2017.1.16-20 QIP2017 Seattle, USA

Threshold theorem for quantum supremacy

arXiv:1610.03632

Keisuke Fujii
Photon Science Center, The University of Tokyo
/PRESTO, JST

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arXiv:1610.03632 (ascendancy)

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東京大学 THE UNIVERSITY OF TOKYO
JST
さきがけ PRESTO
ERATO
MQM MACROSCOPIC QUANTUM MACHINES
CREST

Outline

- Motivations
- Hardness proof by postselection
- Threshold theorem for quantum supremacy
- Applications: 3D topological cluster computation & 2D surface code
- Summary

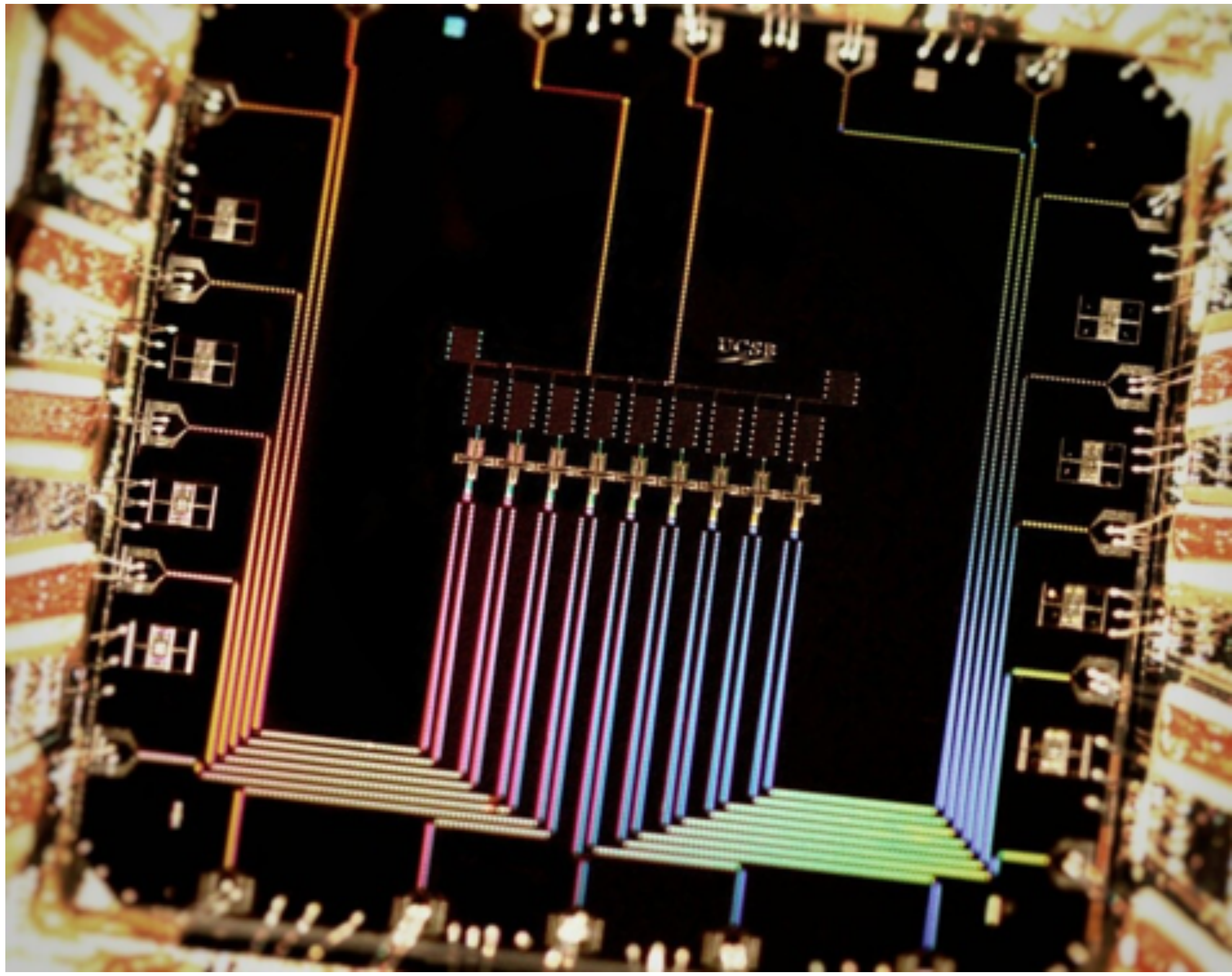
Quantum supremacy with near-term quantum devices

"QUANTUM COMPUTING AND THE ENTANGLEMENT FRONTIER" by J Preskill

The 25th Solvay Conference on Physics
19-22 October 2011; arXiv:1203.5813

How can we best achieve quantum supremacy with the *relatively small systems that may be experimentally accessible fairly soon*, systems with of order 100 qubits?

and Talk by S. Boixo et al



https://www.technologyreview.com/s/601668/google-reports-progress-on-a-shortcut-to-quantum-supremacy/

Intermediate models for non-universal quantum computation

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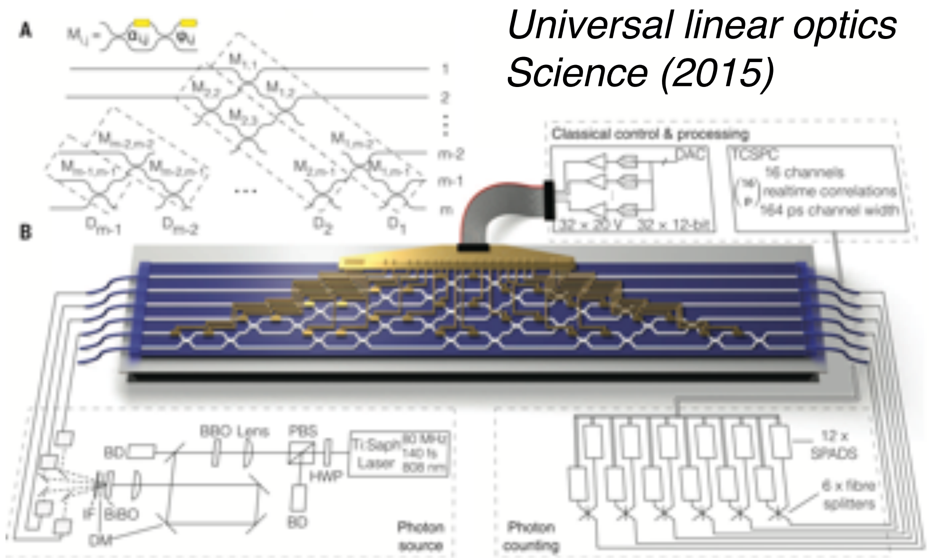
Boson Sampling

Aaronson-Arkhipov '13

Universal linear optics
Science (2015)



Linear optical quantum computation

Experimental demonstrations

J. B. Spring *et al*. Science **339**, 798 (2013)
M. A. Broome, Science **339**, 794 (2013)
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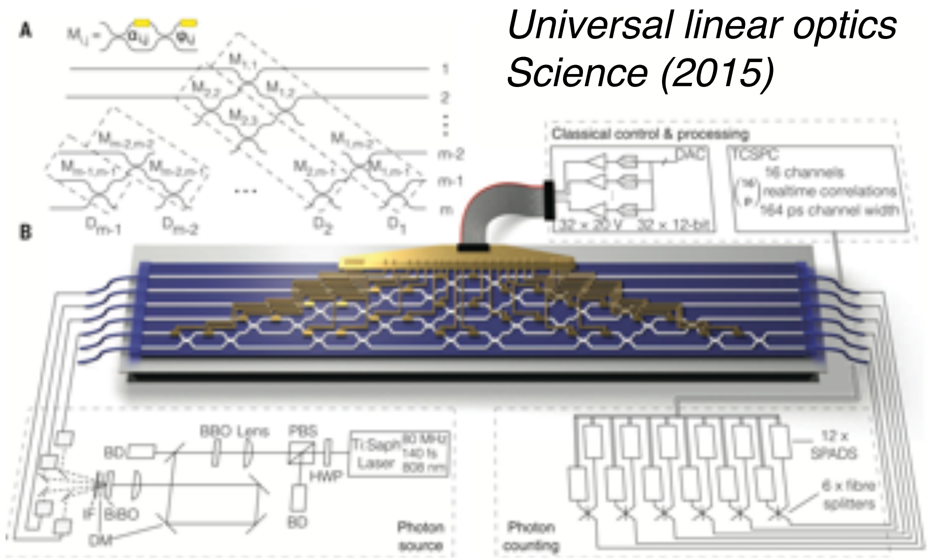

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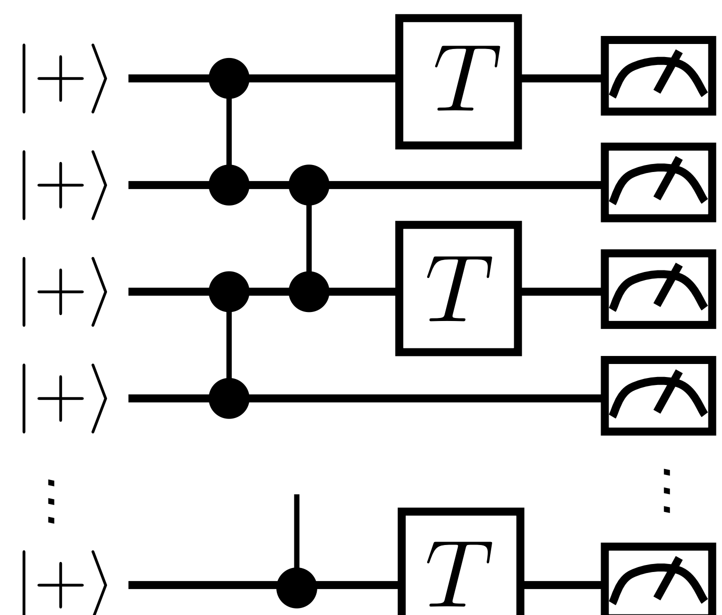
IQP (commuting circuits)

Bremner-Jozsa-Shepherd '11



Ising type interaction

KF-Morimae '13
Bremner-Montanaro-Shepherd '15
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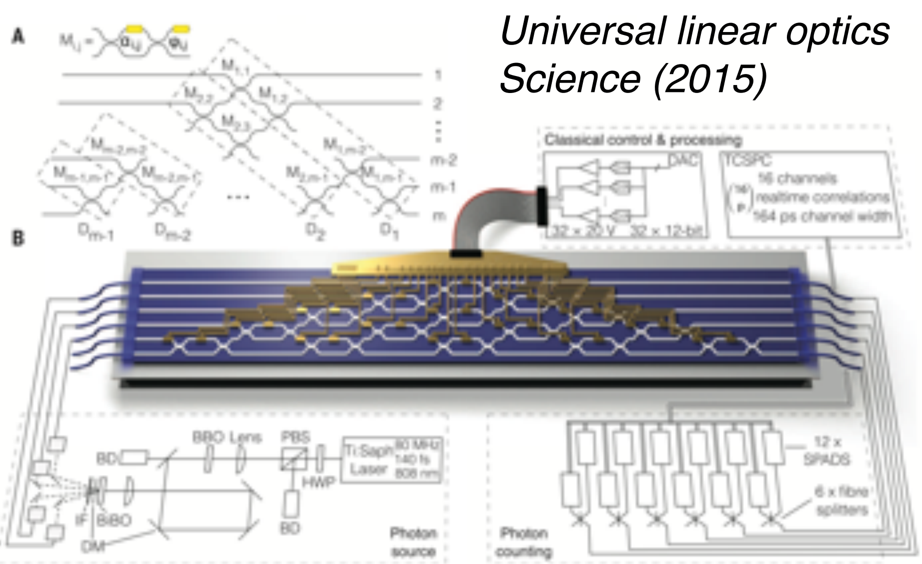

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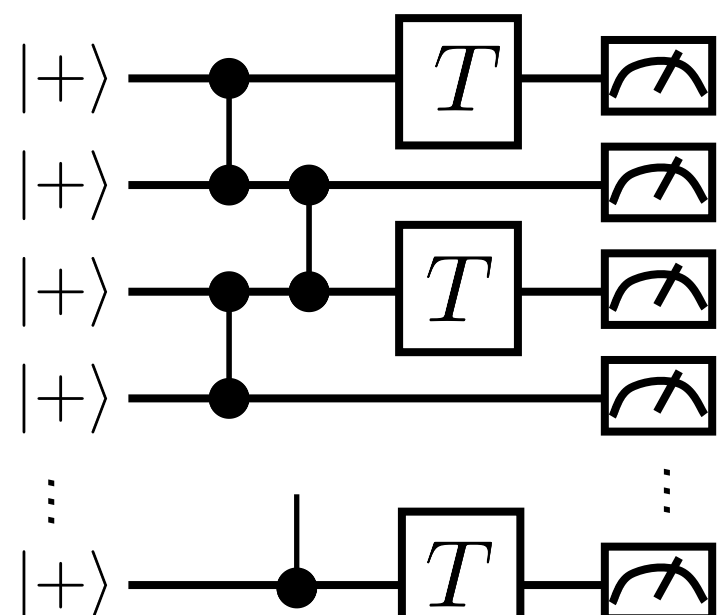

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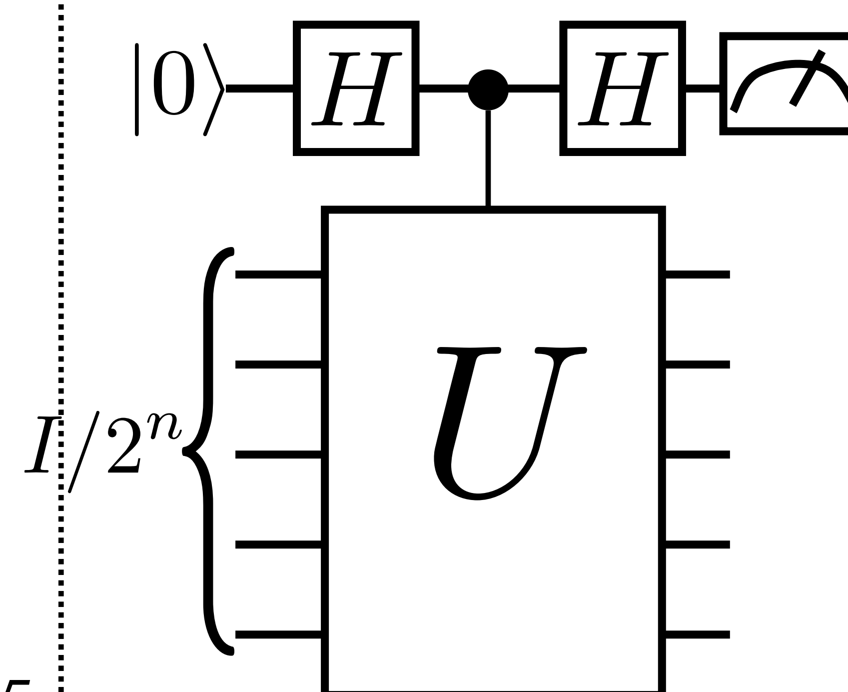
DQC1 (one-clean qubit model)

Knill-Laflamme '98
Morimae-KF-Fitzsimons '14
KF et al, '16



NMR spin ensemble

Intermediate models for non-universal quantum computation

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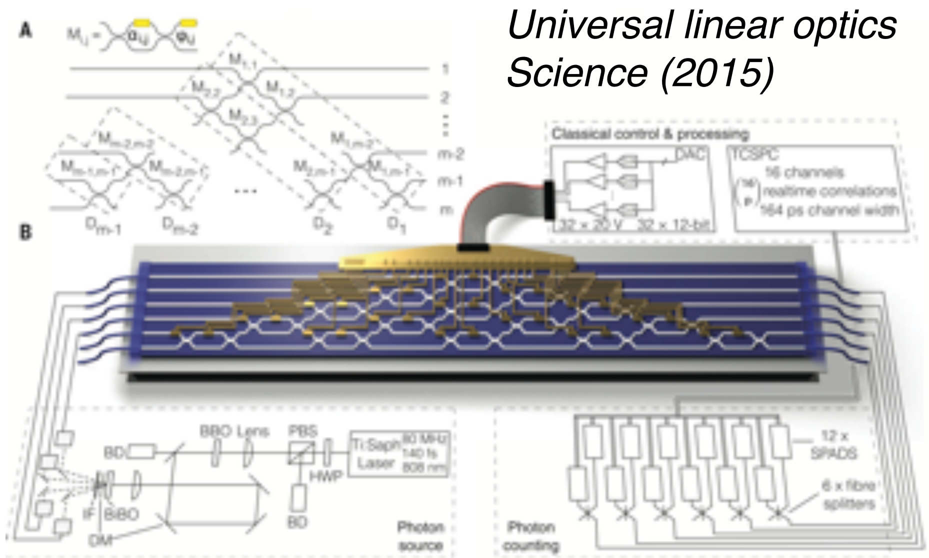

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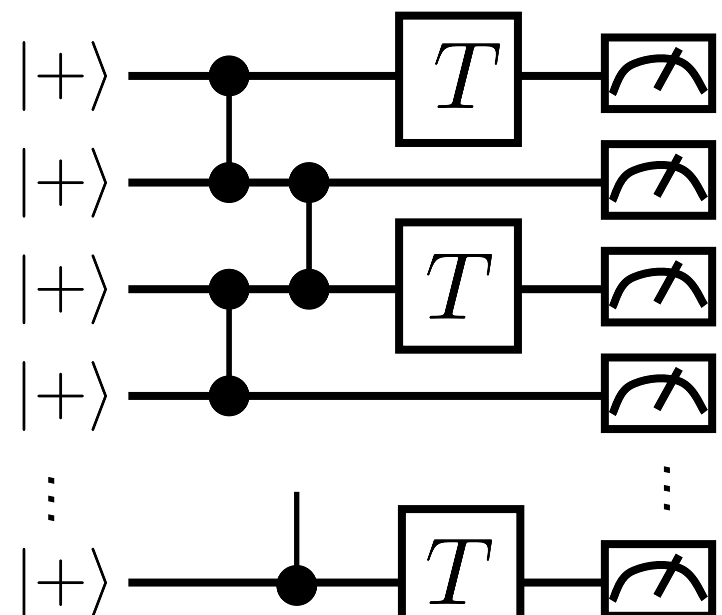

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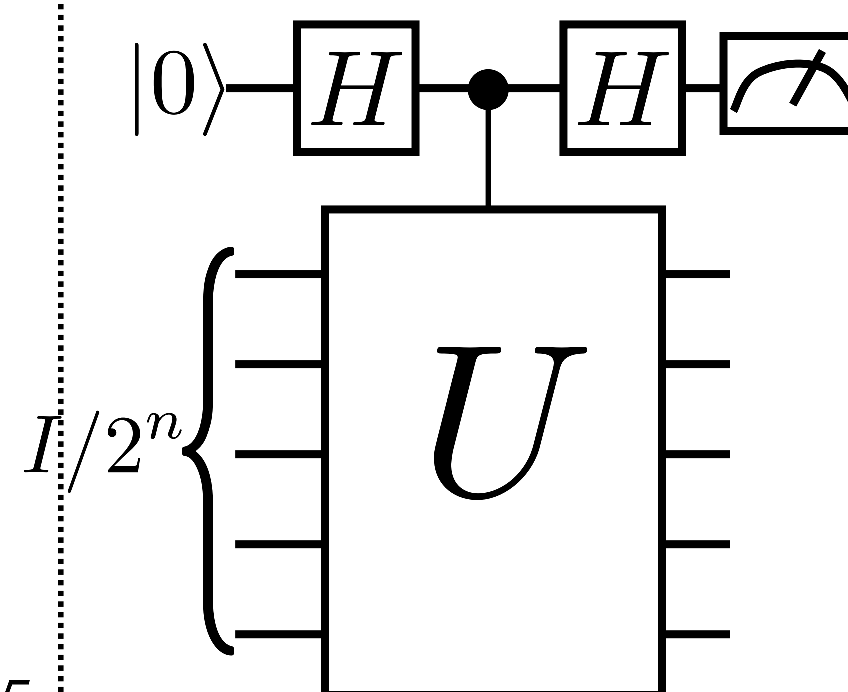


NMR spin ensemble

The purpose of this study:

→ universal but (very) noisy quantum circuits

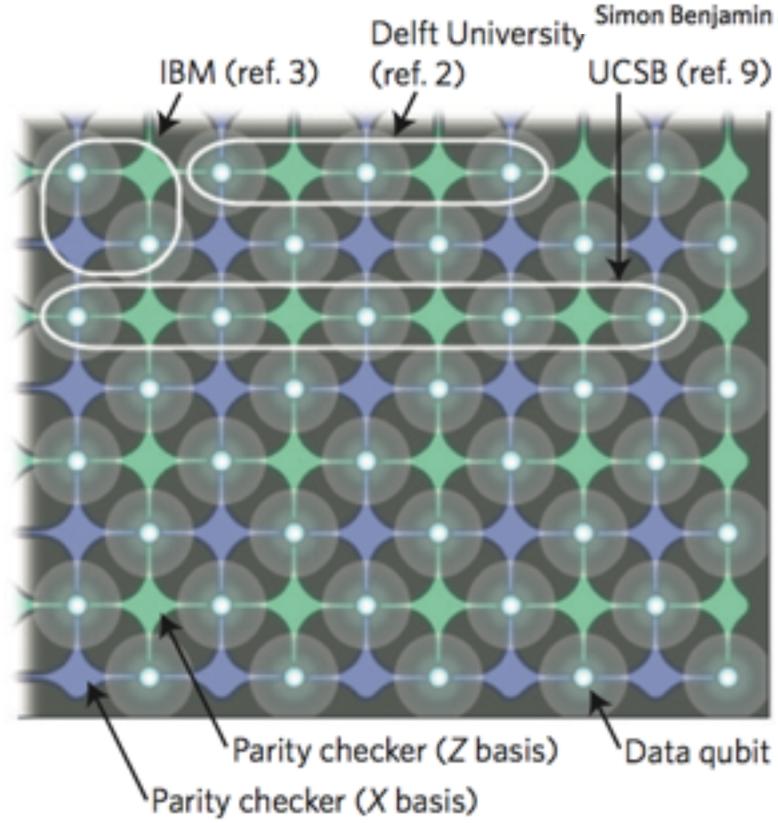
Noisy quantum circuits approaching fault-tolerance threshold

SUPERCONDUCTING QUBITS

Solving a wonderful problem

Superconducting qubits are used to demonstrate features of quantum fault tolerance, making an important step towards the realization of a practical quantum machine.

Simon Benjamin and Julian Kelly



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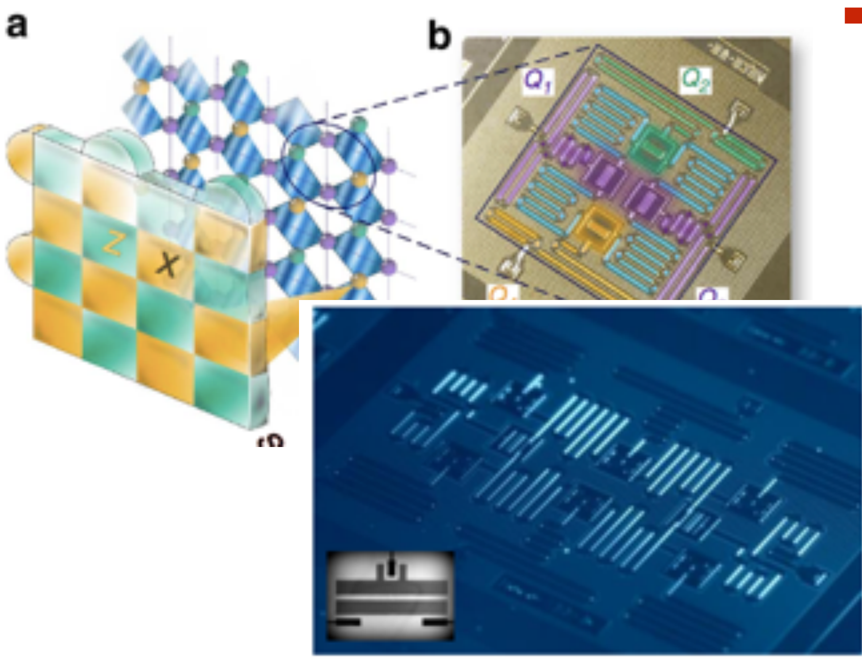

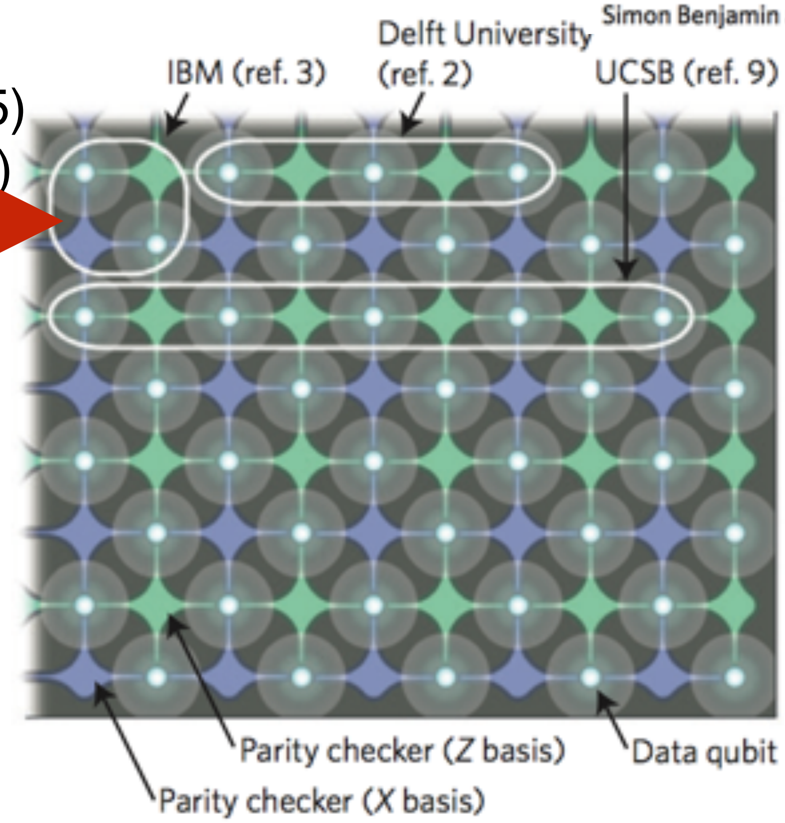

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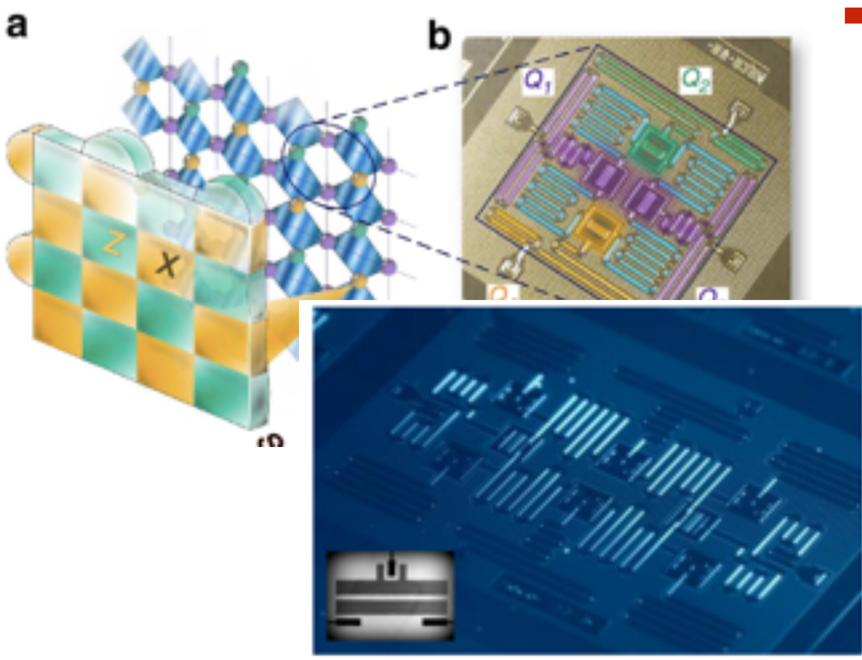

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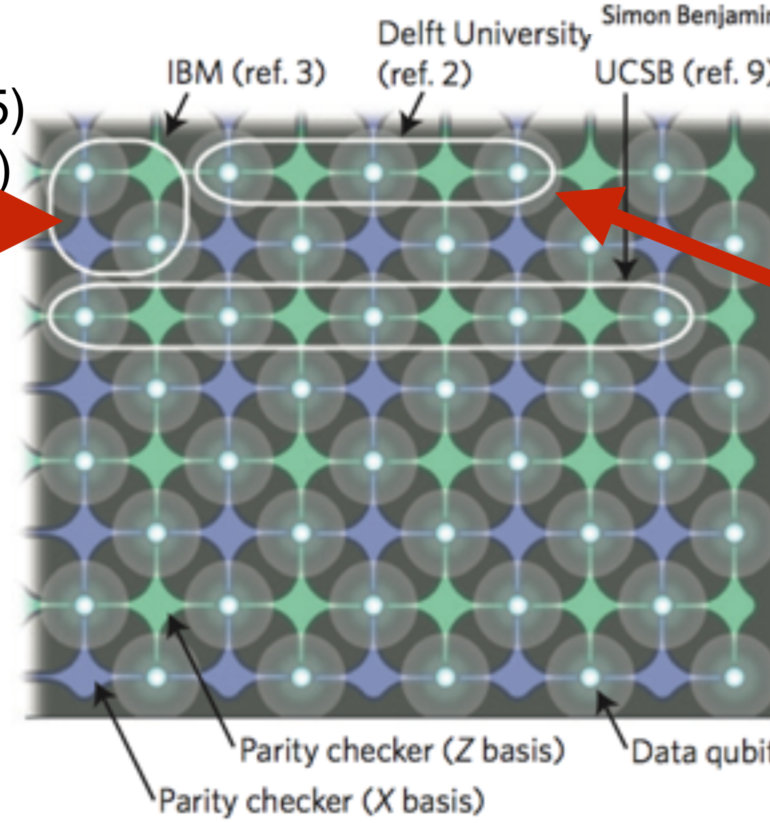

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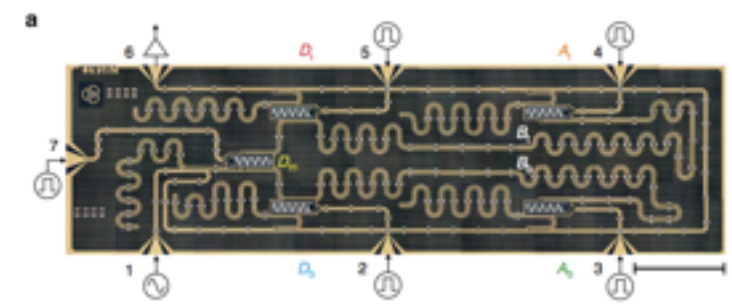
Riste et al., Nat. Comm. **6** 6983 (2015



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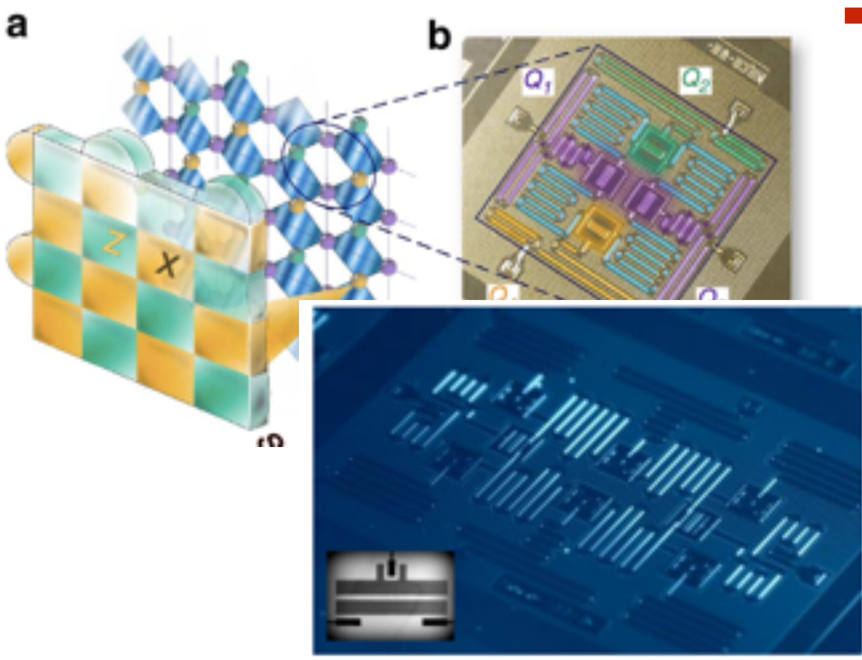

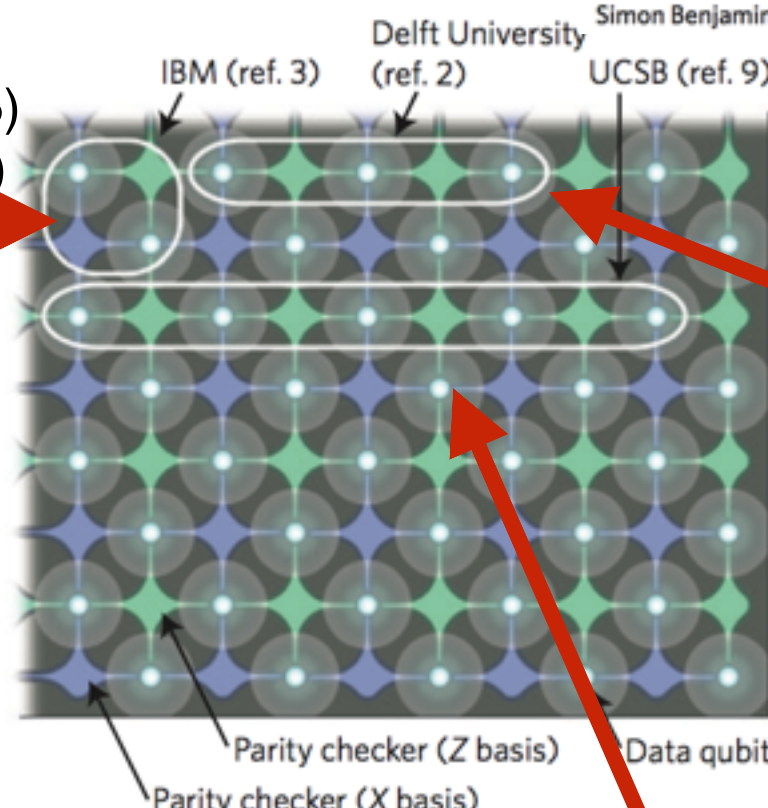

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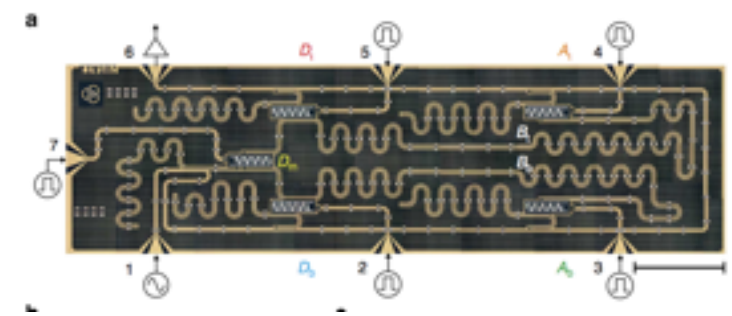

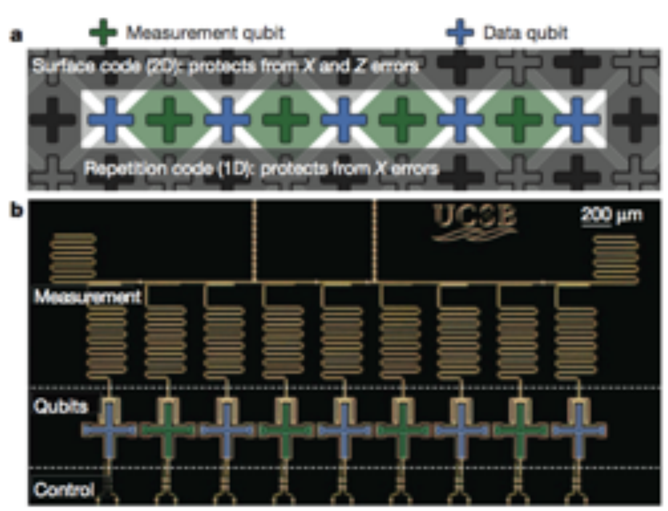

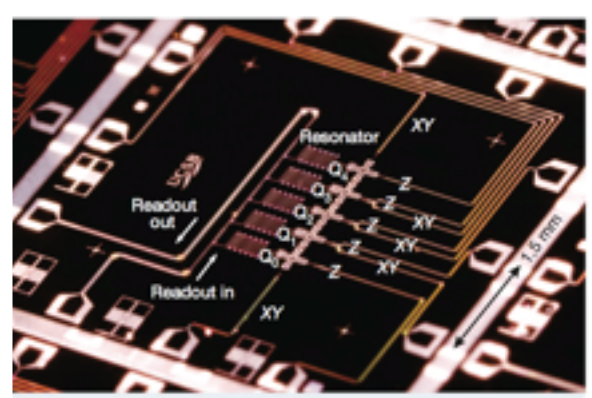

UCSB(Martinis)+Google:

Kelly et al., Nature **519,** 66 (2015)
Barends et al., Nature **508**, 500 (2014)

[fidelities]
single-qubit gate: 99.92%
two-qubit gate: 99.4%
measurement: 99%

Noisy quantum circuits above standard noise threshold

Threshold theorem*:* if the noise strength is smaller than a certain constant threshold value, quantum computation can be performed with an arbitrary accuracy.

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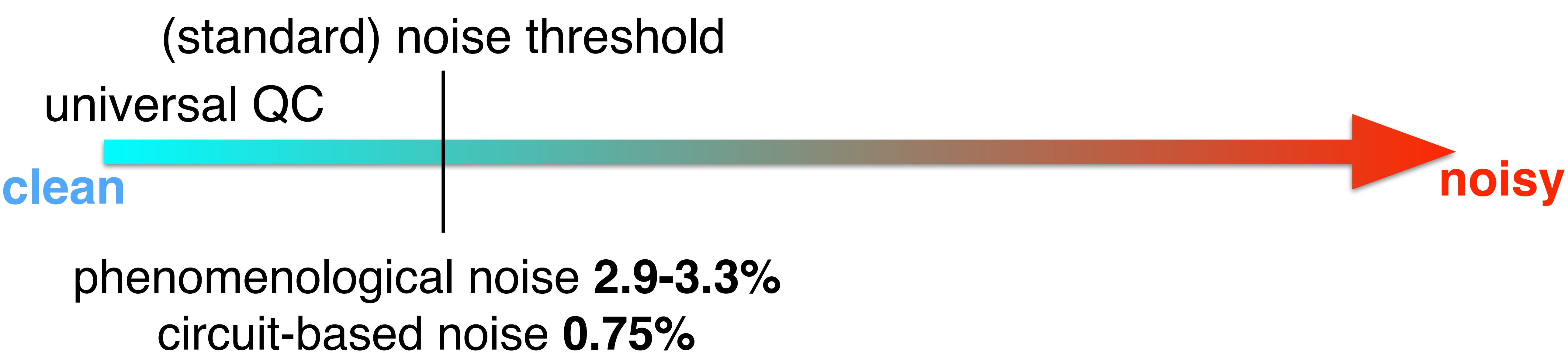

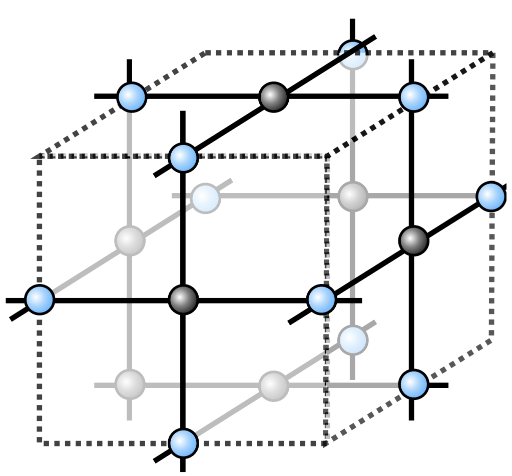

Topological fault-tolerance in cluster state quantum computation

R Raussendorf, J Harrington and K Goyal

New Journal of Physics **9** (2007) 199

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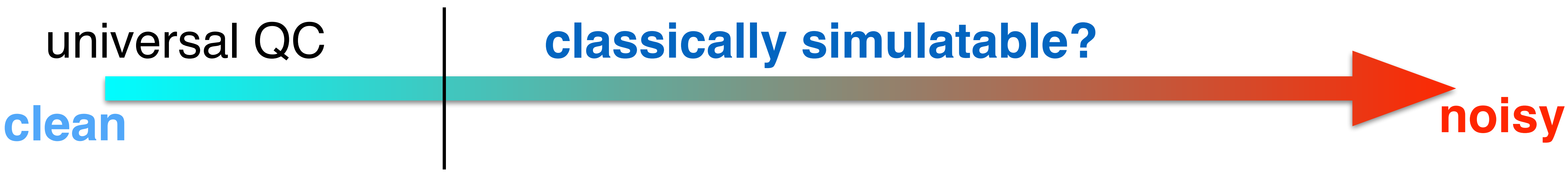
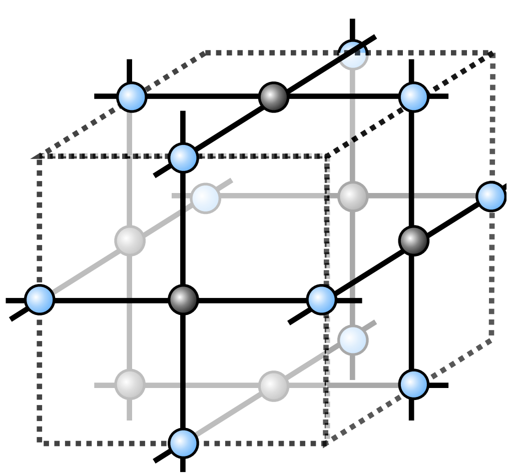
Threshold theorem: if the noise strength is smaller than a certain constant threshold value, quantum computation can be performed with an arbitrary accuracy.

(standard) noise threshold



phenomenological noise **2.9-3.3%**
circuit-based noise **0.75%**



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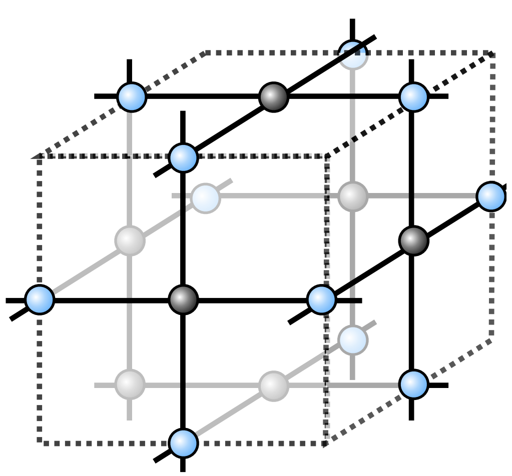
phenomenological noise **14.6%**



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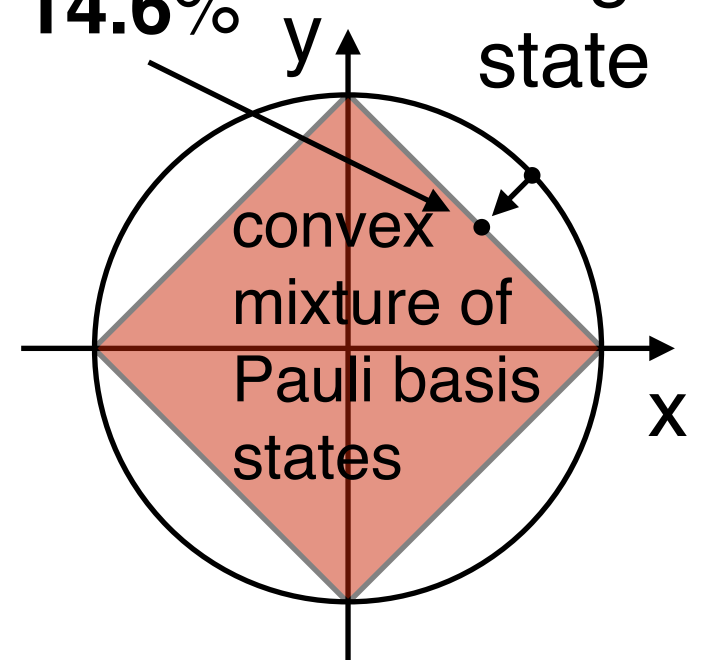

Outline

- Motivations
- Hardness proof by postselection
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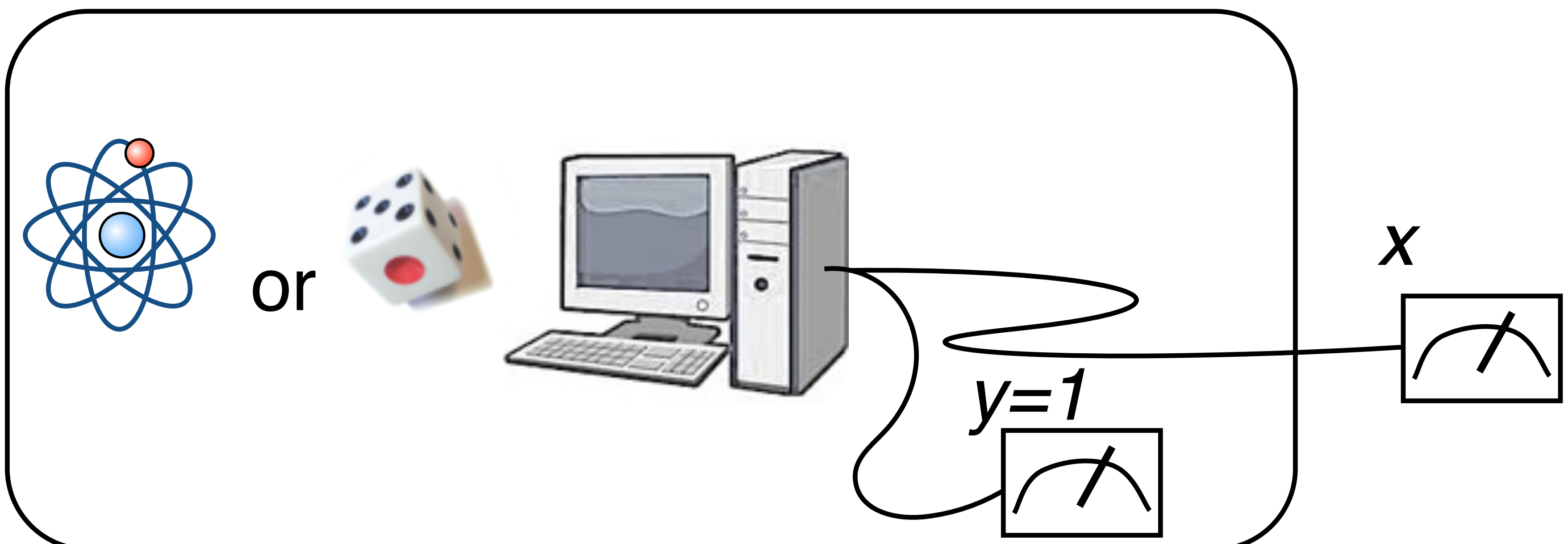
Postselected computation

= solving a decision problem by using conditional probability distribution.

while(y==1)



$$p(x|y=1) = p(x,y)/p(y=1)$$

not zero but can be exponentially small

yes: $p(x=1|y=1) \geq 2/3$

no: $p(x=0|y=1) \geq 2/3$

Aaronson, Proc. of the Royal Society A: Math., Phys. and Eng. Sci. **461**, 3473 (2005).

Hardness proof
via postBQP = PP

A (fictitious) ability to postselect a possibly exponentially rare events allows us to distinguish quantum and classical tasks!

classical+ postselection

quantum+ postselection

Bremner-Jozsa-Shepherd, Proc. Royal Soc. A: Math. Phys. and Eng. Sic. 467, 2126 (2011)
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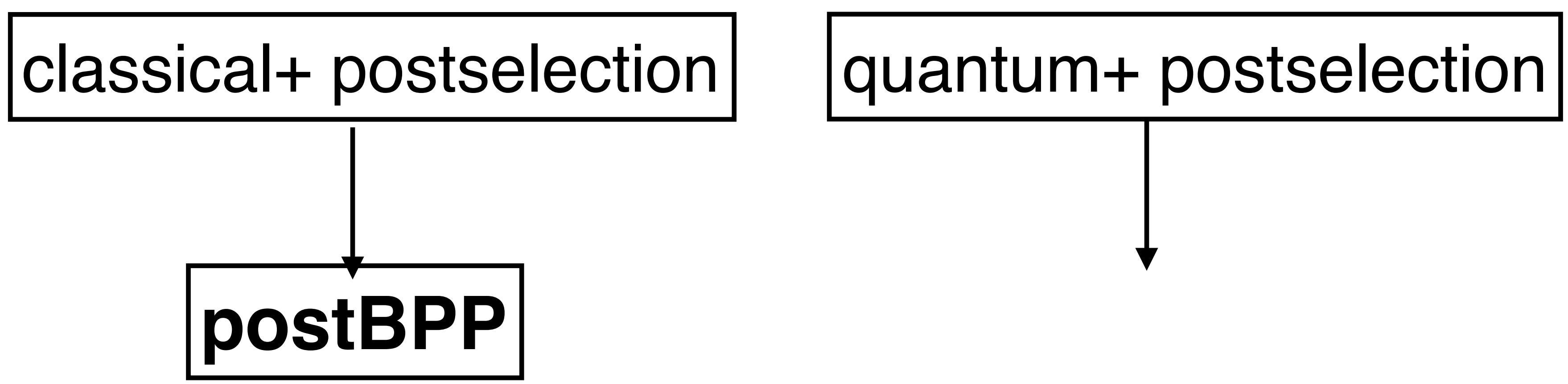

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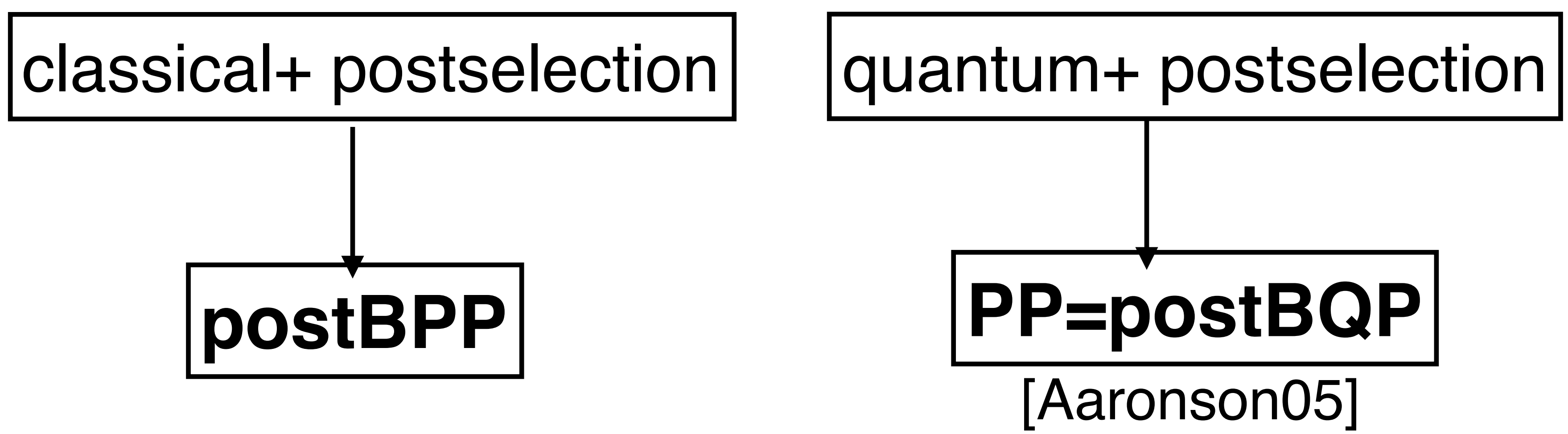

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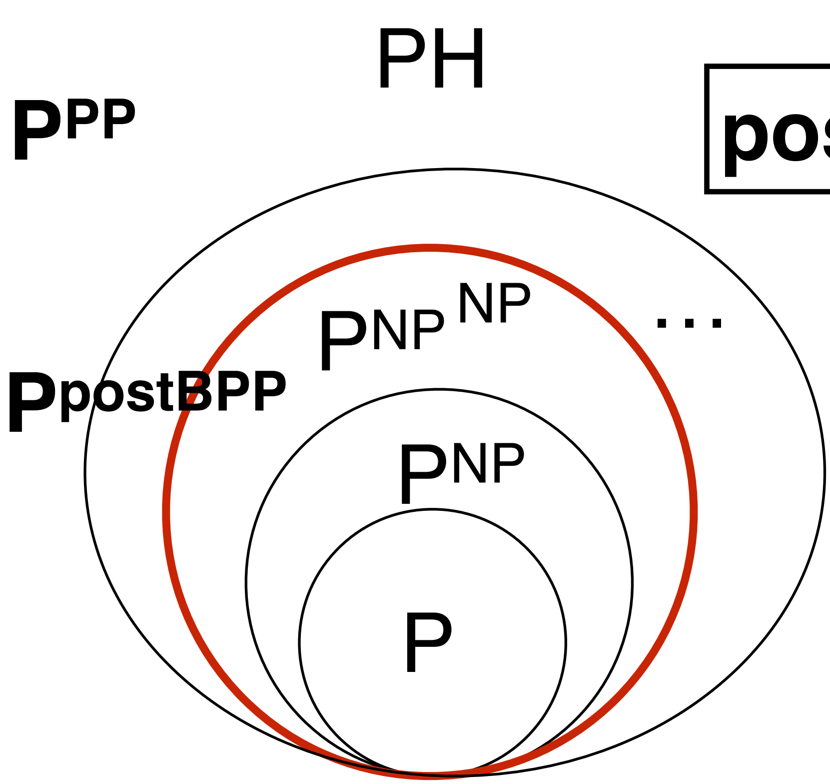
A (fictitious) ability to postselect a possibly exponentially rare events allows us to distinguish quantum and classical tasks!

classical+ postselection → **postBPP** ≠ **PP=postBQP** ← quantum+ postselection

[Aaronson05]



unless the PH collapses to the 3rd level.

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Hardness proof via postBQP = PP

A (fictitious) ability to postselect a possibly exponentially rare events allows us to distinguish quantum and classical tasks!

classical+ postselection → **postBPP** ≠ quantum+ postselection → **PP=postBQP** [Aaronson05]

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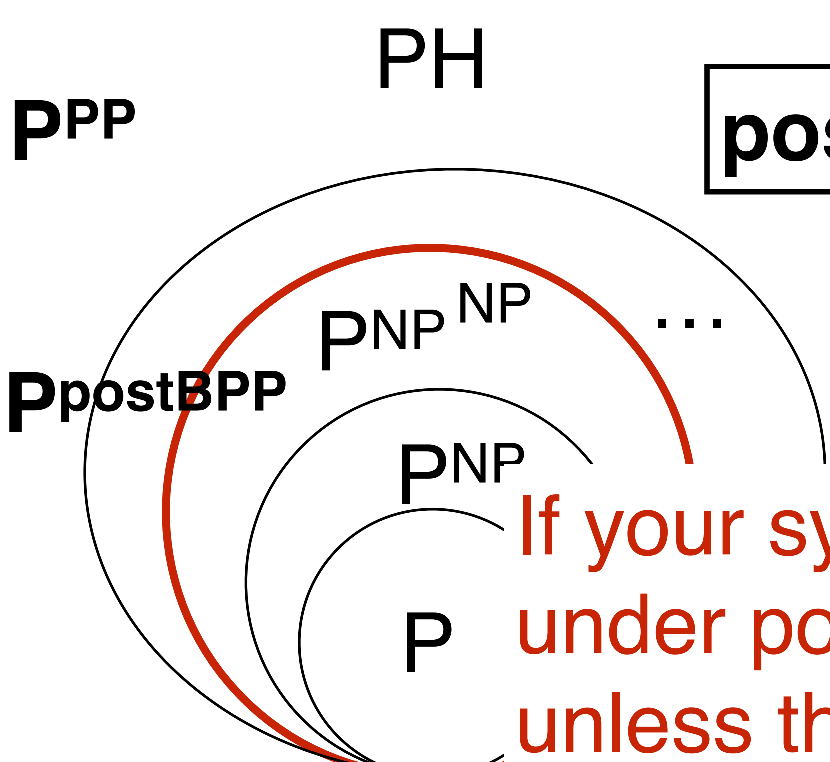

If your system is potentially as powerful as postBQP under postselection, then its classical simulation is hard unless the PH collapses!

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Difficulty of quantum supremacy with noisy sampling

- multiplicative error (or exponentially small additive error)

$$\frac{1}{c}p^{\mathrm{ideal}}(x) < p^{\mathrm{samp}}(x) < cp^{\mathrm{ideal}}(x) \qquad (c > 1)$$

[Bremner-Jozsa-Shepherd, '11]

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[Bremner-Jozsa-Shepherd, '11]

- constant additive error with l_1-norm

$$\|p^{\mathrm{samp}}(x) - p^{\mathrm{ideal}}(x)\|_1 = \sum_x |p^{\mathrm{samp}}(x) - p^{\mathrm{ideal}}(x)| < c$$

[Aaronson-Arkhipov, '11, Bremner-Montanaro-Shepherd '16]

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[Aaronson-Arkhipov, '11, Bremner-Montanaro-Shepherd '16]

Small amount of noise can easily break these conditions.

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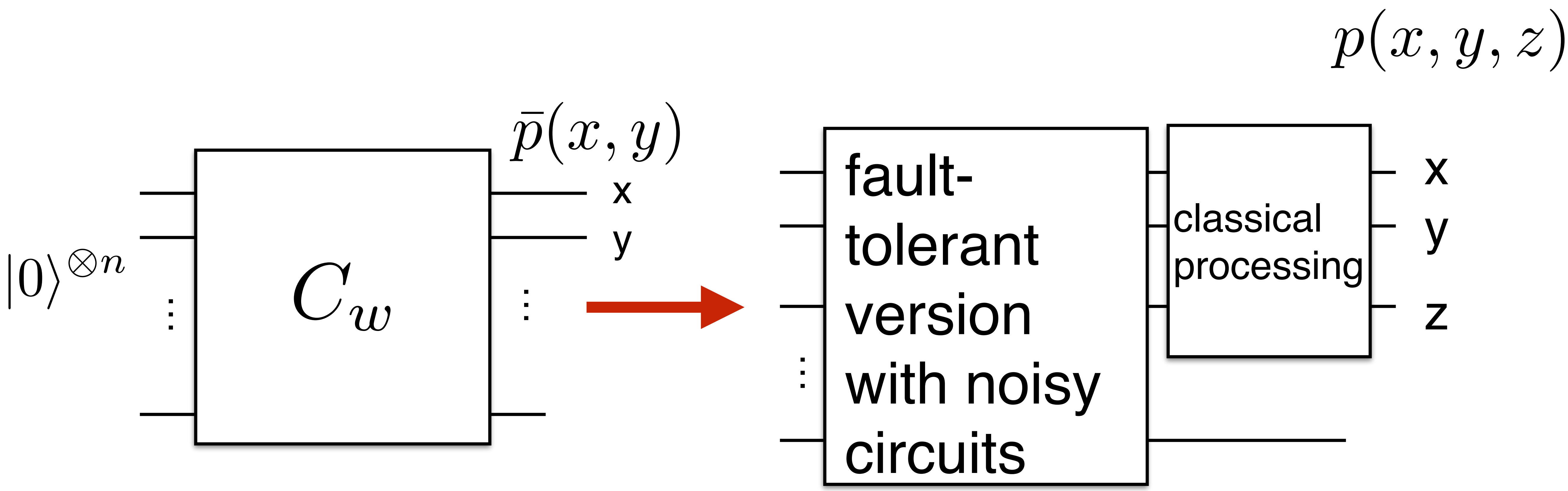
Main idea: simulation of fault-tolerant quantum computation under postselection



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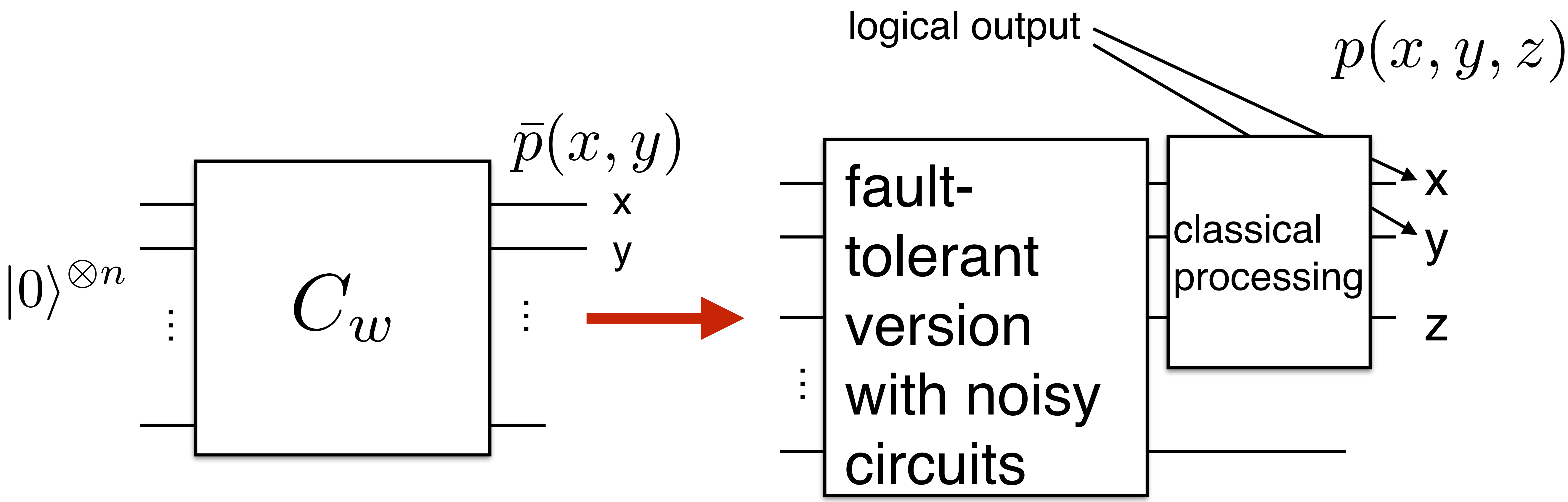

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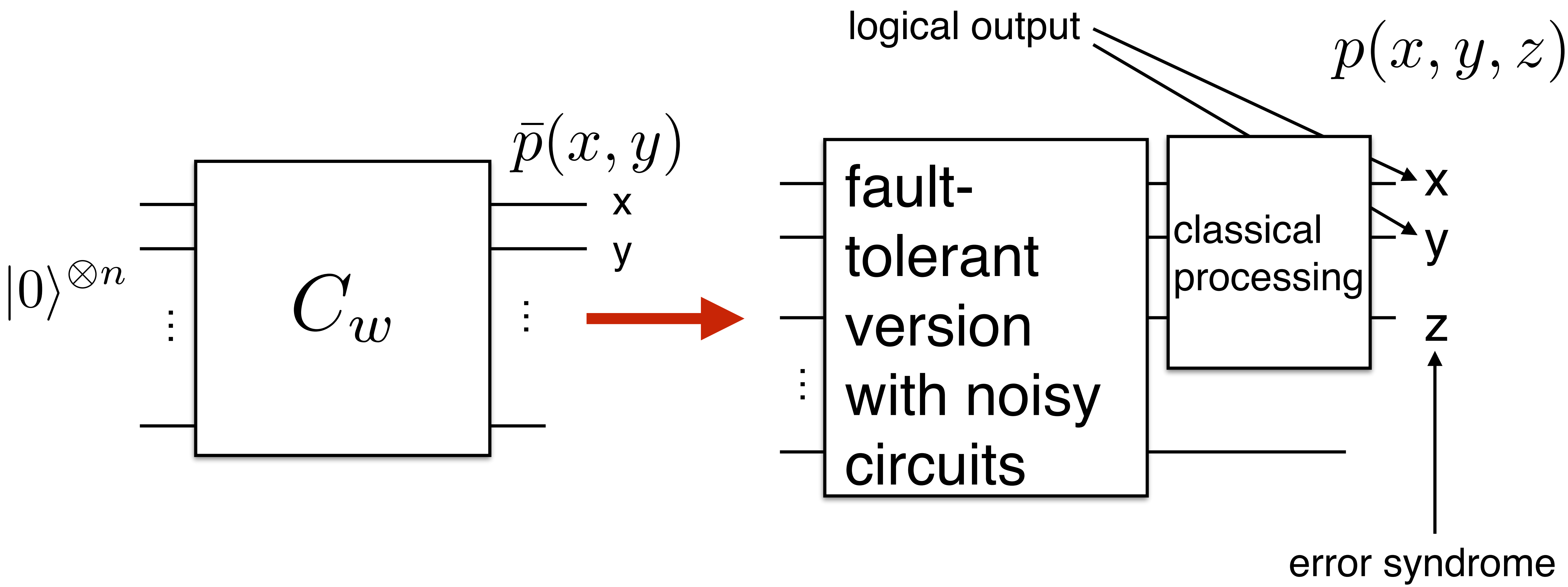

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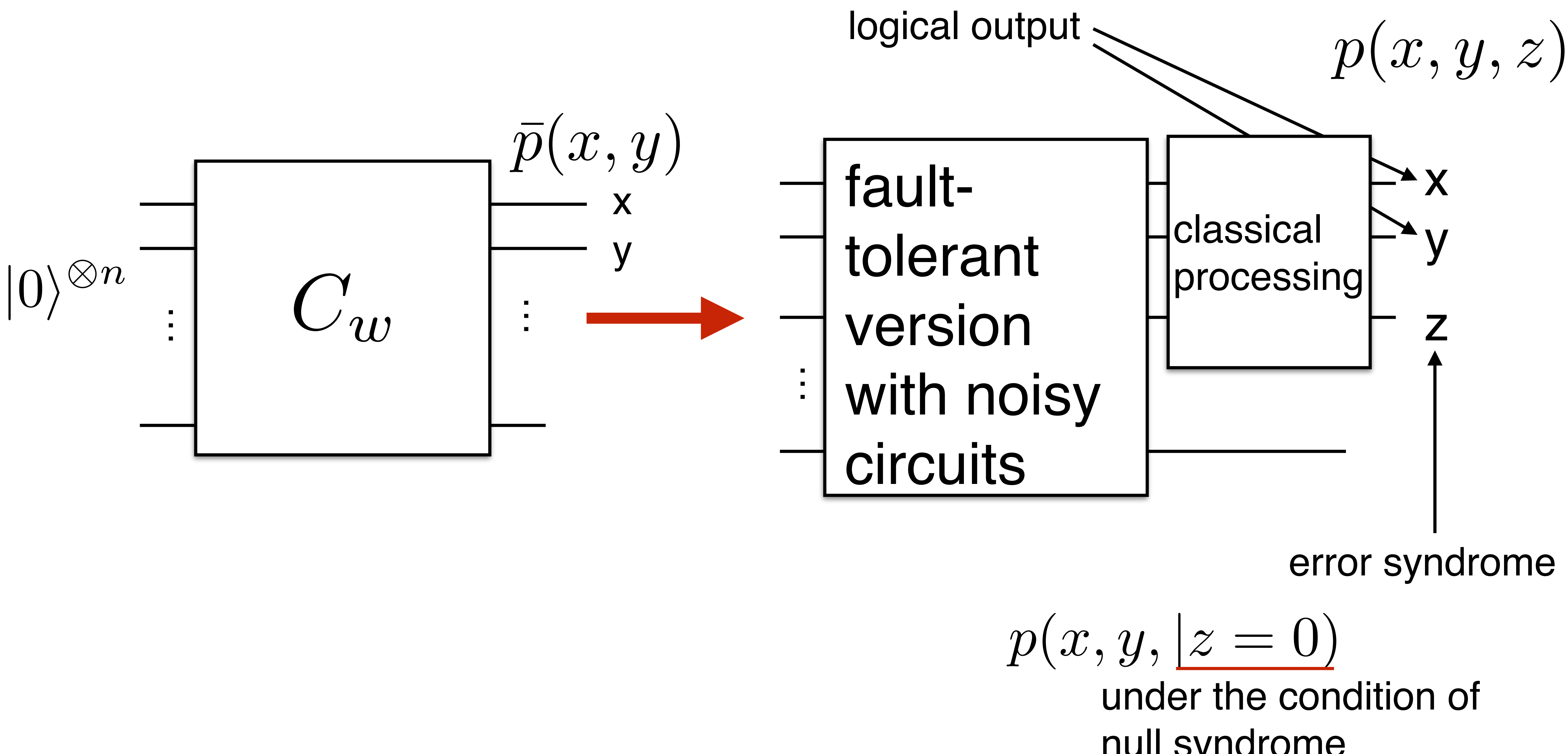

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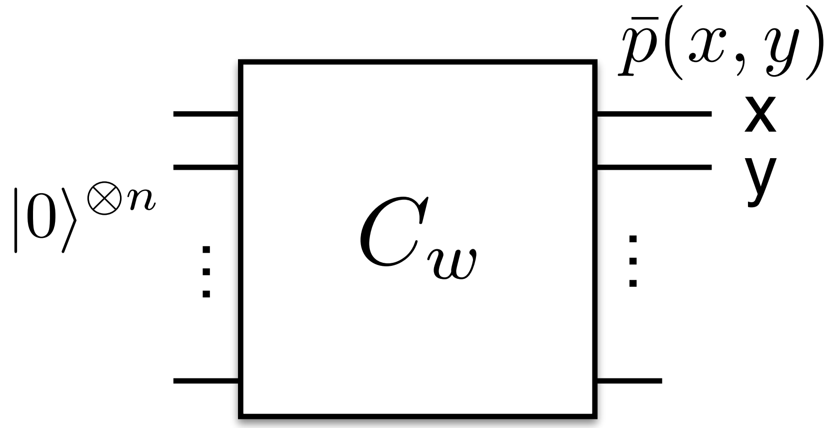
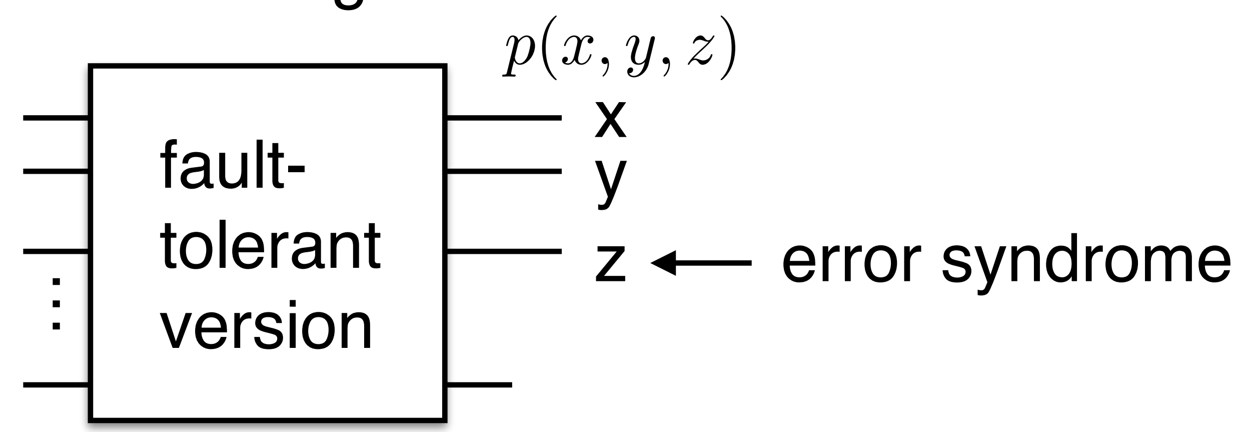
Threshold theorem for quantum supremacy

- Part1: An exponentially small additive error is enough.





$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

where the overhead is polynomial in (n,κ). Then, classical simulation of $p(x,y,z)$ with a multiplicative error $1 < c < \sqrt{2}$ is hard.

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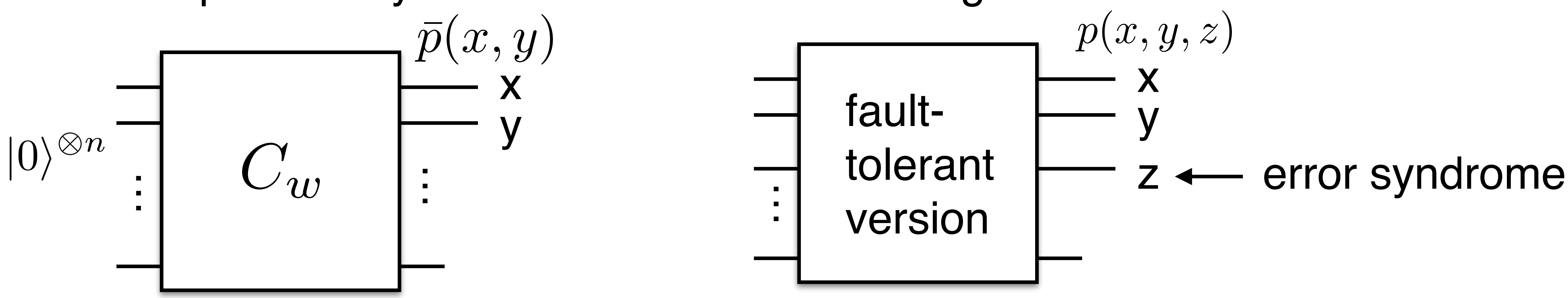

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- Part2: The exponentially small additive error

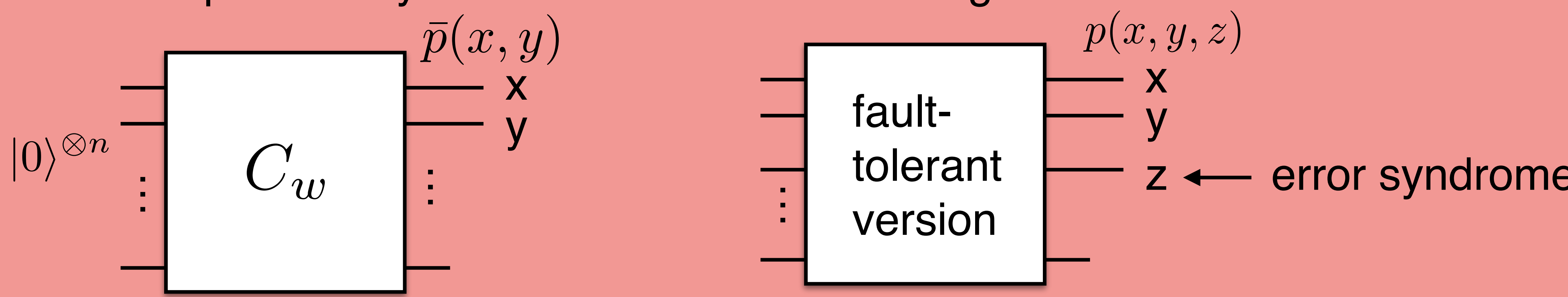
$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

is achievable by quantum error correction under postselection.

arXiv:1610.03632

Threshold theorem for quantum supremacy

- Part1: An exponentially small additive error is enough.



$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

where the overhead is polynomial in (n,κ). Then, classical simulation of $p(x,y,z)$ with a multiplicative error $1 < c < \sqrt{2}$ is hard.

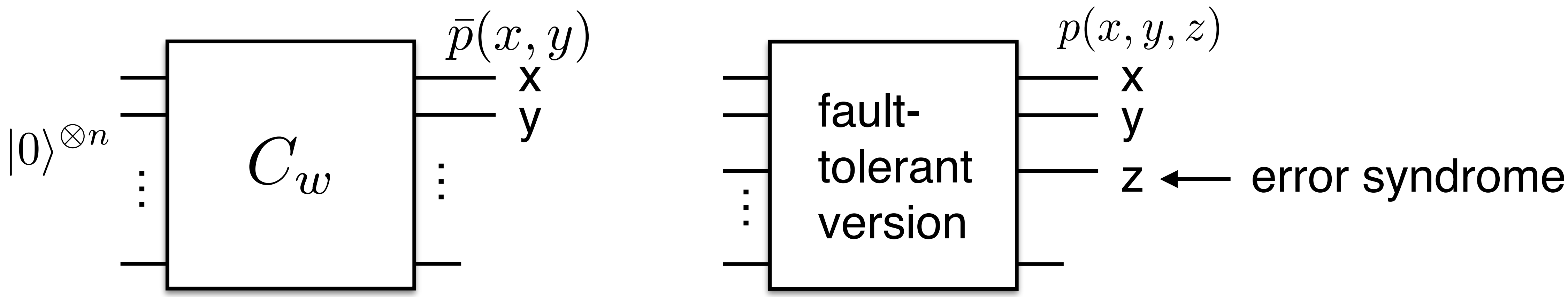
- Part2: The exponentially small additive error

$$|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$$

is achievable by quantum error correction under postselection.

arXiv:1610.03632

Part1: an exponential small additive error is enough



Solve a PP-complete problem (**MAJSAT**) using $\bar{p}(x|y)$ as in [Aaronson05]

→ probability for postselection: $\bar{p}(y=0) > 2^{-6n-4}$

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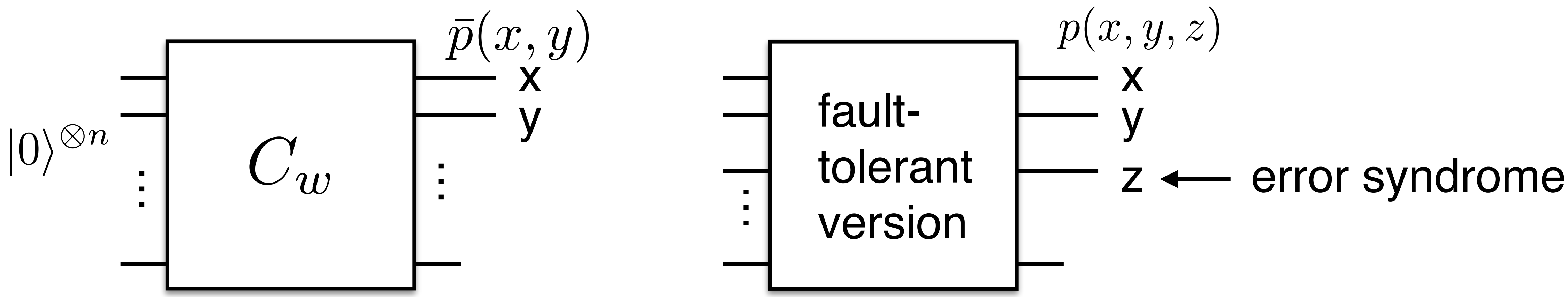

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Therefore, if $|\bar{p}(x,y) - p(x,y|z=0)| < e^{-\kappa}$ with $\kappa = \mathrm{poly}(n)$

then we have

$$|\bar{p}(x|y=0) - p(x|y=0,z=0)| < 1/2$$

→ $p(x,y,z)$ can solve the PP-complete problem under postselection.

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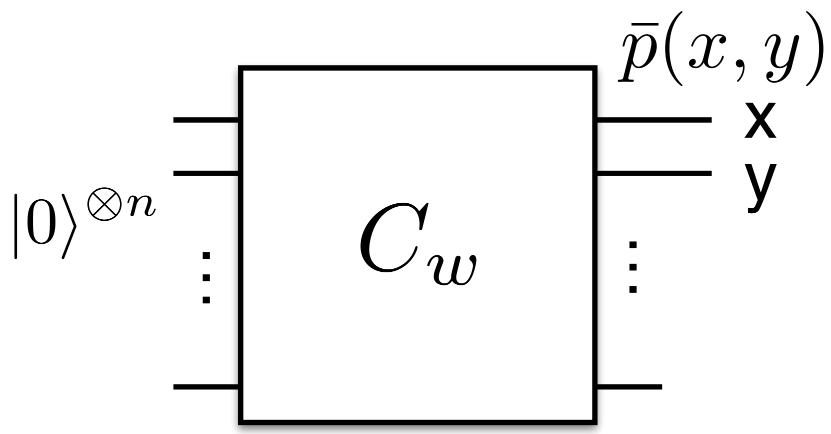

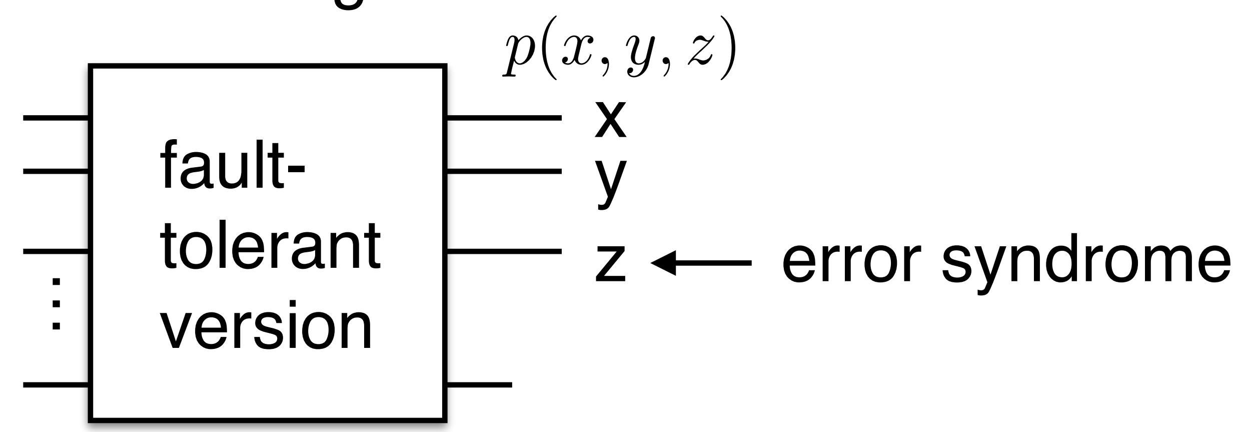

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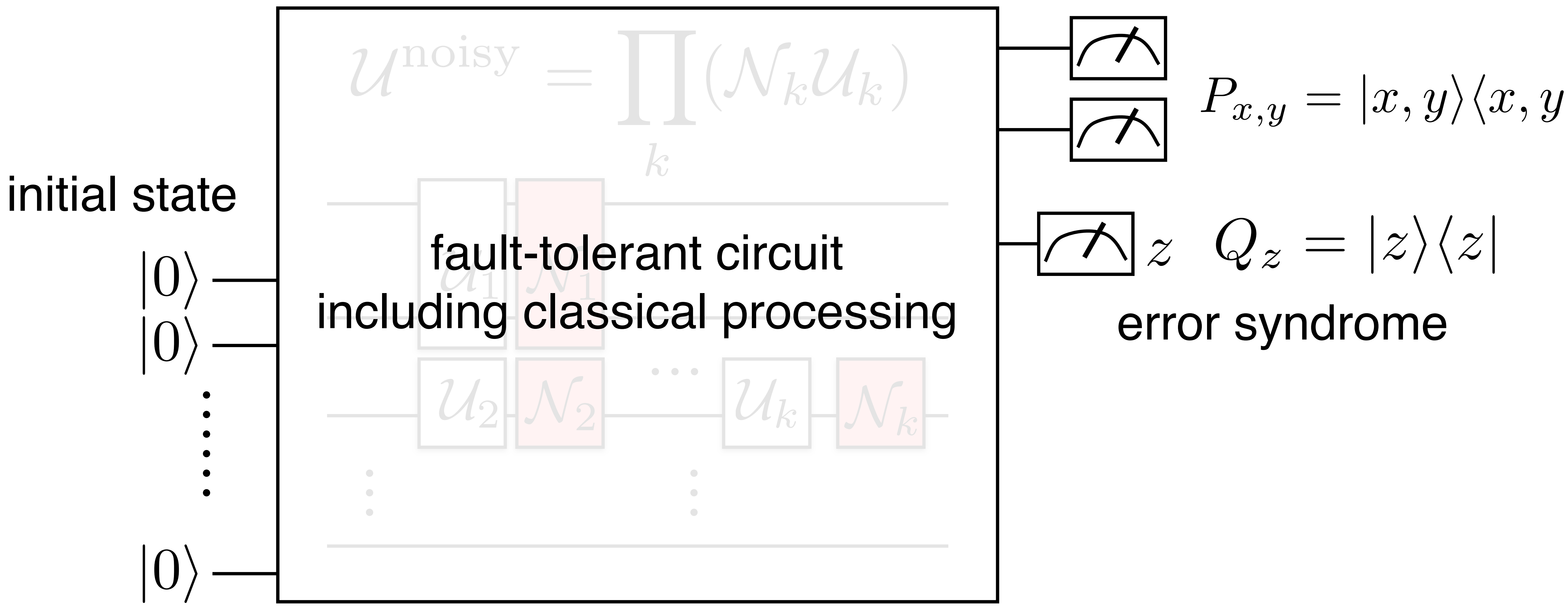
Part2: error reduction under postselection (sketch)



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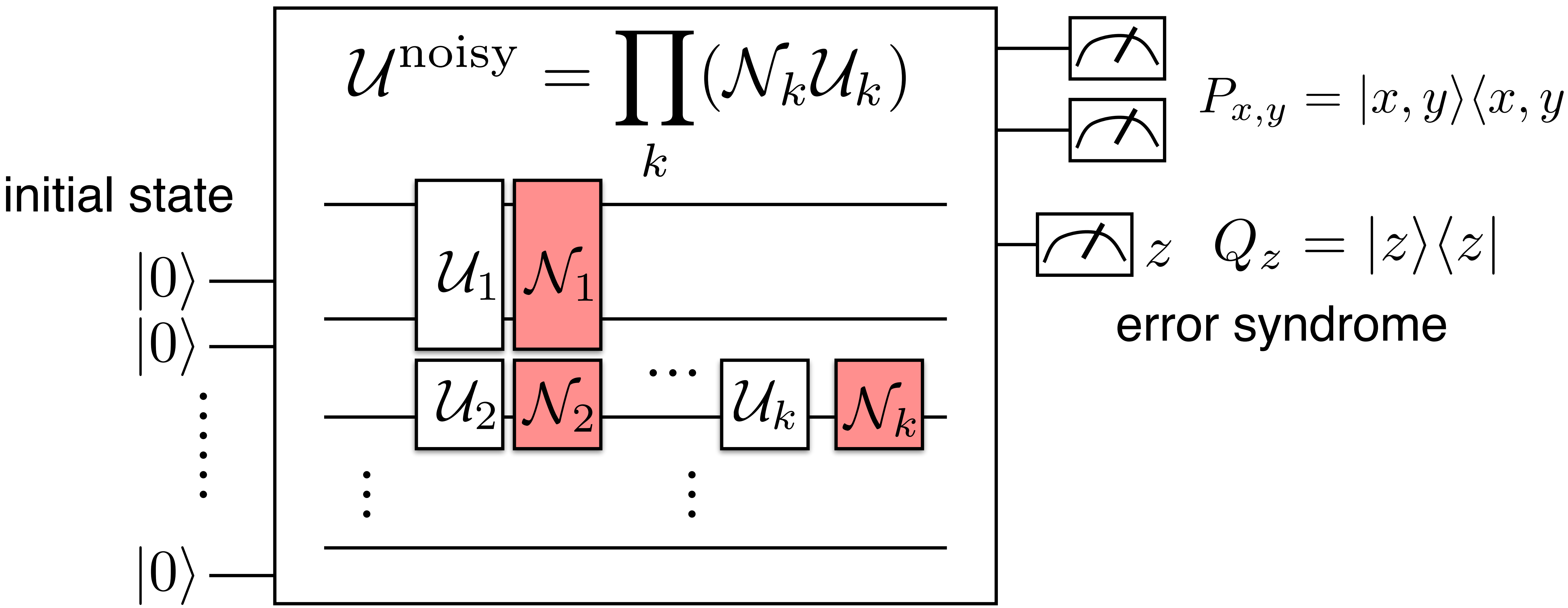

fault-tolerant circuit
including classical processing

arXiv:1610.03632

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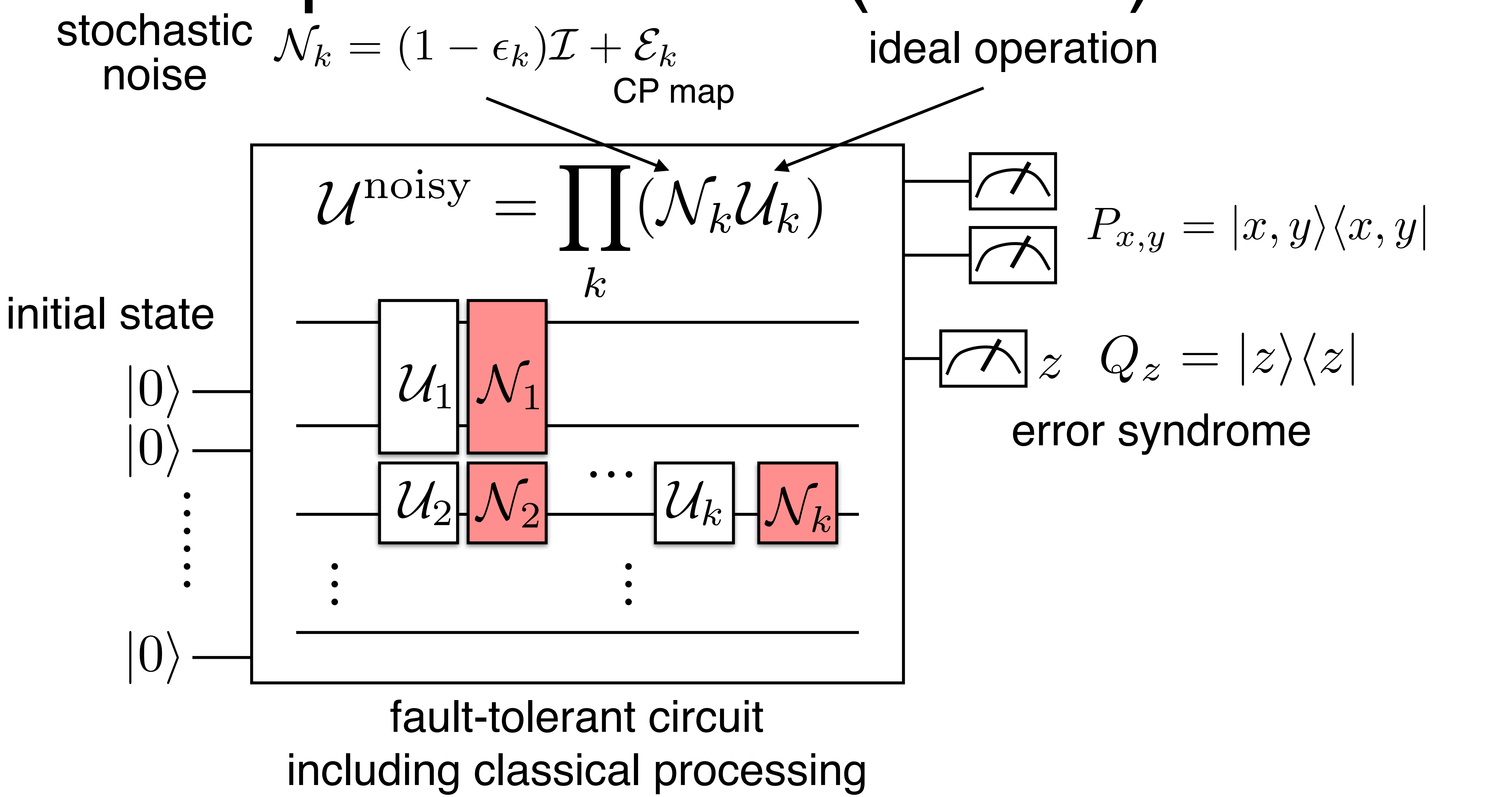

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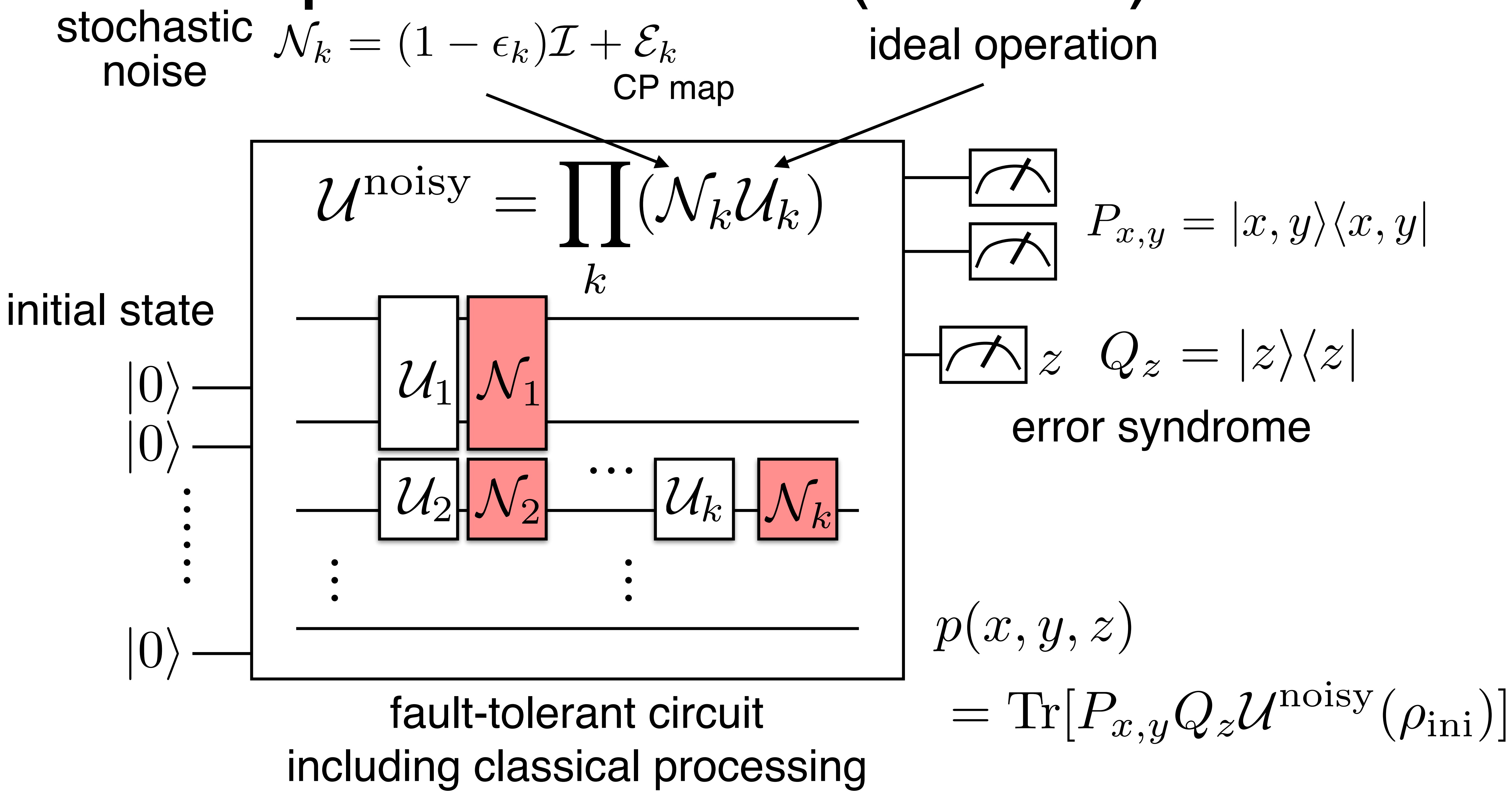

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Part2: error reduction under postselection (sketch)

Using $\mathcal{N}_k = (1-\epsilon_k)\mathcal{I} + \mathcal{E}_k$, we decompose $\mathcal{U}^{\text{noisy}}$ into

$$\mathcal{U}^{\text{noisy}}(\rho_{\text{ini}}) = \rho_{\text{sparse}} + \rho_{\text{faulty}}$$

such that $\bar{p}(x,y) \propto \text{Tr}[P_{x,y} Q_{z=0} \rho_{\text{sparse}}]$.

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Then we can show that

$$\|\bar{p}(x,y) - p(x,y|z=0)\|_1 < 2\|\rho_{\text{faulty}}\|_1 / q_{z=0}$$

where $q_{z=0} \equiv \text{Tr}[Q_{z=0}\mathcal{U}^{\text{noisy}}(\rho_{\text{ini}})]$. (prob. of null syndrome measurement)

$$< 2\sum_{r\geq d} C(r)\left(\frac{\epsilon}{1-\epsilon}\right)^r \quad (\epsilon \equiv \max_k \epsilon_k)$$

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There is a constant threshold $\varepsilon_{\mathrm{th}}$ below which the output $p(x,y,z) = \mathrm{Tr}[P_{x,y} Q_z \mathcal{U}^{\mathrm{noisy}}(\rho_{\mathrm{ini}})]$ from the noisy quantum circuits cannot be simulated efficiently on a classical computer unless the PH collapses to the 3rd level.

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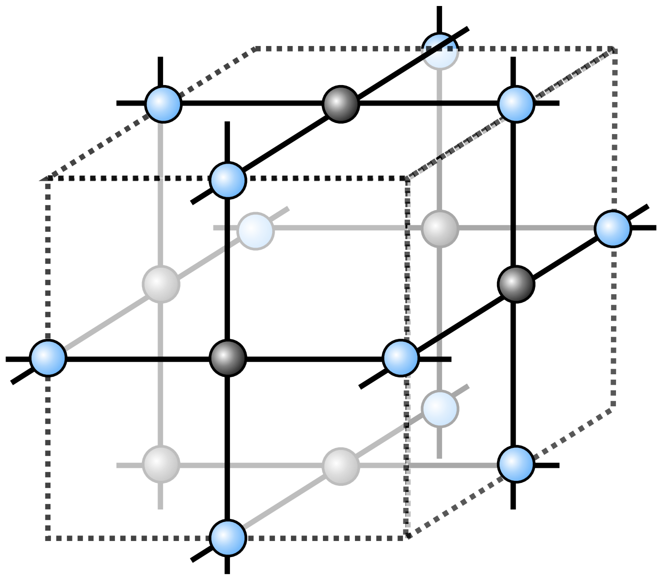
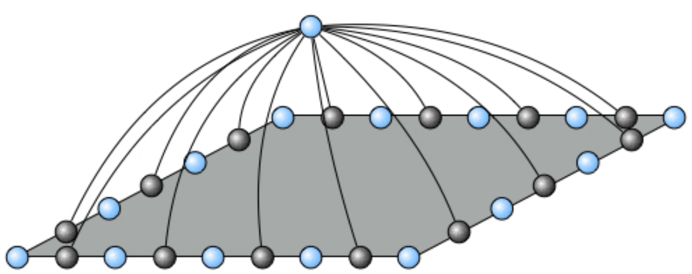
Outline

- Motivations
- Hardness proof by postselection
- Threshold theorem for quantum supremacy
- Applications: 3D topological cluster computation & 2D surface code
- Summary

Topological MBQC on a 3D cluster state





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  (corresponds to commuting circuits of depth log(n))

see also KF-Tamate '16

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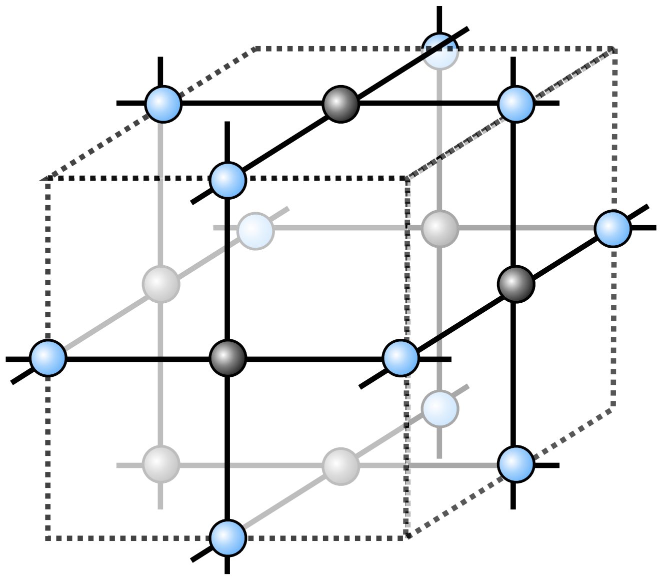

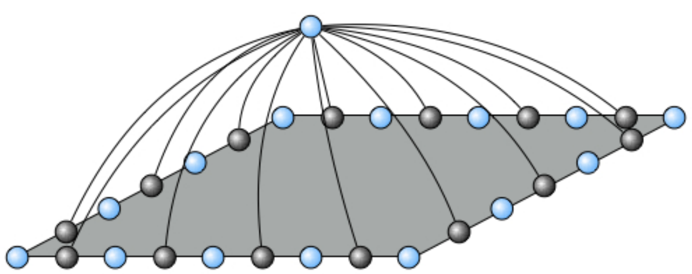

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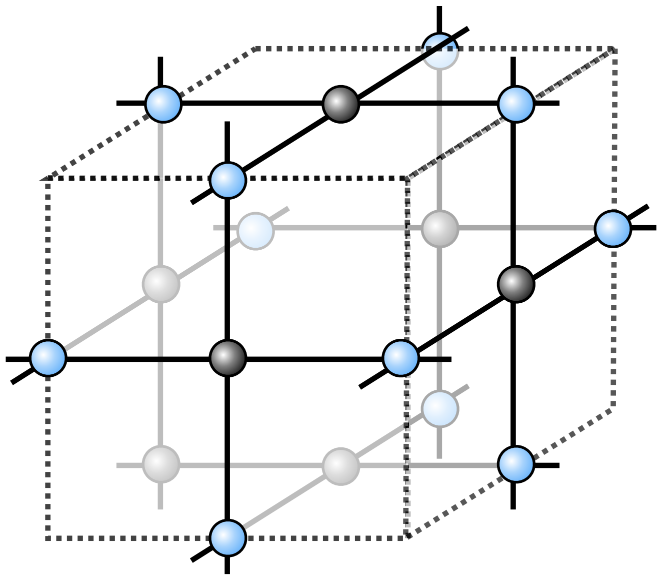

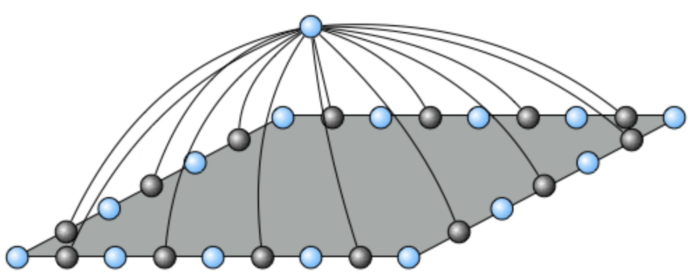

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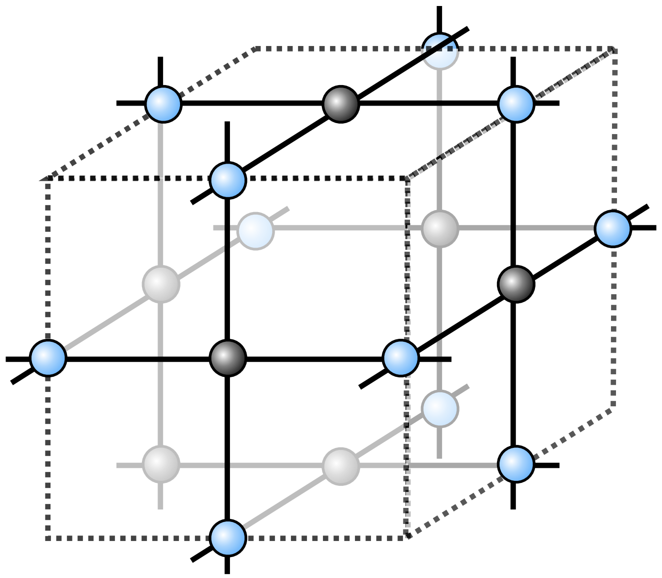

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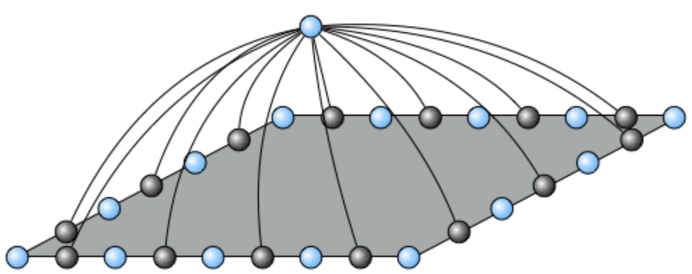

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Clifford operations
(counting # of self-avoiding walks: Dennis et al '02)

$$\frac{12}{5}\text{poly}(n)\left(\frac{5\epsilon}{1-\epsilon}\right)^{d} \longrightarrow \epsilon_{\text{cl}} = 0.167$$

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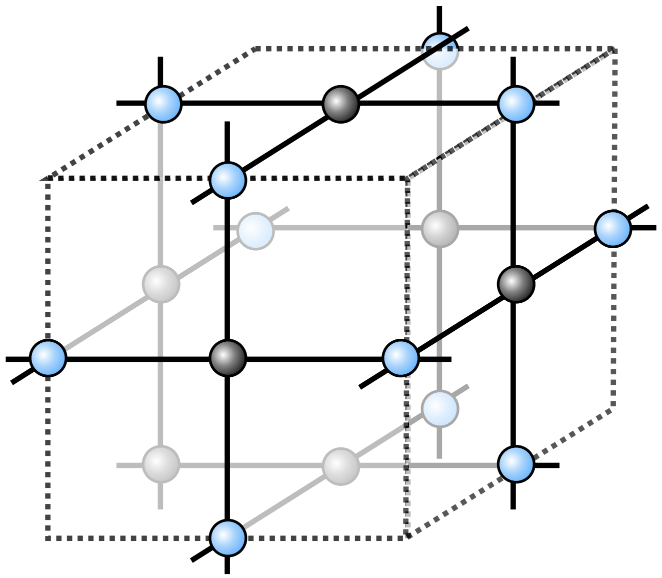

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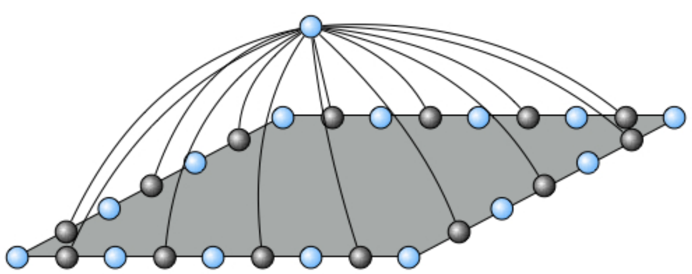

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magic state distillation $\longrightarrow$ $\epsilon_{\mathrm{magic}} = 0.146$

(Bravyi-Kitaev '05; Reichardt '06)

see also KF-Tamate '16

Noisy quantum circuits above standard noise threshold

Threshold theorem: if the noise strength is smaller than a certain constant threshold value, quantum computation can be done with an arbitrary accuracy poly(logarithmic) overhead.

universal QC

noise threshold

distillability of of magic state

classically simulatable by GK theorem

clean

noisy

classical simulation is hard!

phenomenological noise **2.9-3.3%**
circuit-based noise **0.75%**

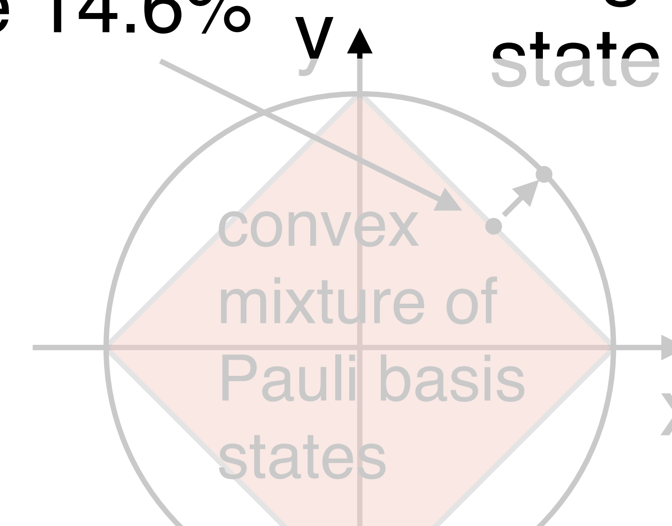
phenomenological noise 14.6%

Topological fault-tolerance in cluster state quantum computation

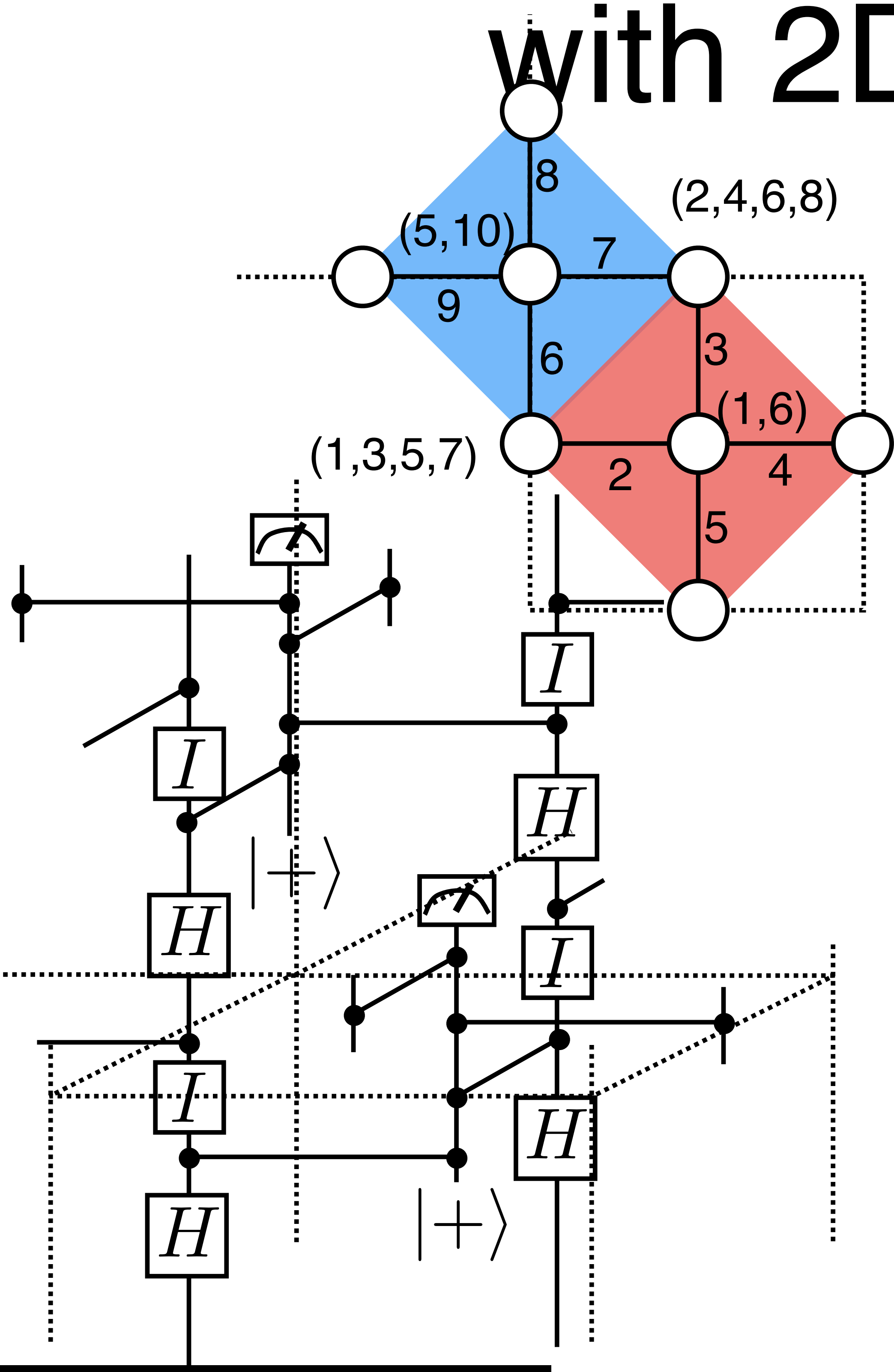
R Raussendorf, J Harrington and K Goyal

New Journal of Physics **9** (2007) 199

Ann. Phys. ***321*** *2242* (2006)



Circuit-based noise model with 2D surface code



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arXiv:1610.03632

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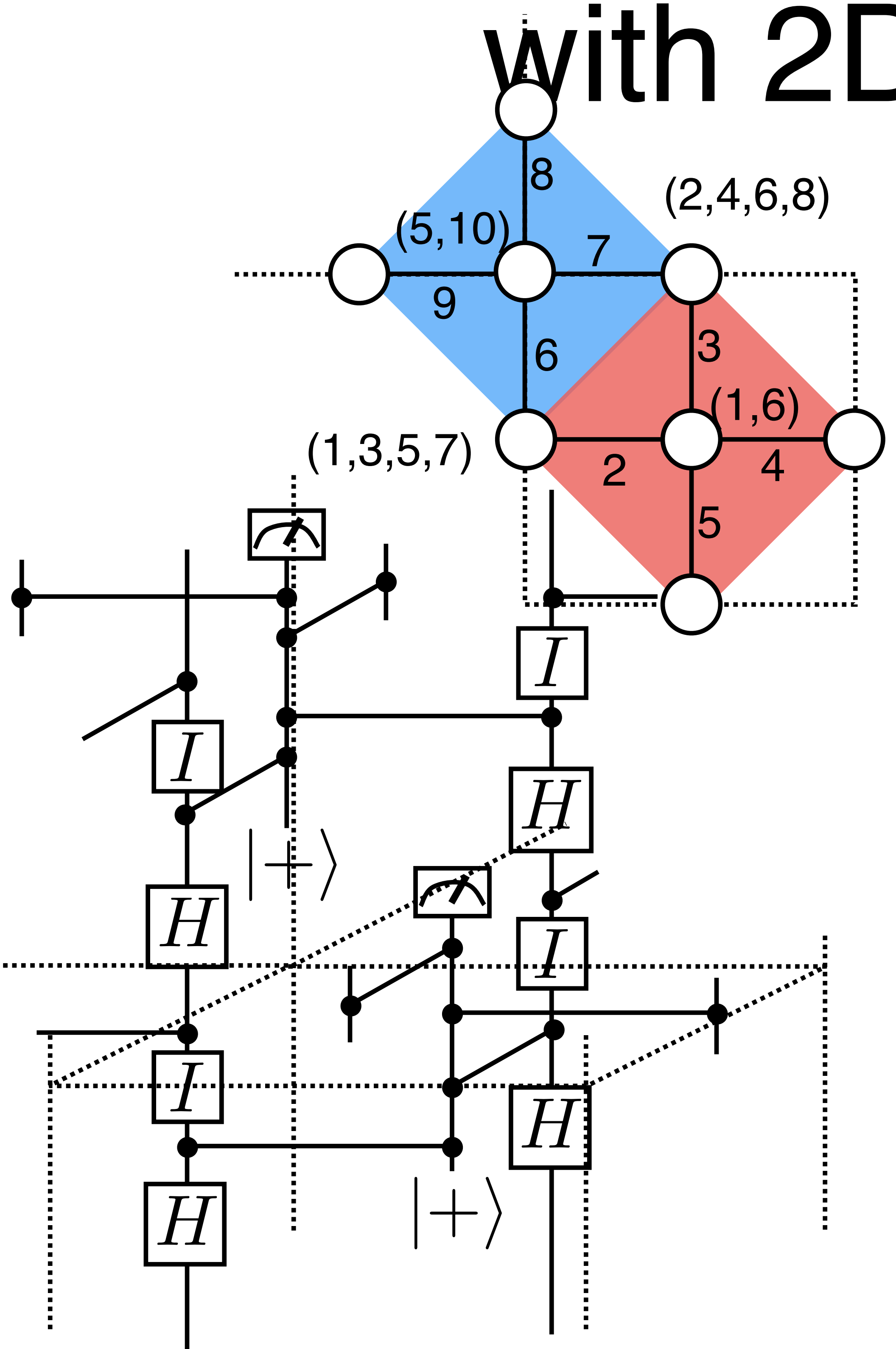

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ν: independent error rate

μ: correlated error rate

$\nu = 54p/15, \mu = 6p/5$

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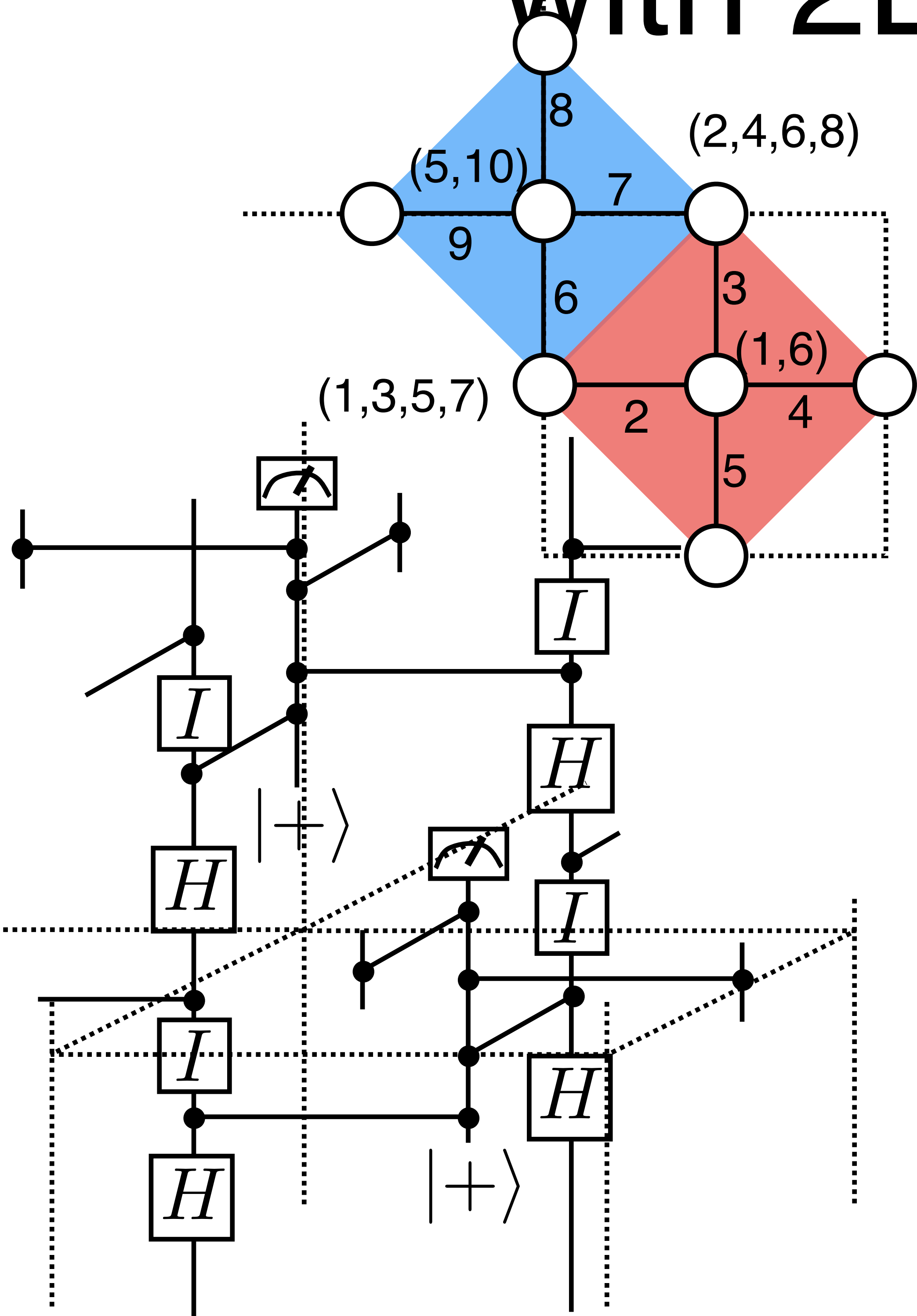

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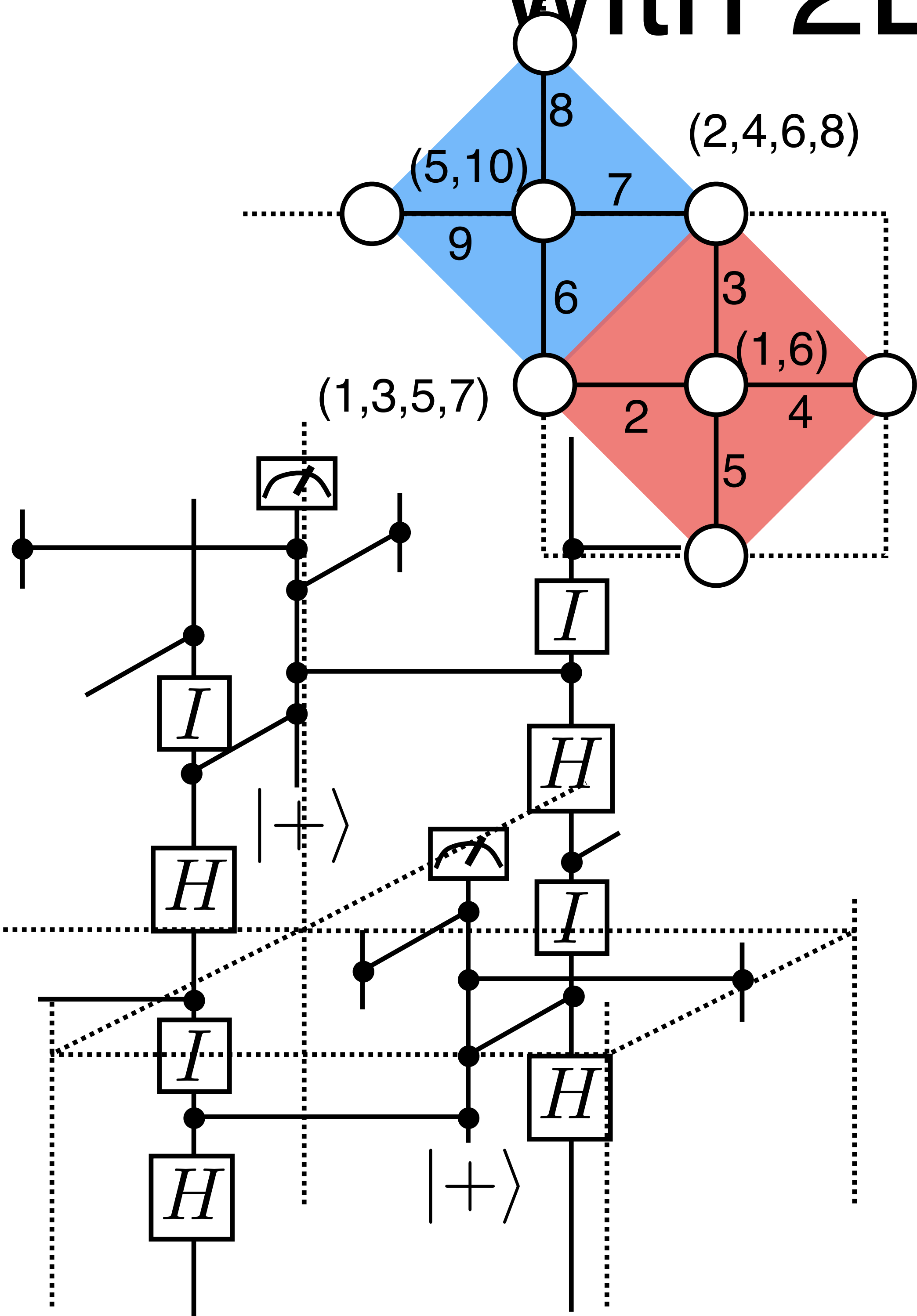

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- threshold value: p=2.84% (distillability of magic state)
- higher than the standard threshold 0.75%

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Summary

- Sampling with noisy quantum circuits can exhibit quantum supremacy.
- The threshold for supremacy is much higher than that for universal fault-tolerant quantum computation.
- The threshold is determined purely by distillability of the magic state (in a phenomenological model it sharply separate classically simulatable and not-simulatable regions).
- Can we directly verify or identify quantum supremacy of the near-term noisy quantum devices in a pre-fault-tolerant region?

Thank you for your attention!

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