantify model uncertainty?
- How to combine expert knowledge with data-driven evidence?
- How to learn task-meaningful representations?

ACKNOWLEDGEMENTS

Special thanks to:

- Beau Coker
- Finale Doshi-Velez
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- Oscar Puig
- Francesca Milletti
- Fernando Perez-Cruz

- Isabel Valera
- Maria Lomeli
- Zoubin Ghahramani



CRCS Center for Research on
Computation and Society

at Harvard John A. Paulson School of Engineering and Applied Sciences



HDSI | Harvard Data
Science Initiative



THANK YOU FOR LISTENING!

