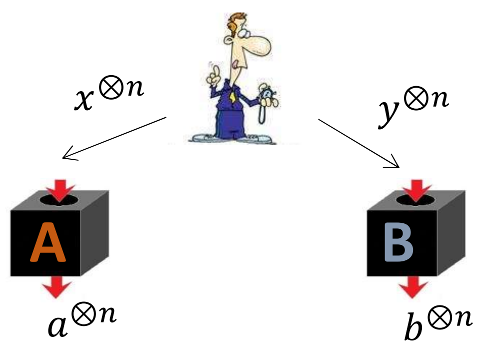
Concluding remark

- Note that there is a natural isomorphism between states of n pairs of qubits and states of a single pair of qu-Dits, for $D = 2^n$.
- If we are able to self-test $|\psi\rangle = \bigotimes_{i=1}^{n} \left(\cos\theta_i |00\rangle + \sin\theta_i |11\rangle \right)$, then we can also self-test some state of a single pair of qu-Dits.
- Hence, as a corollary of our result, we deduce that we can self-test an n dimensional subfamily of the family of all partially entangled states of two qu-Dits, for $D = 2^n$.
- With a different approach, C. & Goh & Scarani show that all pure bipartite entangled states can be self-tested[8].

THANK YOU!

[8] A. Coladangelo, K. T. Goh and V. Scarani (2016). All pure bipartite entangled states can be self-tested.

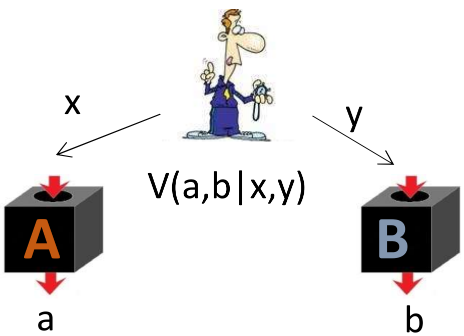
Rigidity of The Parallel Repeated Magic Square Game



$V(a^{\otimes n}, b^{\otimes n} | \, x^{\otimes n} \, y^{\otimes n})$

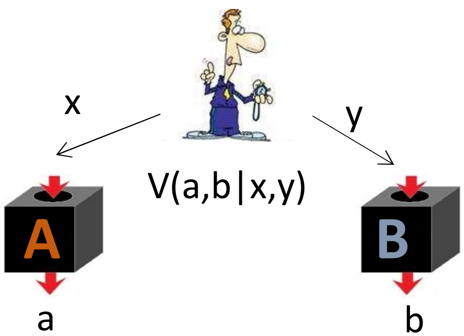
Matthew Coudron, Anand Natarajan
MIT EECS/CSAIL, MIT CTP

QIP '17



The Magic Square Game

	Column 1	Column 2	Column3
Row 1			
Row 2			
Row3			



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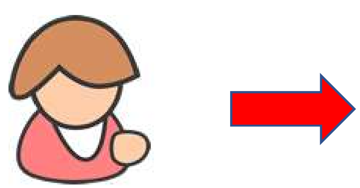

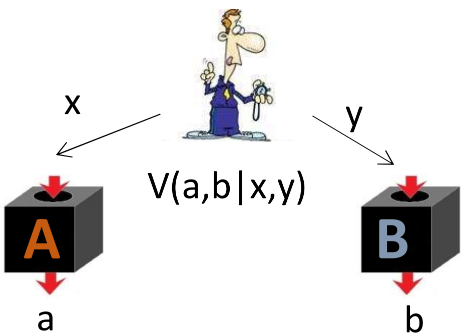

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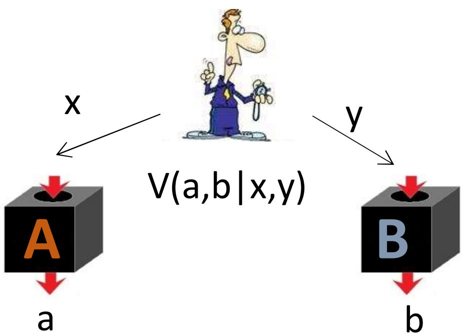

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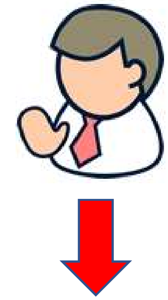
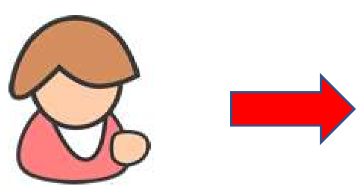
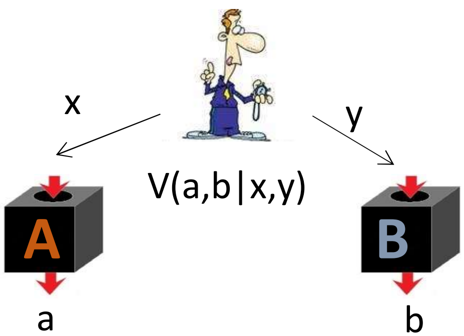


	Column 1	Column 2	Column3
Row 1			
Row 2			
Row3	1	1	-1





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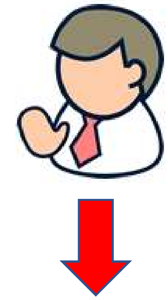

	Column 1	Column 2	Column3
Row 1			-1
Row 2			1
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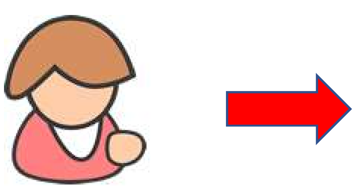
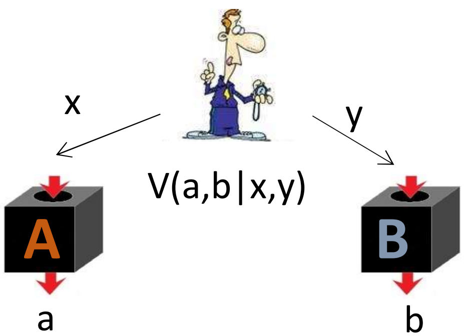
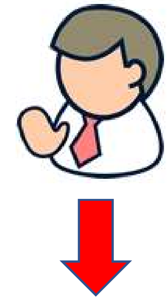
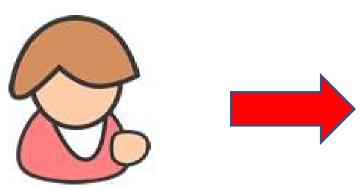




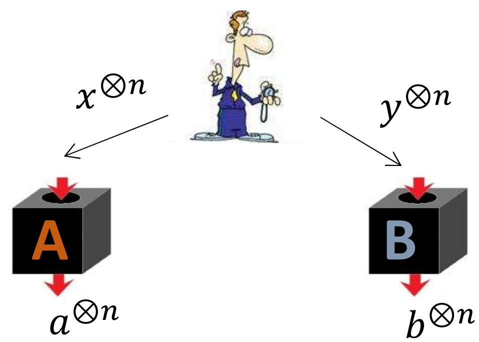
The Magic Square Game: The Ideal Strategy



	Column 1	Column 2	Column3
Row 1	$I \otimes \sigma_Z$	$\sigma_Z \otimes I$	$-\sigma_Z \otimes \sigma_Z$
Row 2	$\sigma_X \otimes I$	$I \otimes \sigma_X$	$-\sigma_X \otimes \sigma_X$
Row3	$\sigma_X \otimes \sigma_Z$	$\sigma_Z \otimes \sigma_X$	$-\sigma_Y \otimes \sigma_Y$



Main Theorem



$$V(a^{\otimes n}, b^{\otimes n} \mid x^{\otimes n}\, y^{\otimes n})$$

Rigidity of the n-round parallel repetition of the Magic Square game:

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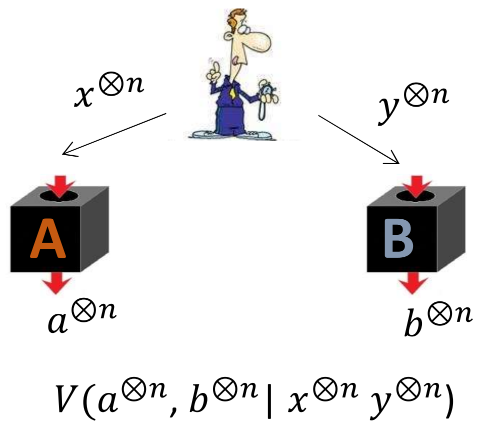

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- For any entangled strategy succeeding with probability 1 − ε, the players' shared state is O(poly(nε))-close to 2n EPR pairs under a local isometry.

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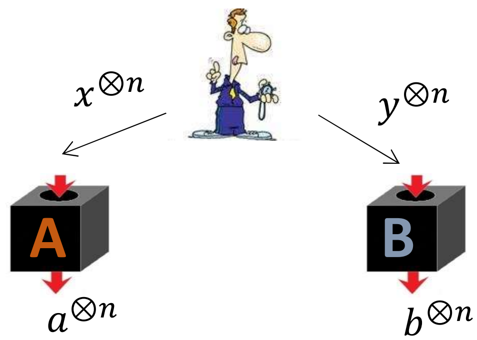

$V(a^{\otimes n}, b^{\otimes n} | x^{\otimes n} y^{\otimes n})$

Rigidity of the n-round parallel repetition of the Magic Square game:

- For any entangled strategy succeeding with probability 1 − ε, the players' shared state is O(poly(nε))-close to 2n EPR pairs under a local isometry.
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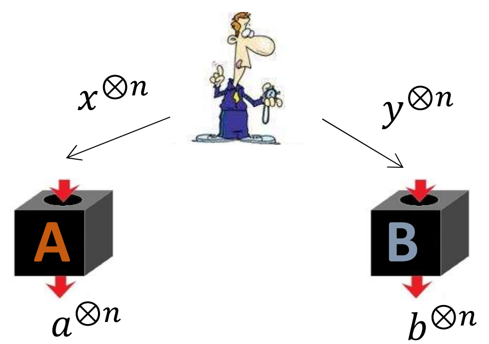
Background and Intuition

- Self-testing results for large games established in by McKague [McK15- "Self-testing in Parallel"]



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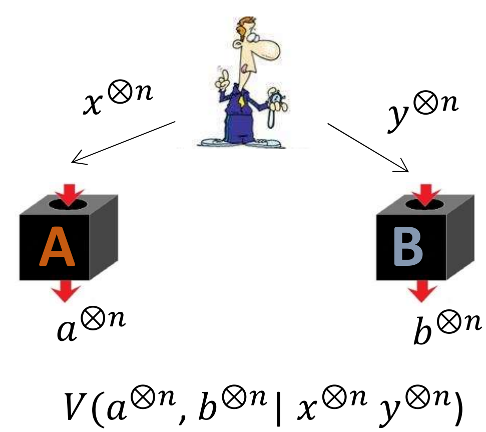

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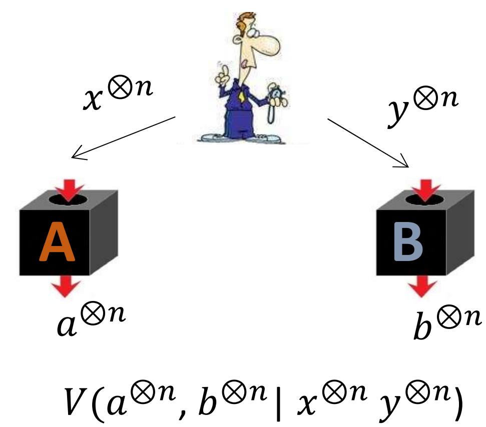

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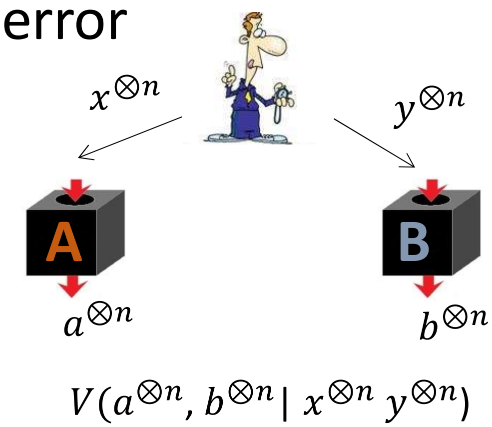

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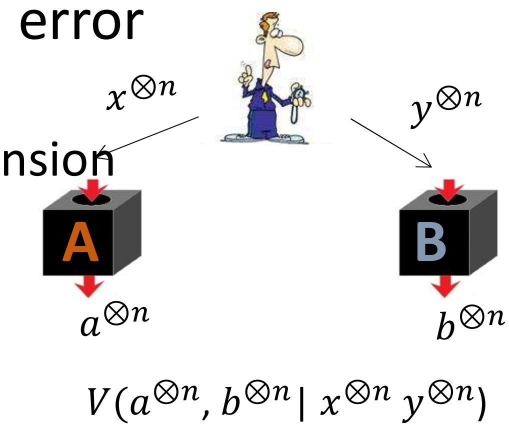

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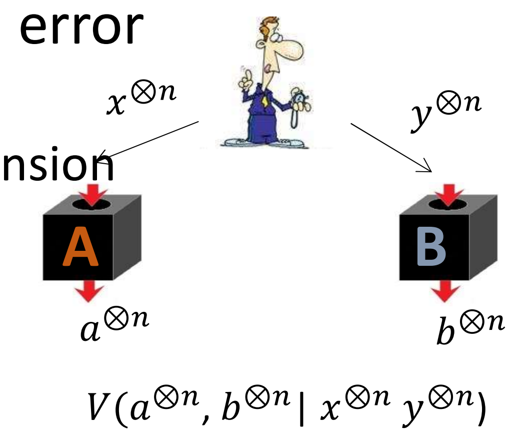

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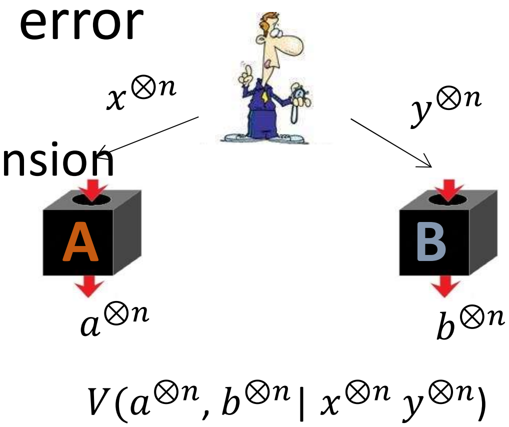
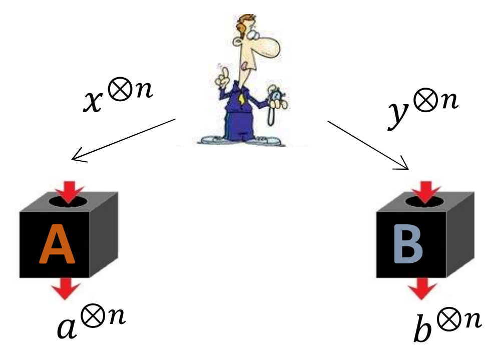


Proof Structure

Theorem A: Commutation and Anti-Commutation



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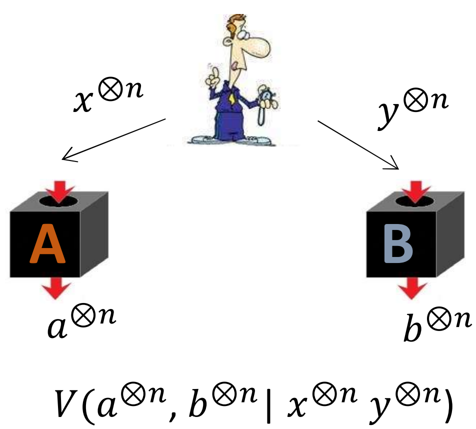
Theorem A: Commutation and Anti-Commutation

There exists a method for assembling Alice's projectors into unitaries $\tilde{A}^c_{r,k}$ (resp. $\tilde{B}^c_{r,k}$), for $k \in [n]$ such that:



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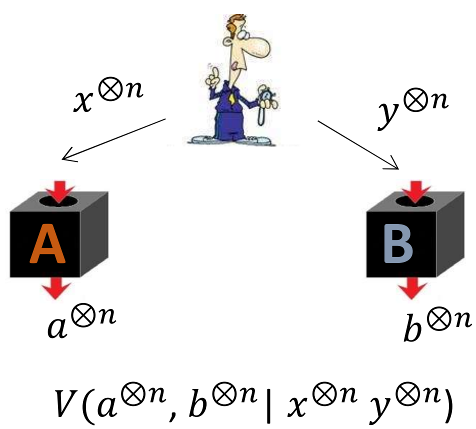
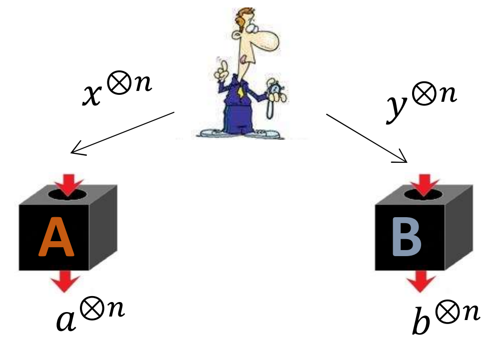
and

$$d_{\psi'}(\tilde{A}^c_{r,k}\tilde{A}^{c'}_{r',k'}, \tilde{A}^{c'}_{r',k'}\tilde{A}^c_{r,k}) \leq O(\sqrt{\epsilon})$$



Proof Structure

Theorem B: The Isometry



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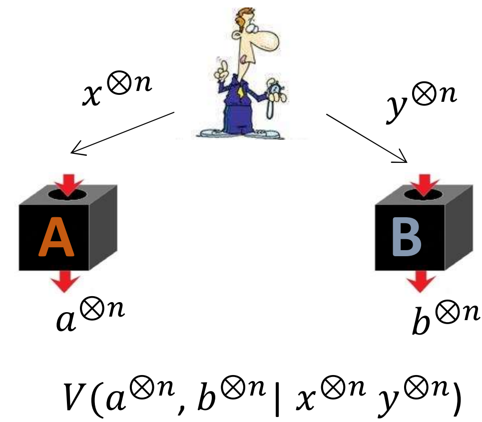

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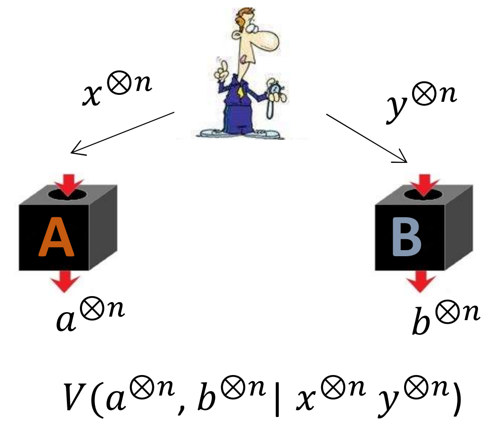

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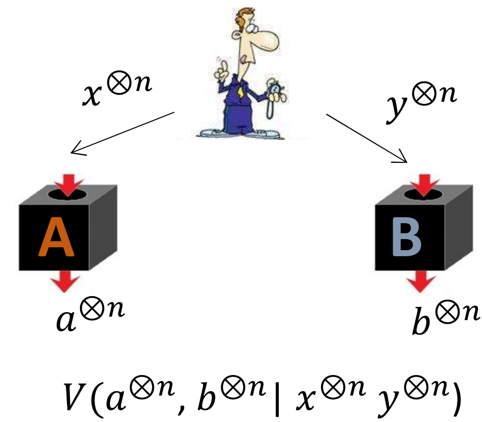

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- This type of isometry was pioneered in works of McKague [McKague16], [Wu, Bancal, McKague, Scarani 16]

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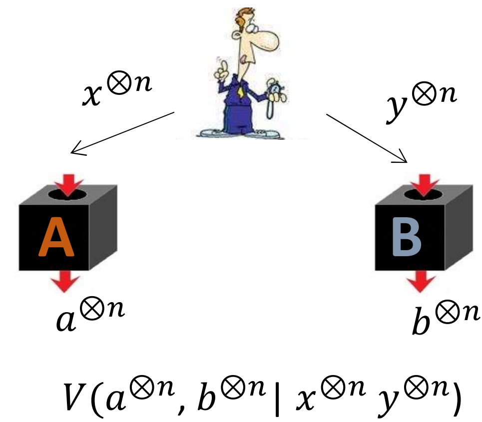

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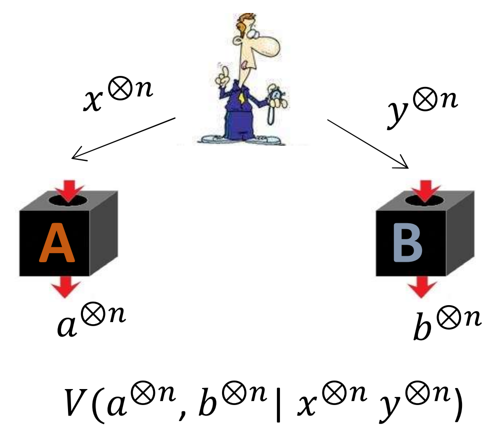
- This theorem overlaps with [Chao, Reichardt, Sutherland, Vidick 16]

Conclusion

- Rigidity theorem for the parallel repeated magic square game which:



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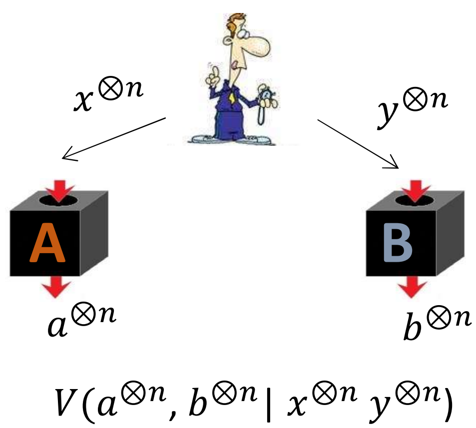

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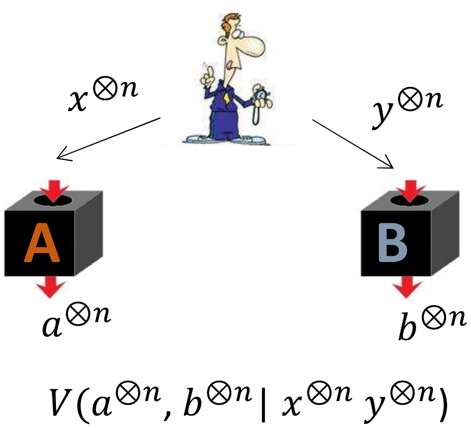

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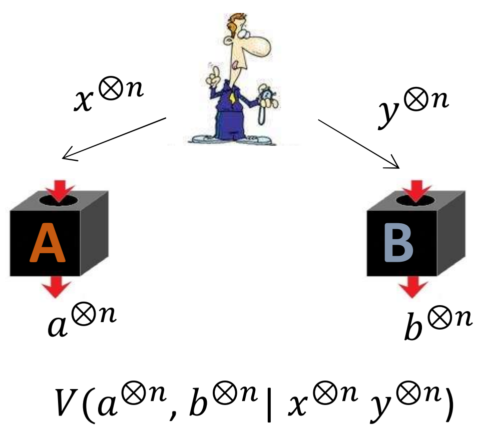

- Open Problems:

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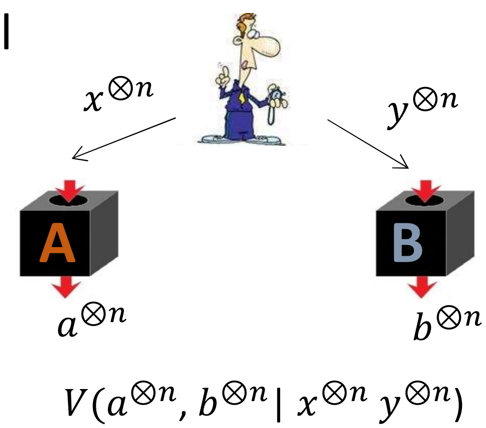

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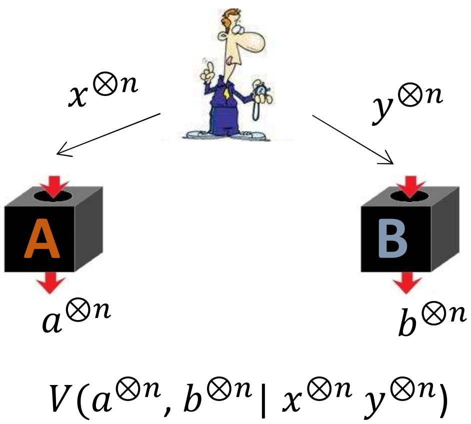

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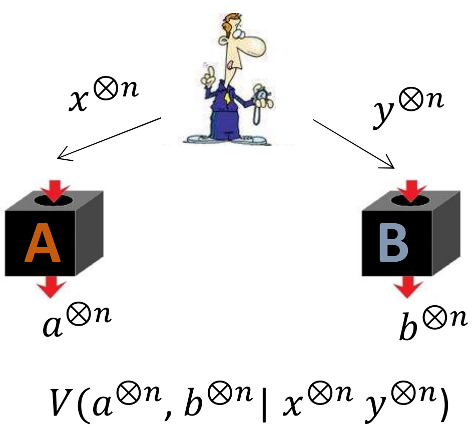

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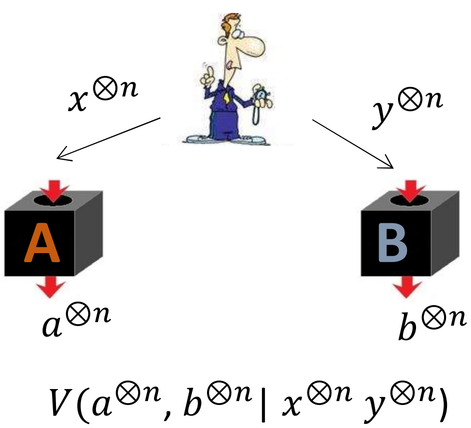

- Open Problems:
  - Reduce error dependence – [NV16]
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  - More applications to delegated quantum computation or interactive proofs for local Hamiltonian, randomness expansion.