

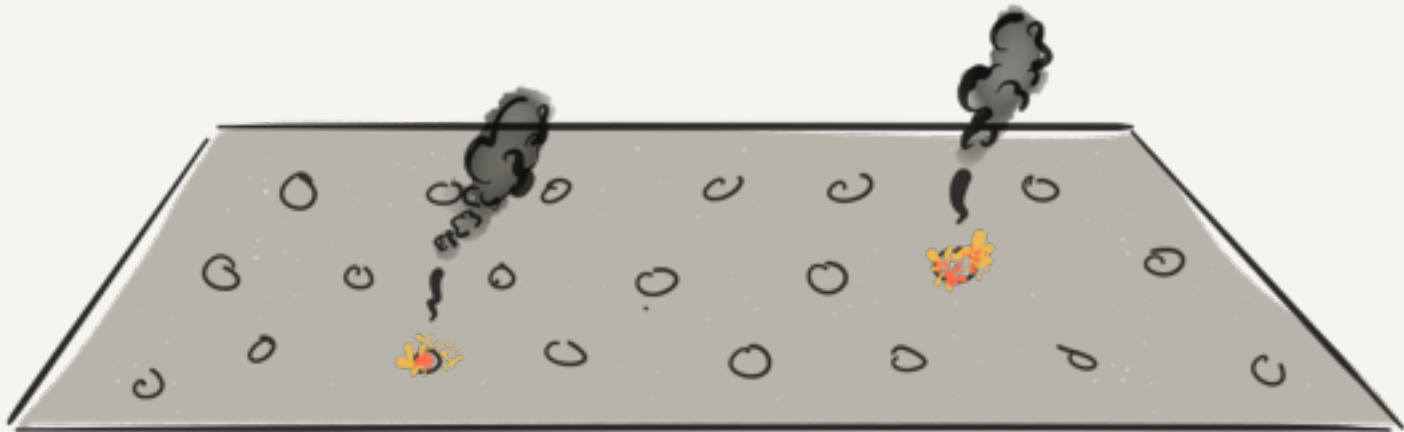
# TIME-CORRELATED NOISE IN QUANTUM COMPUTATION

Motivation

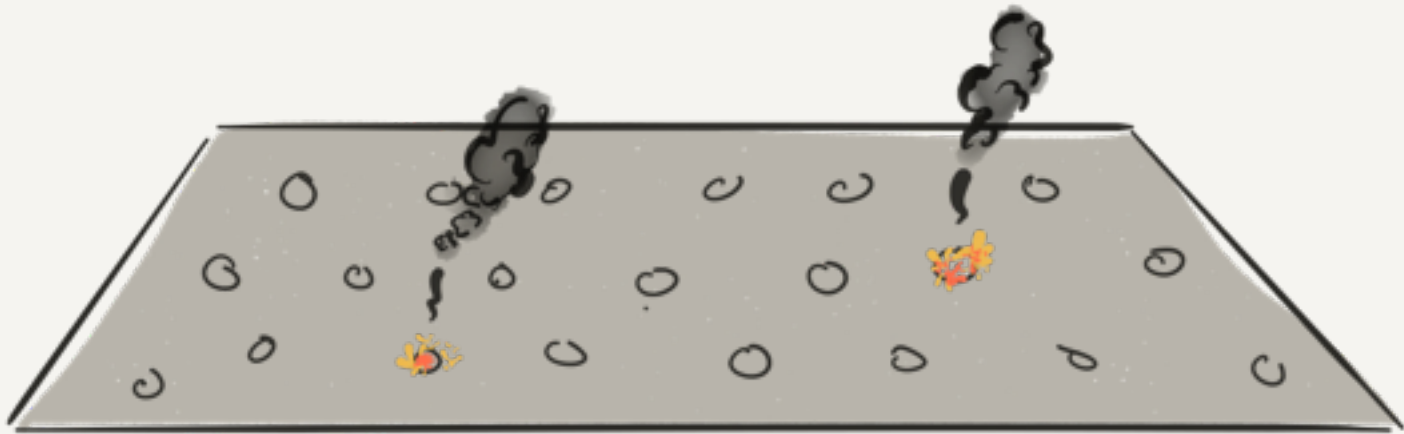
# Fault-tolerant computation

- computing requires isolation & control
- maybe no such qubits occur "naturally"
- fault-tolerance: generic approach
- noise has to be weak & weakly correlated  
in spacetime
- here: arbitrary correlations in time

# Fabrication faults

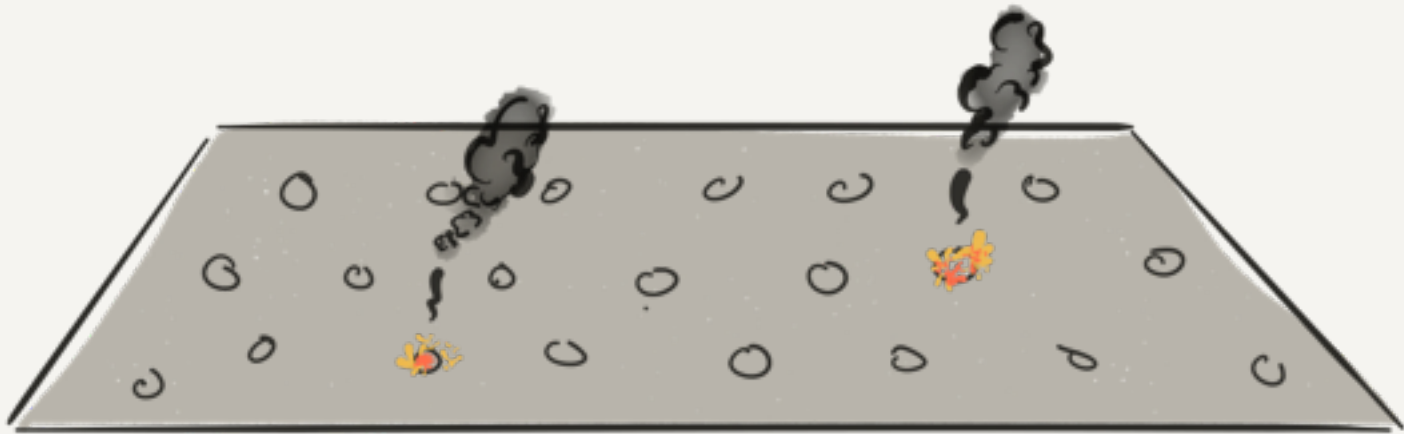


# Fabrication faults



fabrication faults: known / unknown  
operations: flexible / fixed

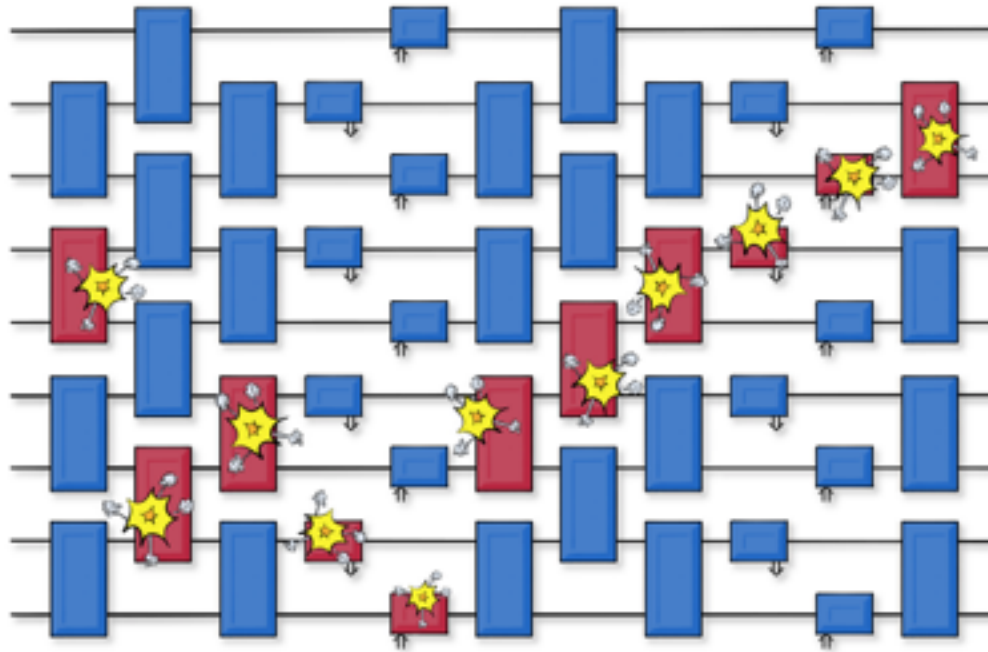
# Fabrication faults



fabrication faults:    known / unknown  
operations:            flexible / fixed

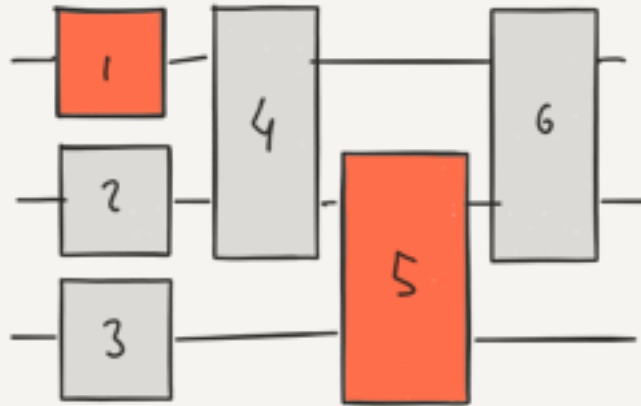
Noise model

# Stochastic noise





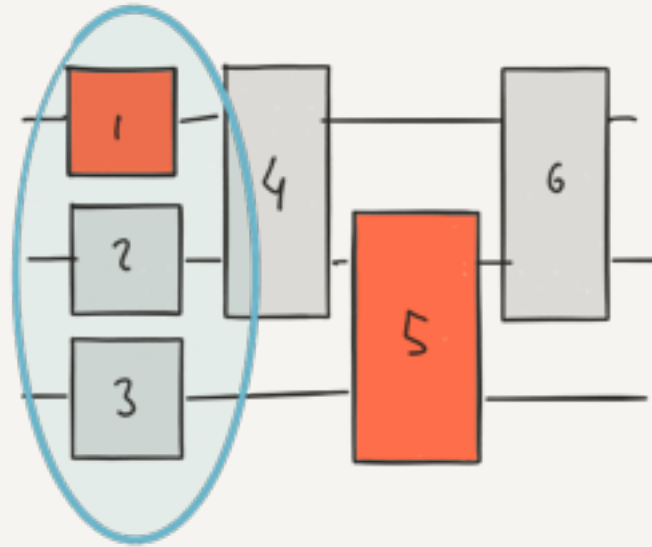
# Local stochastic noise



$$P(\text{1} \wedge \text{5}) \leq \lambda^2$$

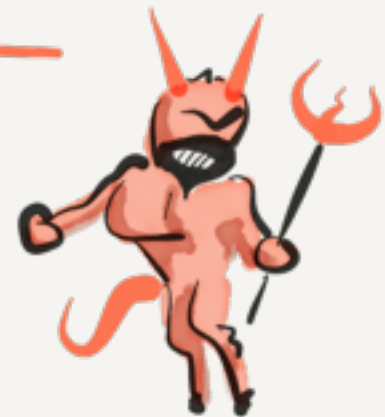


# Spatially local stochastic noise



~~$$P(\text{red} \wedge \text{red}) \leq \lambda^2$$~~

$$P(\text{red} \wedge \text{red}) \leq \lambda^2$$



Quantum memories based on  
single-shot error correction  
exhibit an error threshold under  
spatially local stochastic noise

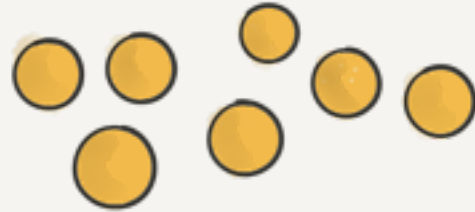
# Single-shot error correction

# Error correction

logical qubit

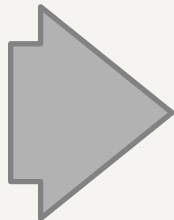


physical qubits

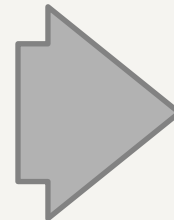


extra d.o.f.  $\rightarrow$  check ops 

syndrome  
extraction

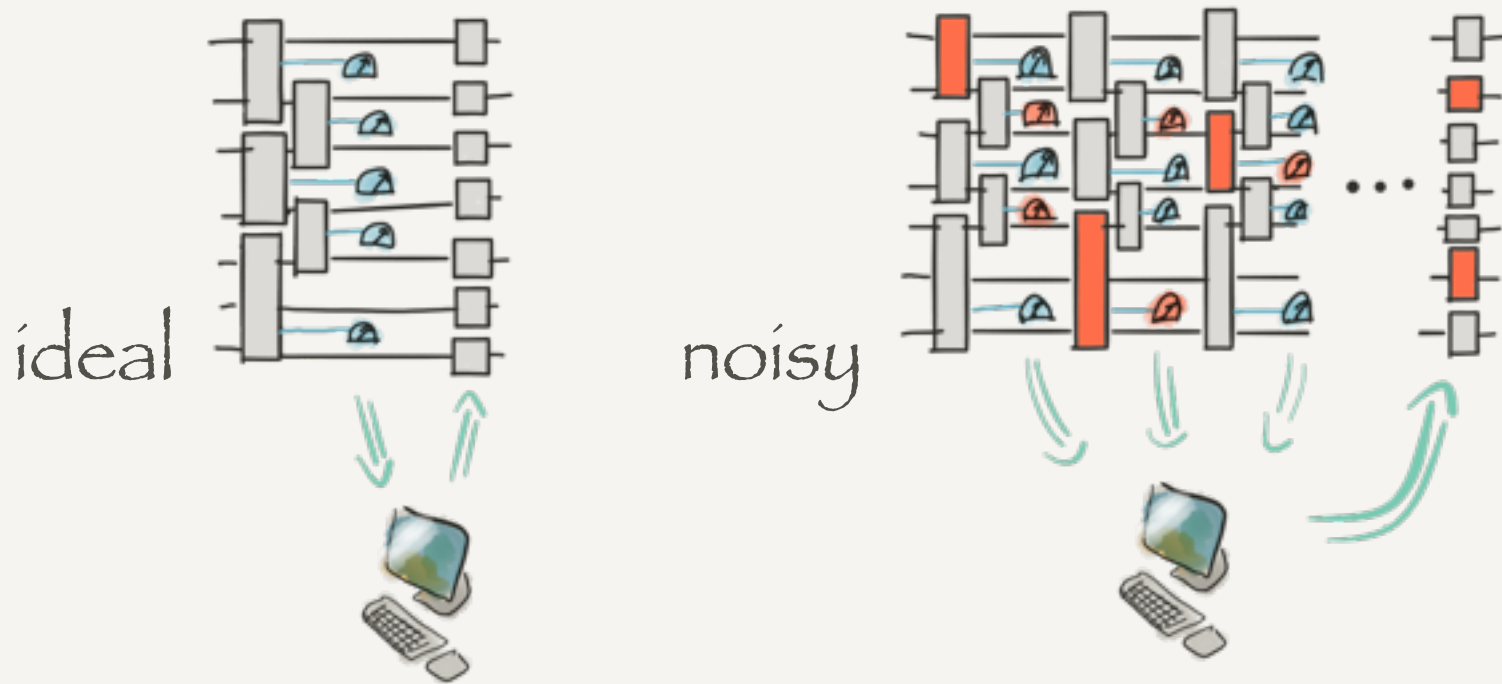


decoding

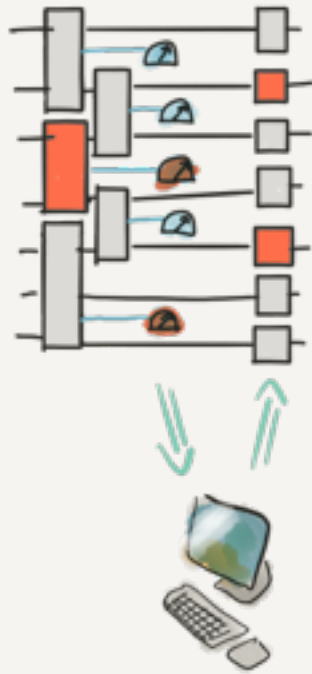


correction

# Error correction



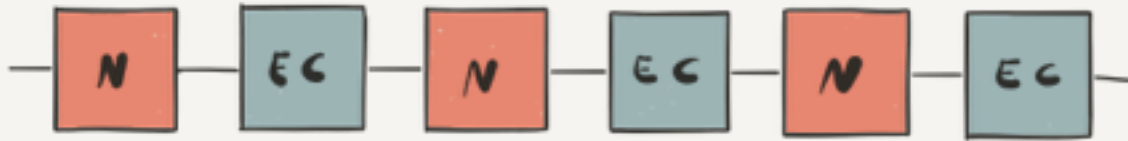
# Error correction



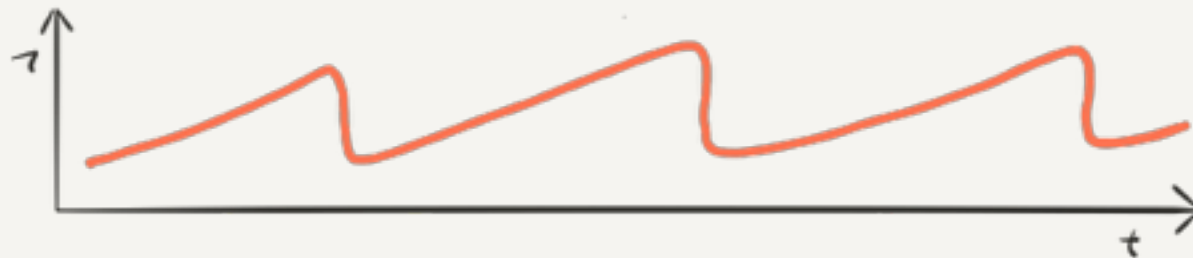
single-shot if  
quantum-local  
(analogous to LOCC)



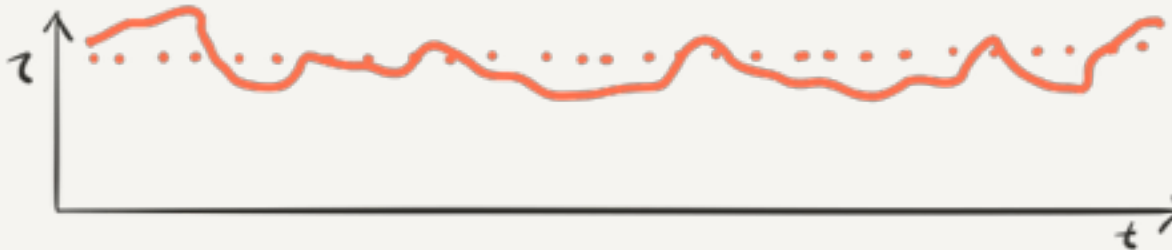
# Error correction



ideal



noisy

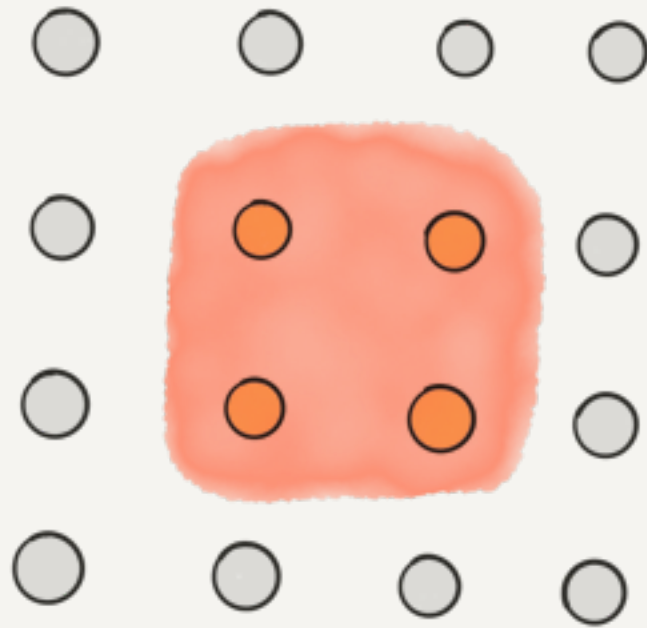


other



Topological codes

# Topological codes



local check  
operators

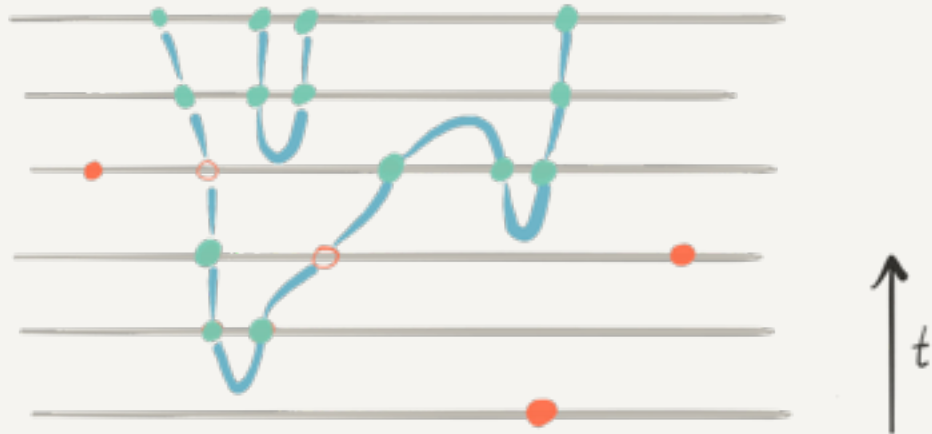


local  
indistinguishability

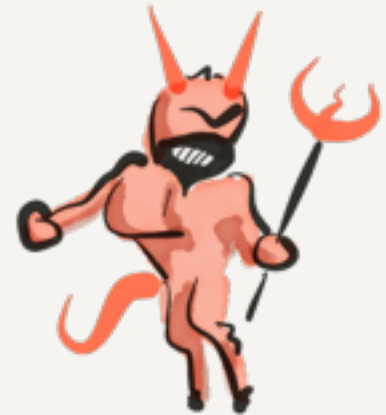
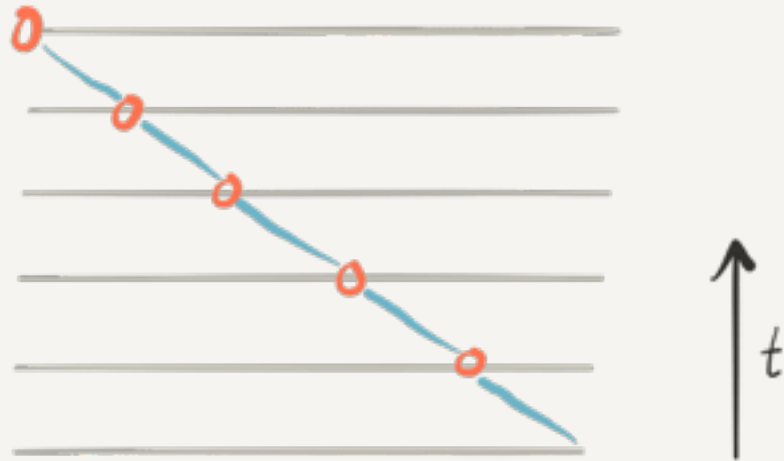
# 2D codes



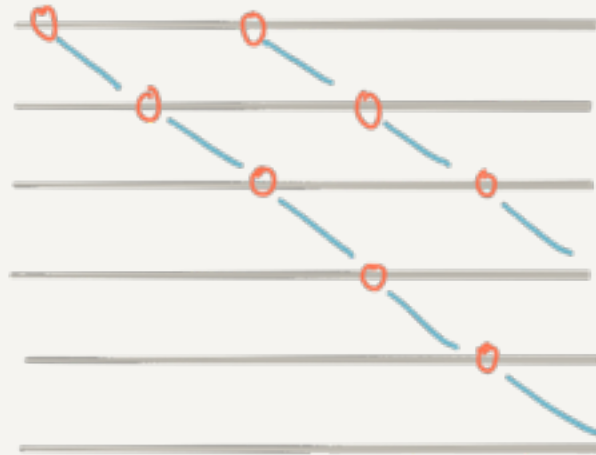
errors: strings  
syndrome: endpoints



# 2D codes



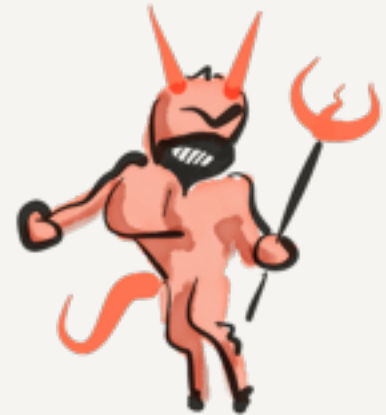
# 2D codes



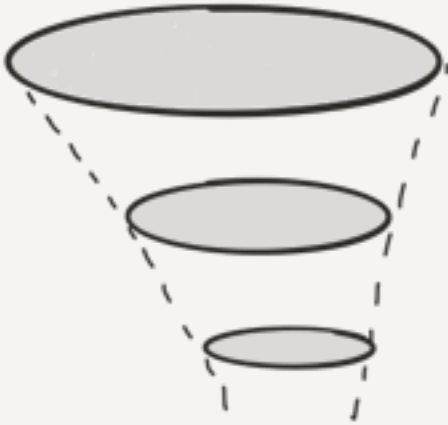
$$P(\text{---}) = \lambda$$

Spatially local (& Markovian), e.g.

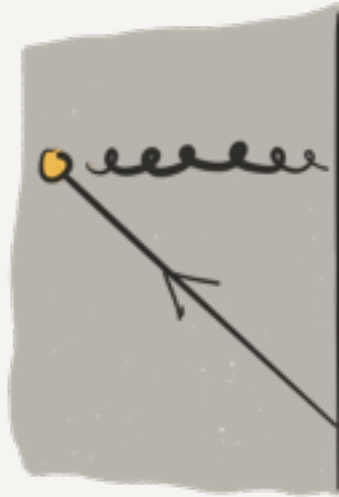
$$P(x_{i,t} \wedge x_{i,t}) = \lambda^2$$



# Single-shot codes



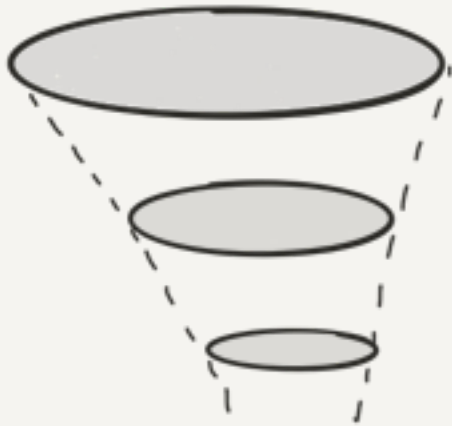
$$D = 4$$



$$D = 3$$



# 4D codes



$$P(\text{loop}) \leq \lambda^l$$

errors: membranes  
syndrome: loops

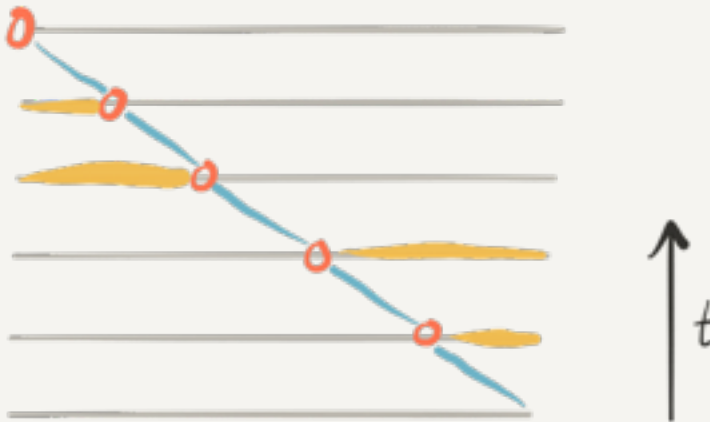


# 3D codes

errors: strings

syndrome: endpoints

$$P(\text{error}) \leq \lambda^e$$



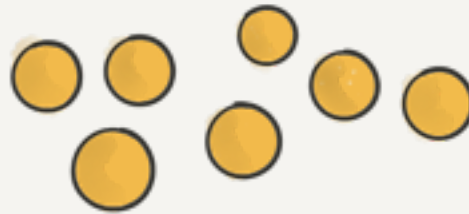


# Subsystem codes

logical qubit



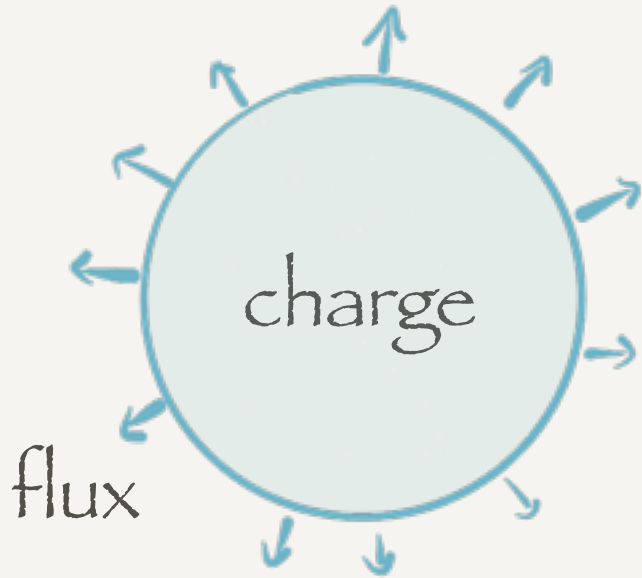
physical qubits



extra d.o.f. {  
check ops  
gauge d.o.f.

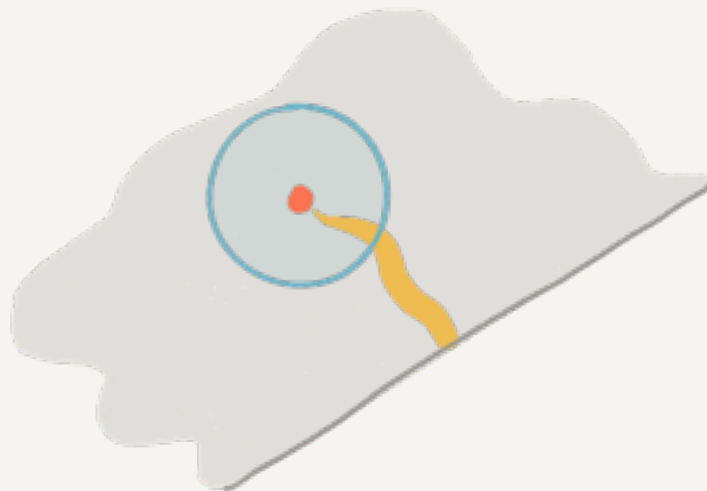


# Gauss law

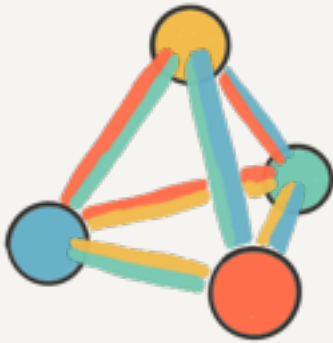


charge → error  
syndrome

field → gauge  
syndrome



# 3D gauge color codes

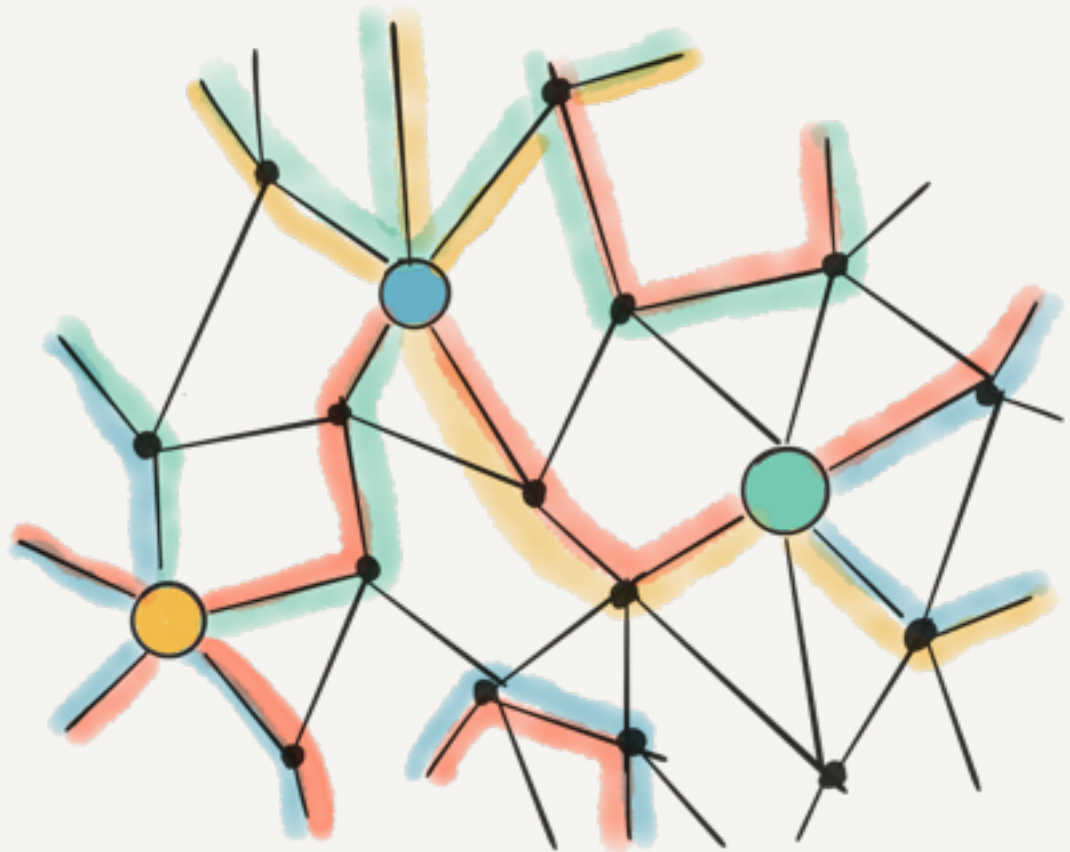


tetrahedron  $\approx$  qubit

edge  $\approx$  gauge op

vertex  $\approx$  check op

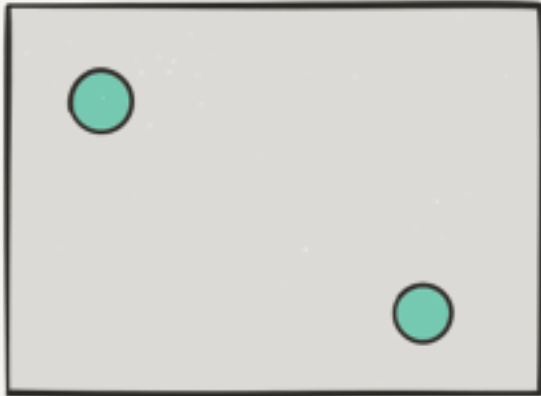
X & Z type



# Confinement



check ops



unconfined



gauge ops



confined!

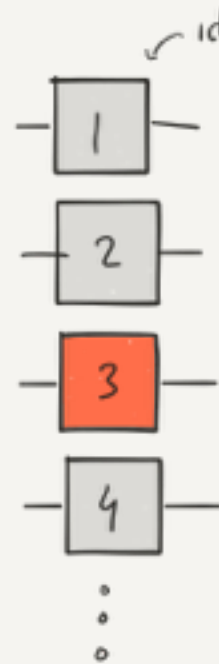
Result

# Quantum memory



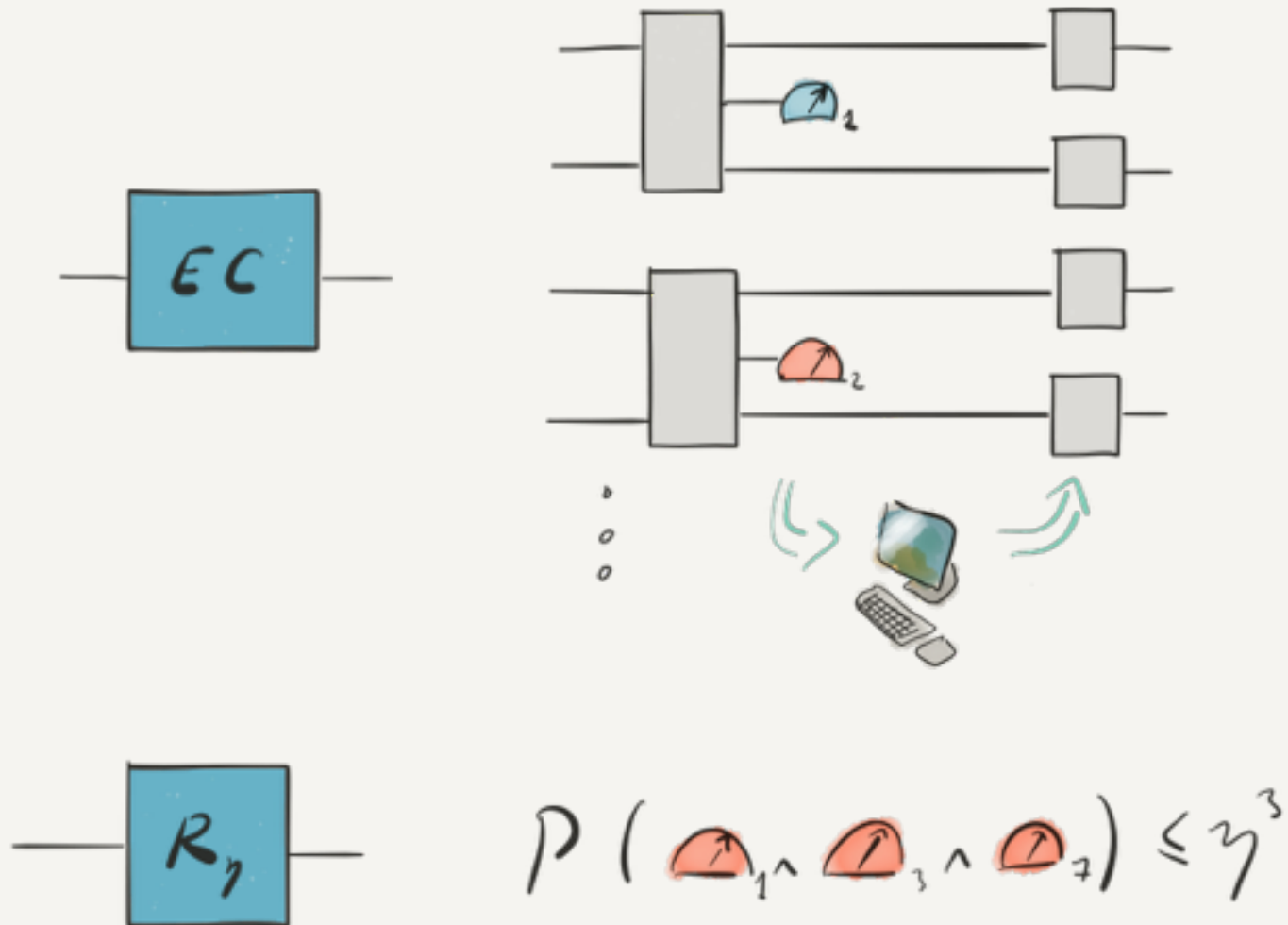
Perfect encoding and decoding to test the  
quality of the quantum memory:  
alternated noise and noisy error correction

# Noise



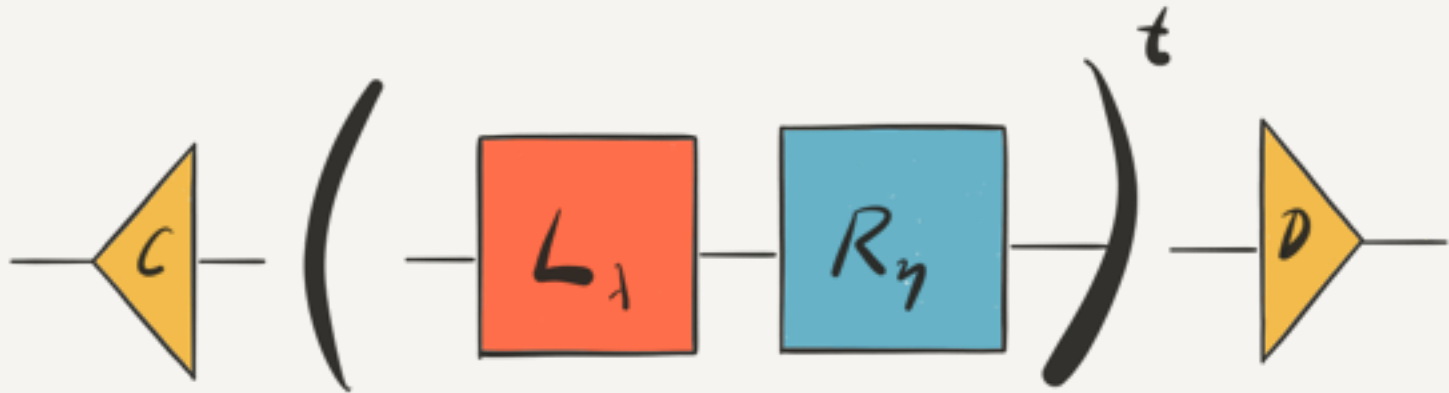
$$P( \boxed{1} \wedge \boxed{3} \wedge \boxed{7} ) \leq \lambda^3$$

# Noisy error correction





# Quantum memory



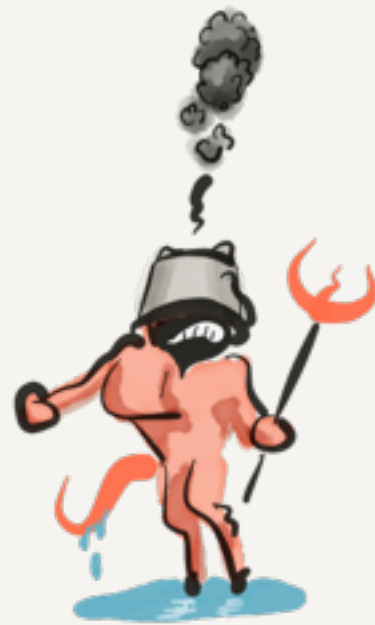
For error rates below a threshold

$$p(\text{error}) \leq at + b$$

where  $a$  &  $b$  decrease exponentially with the system size.

# DISCUSSION

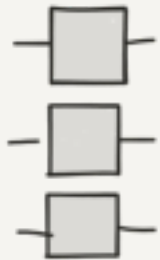
- Universal computation probably straightforward
- $D < 3$
- Known fabrication faults
- Fully local (CA) error correction
- The physics of gauge color codes. Gapless phases? Confinement?



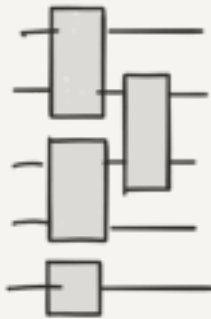


# Local operations

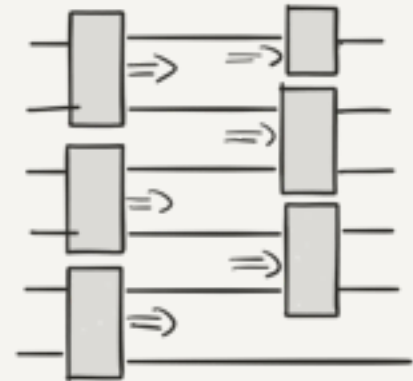
transversal



local



quantum-local




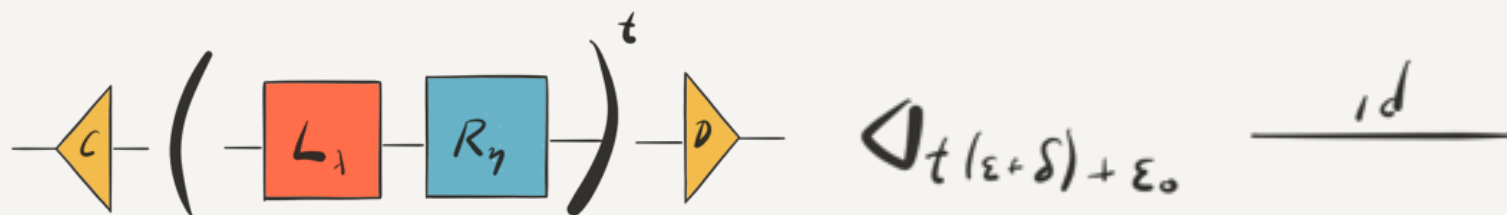
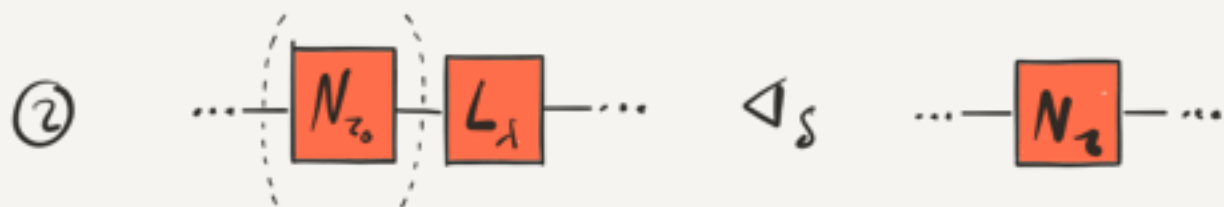
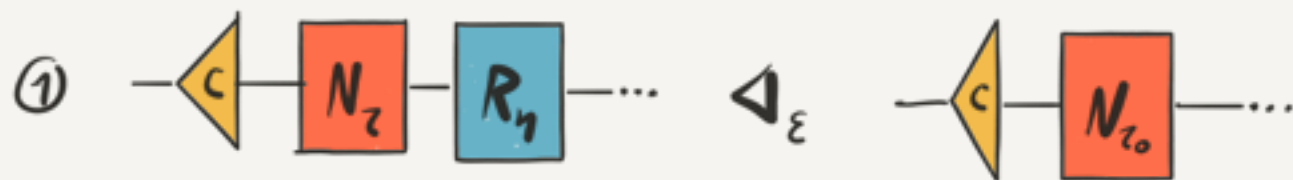
Not universal

not universal?

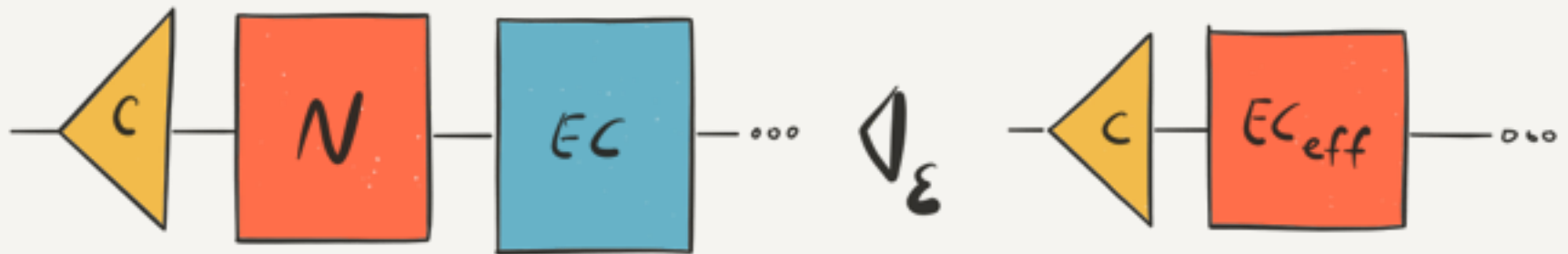
• skull and crossbones icon fault tolerance!

• Universal + EC

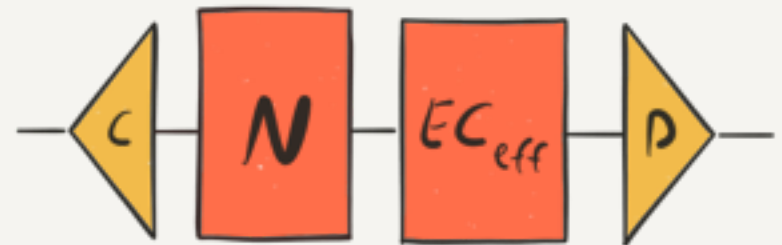
but  + EC = universal

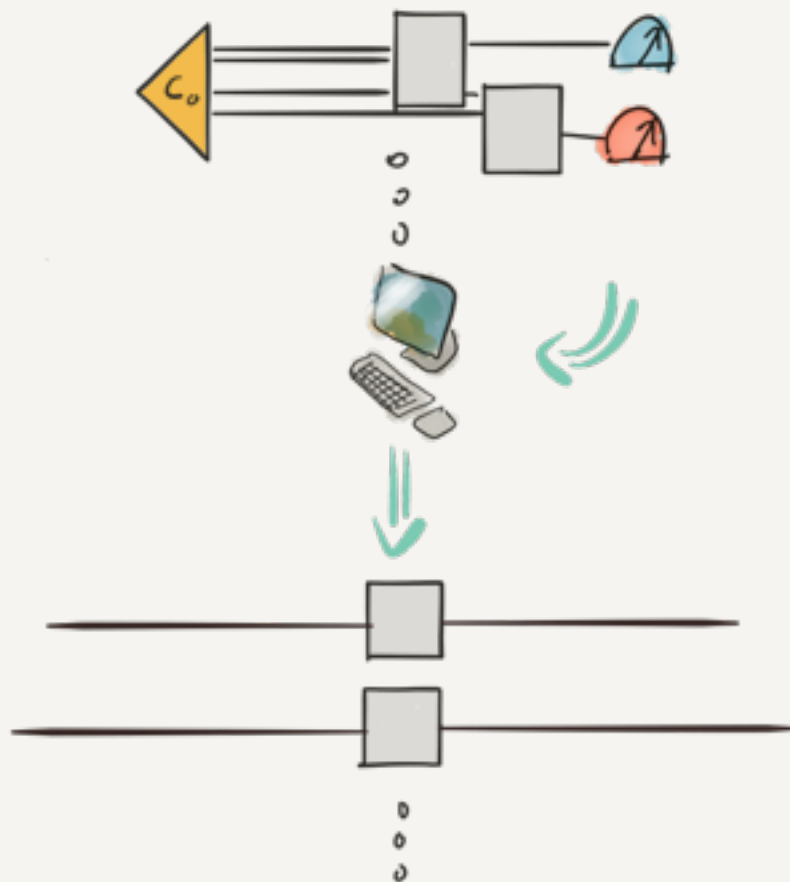
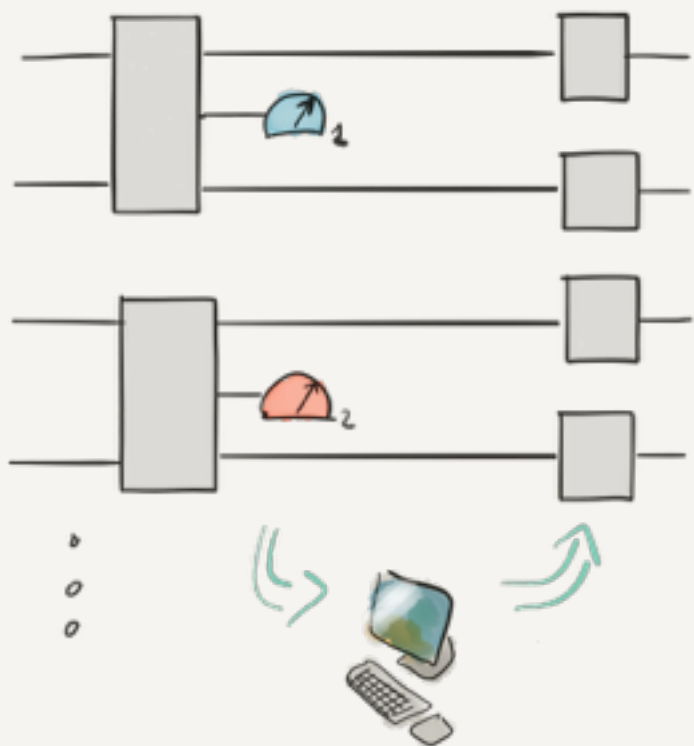


Th.



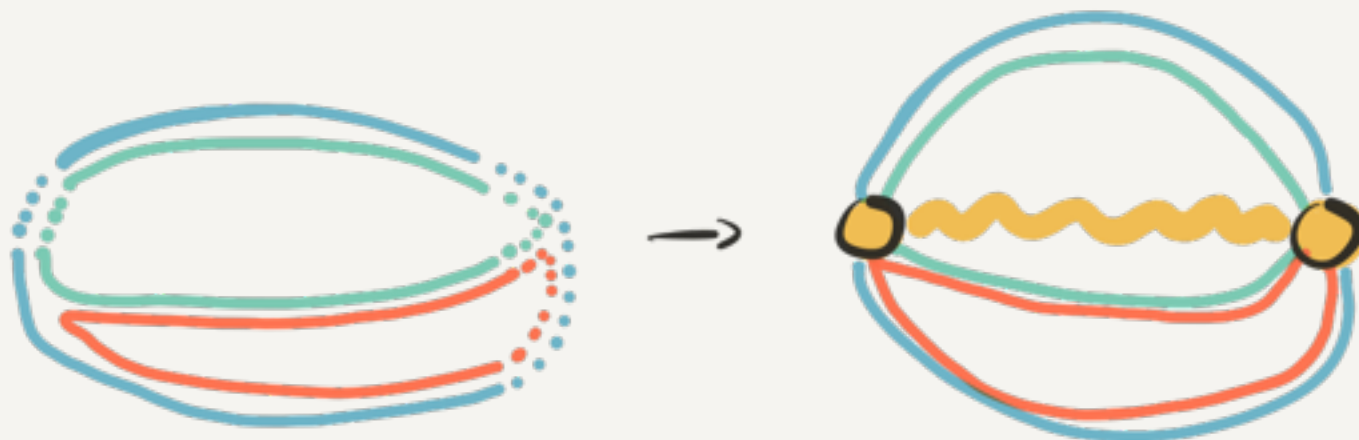
with  $\varepsilon$  the error rate of



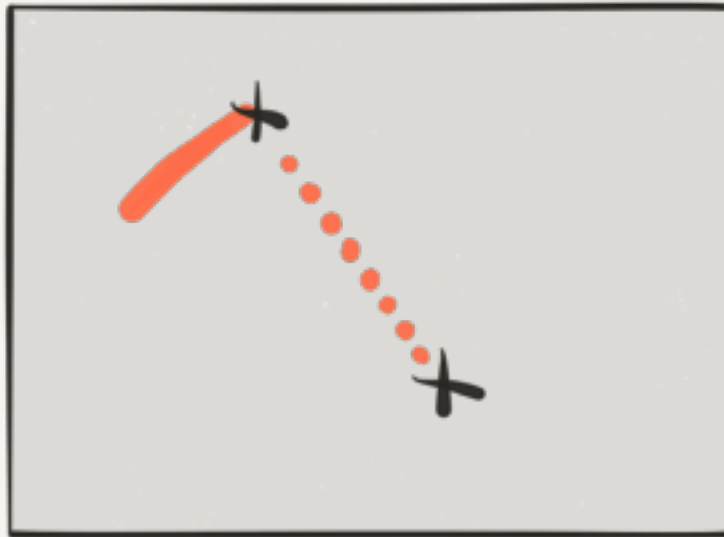




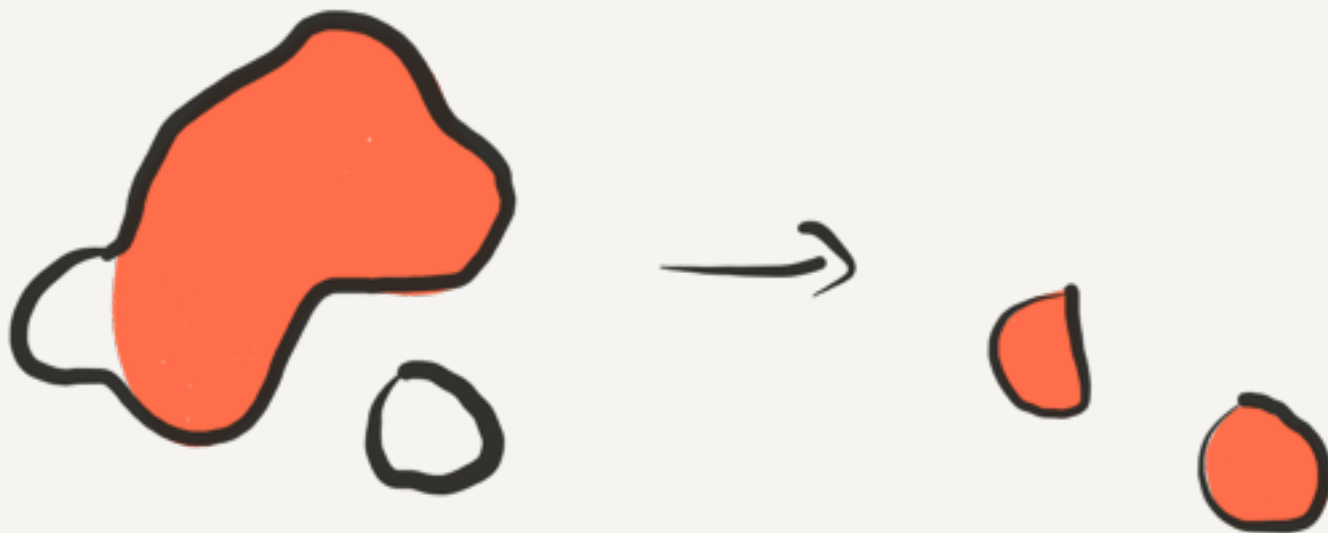
$$- \boxed{E_{\text{eff}}} - \subseteq - \boxed{L_\lambda} -$$



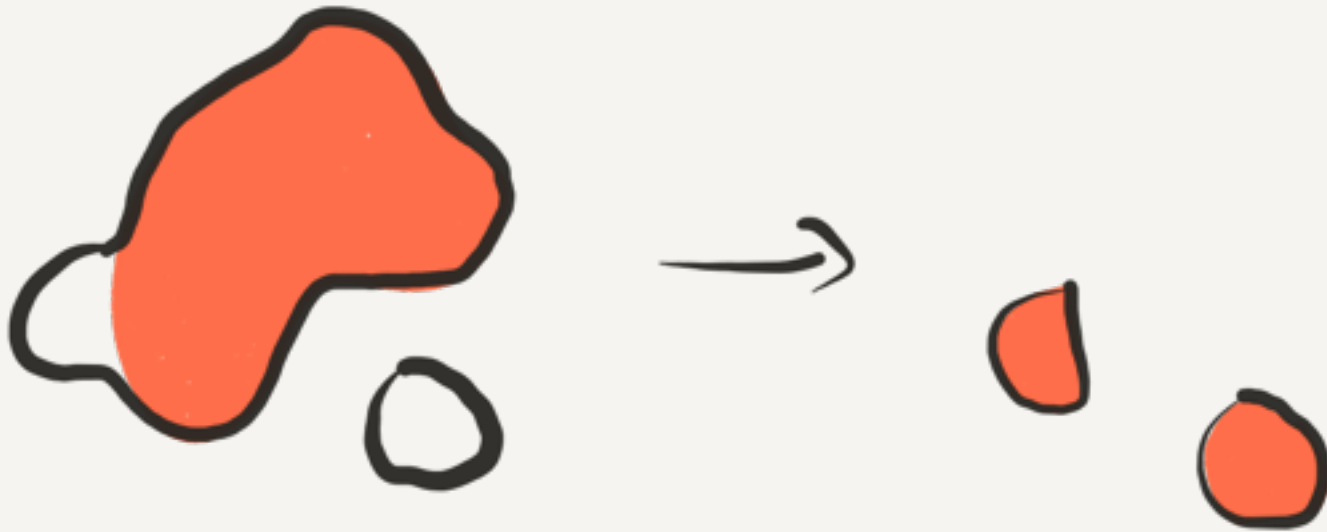
$$d=2$$



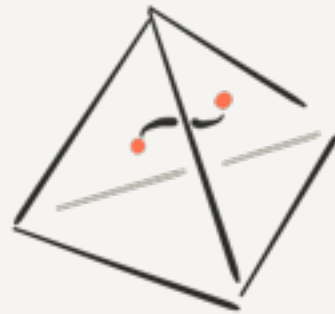
$$d \approx 4$$



$$d \approx 4$$



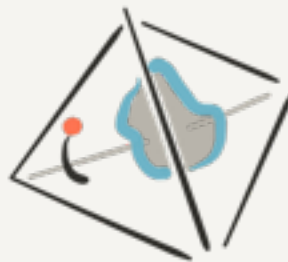
localized measurement errors yield  
localized residual noise



$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{i} \end{bmatrix}$$



$$d=2$$

$$\mathbb{Z}_2^6$$

