QUANTUM HOMOMORPHIC ENCRYPTION FOR POLYNOMIAL-SIZED CIRCUITS

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(joint work with Yfke Dulek and Christian Schaffner)

http://arxiv.org/abs/1603.09717

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QIP 2017, Seattle, Washington, Monday 16 January 2017

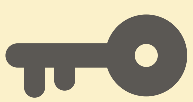
EXAMPLE: IMAGE TAGGING

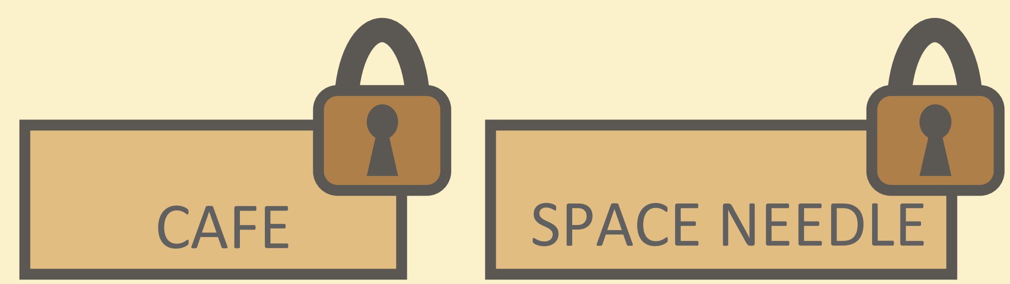
Classical homomorphic encryption: Gentry [2009]

What about quantum?

CAFE

SPACE NEEDLE





1. HOMOMORPHIC ENCRYPTION
2. PREVIOUS RESULTS: CLIFFORD SCHEME
3. NEW SCHEME

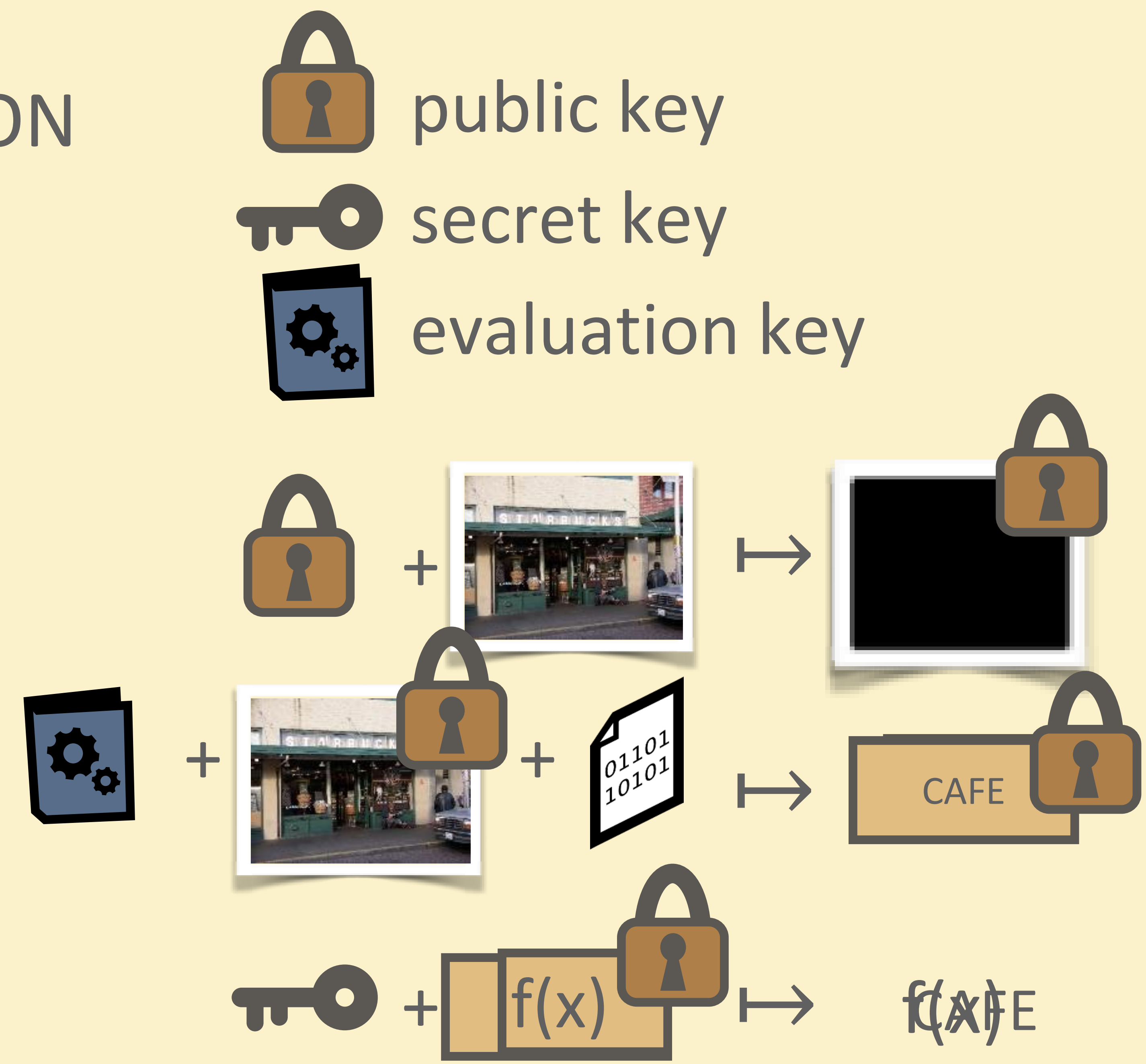
HOMOMORPHIC ENCRYPTION

KEY GENERATION

ENCRYPTION
(secure)

EVALUATION

DECRYPTION



Classical homomorphic encryption: Gentry [2009]

HOMOMORPHIC ENCRYPTION

KEY GENERATION
- public key
- secret key
- quantum ⇒ evaluation key

ENCRYPTION (secure): public key + $|\psi\rangle \mapsto$ encrypted $|\psi\rangle$

EVALUATION: evaluation key + encrypted $|\psi\rangle$ + 01101 10101 $\mapsto$ encrypted $U|\psi\rangle$

DECRYPTION: secret key + encrypted $U|\psi\rangle \mapsto U|\psi\rangle$

✓ HOMOMORPHIC ENCRYPTION

2. PREVIOUS RESULTS: CLIFFORD SCHEME

3. NEW SCHEME

QUANTUM HOMOMORPHIC ENC

	homomorphic for	compactness	security
Not encrypting	Quantum circuits	yes	no
Quantum OTP	no	yes	yes
append evaluation description	Quantum circuits	Complexity of Dec prop to (# gates)	yes
Clifford Scheme	Clifford circuits	yes	computational

Quantum one-time pad:

pick a,b $\in_R$ {0,1}

$|\psi\rangle \mapsto X^a Z^b |\psi\rangle$

THE CLIFFORD GROUP

Generated by $\{H, P, \text{CNOT}\}$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Commutation maps Pauli operators to Paulis (normalizer of Pauli group)

$$HX = ZH \qquad PZ = ZP$$

$$HZ = XH \qquad PX = XZP$$

$$\text{CNOT}(X \otimes I) = (X \otimes X)\text{CNOT}$$
$$\text{CNOT}(I \otimes Z) = (Z \otimes Z)\text{CNOT}$$

Not a universal gate set

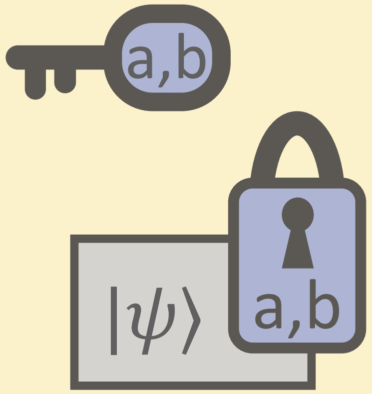
(e.g. efficient classical simulation possible)

CLIFFORD SCHEME

Ingredient 1: quantum one-time pad

encryption: pick a,b $\in_R$ {0,1}

$|\psi\rangle \mapsto X^a Z^b |\psi\rangle$ =

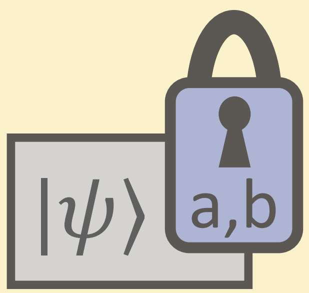


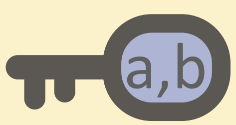
decryption: $X^a Z^b |\psi\rangle \mapsto |\psi\rangle$

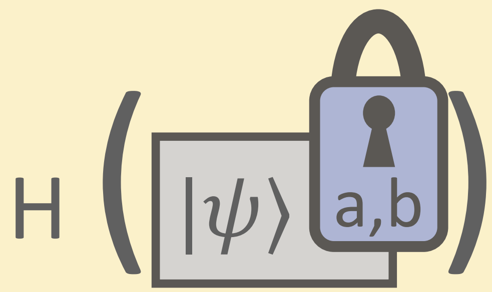
Ingredient 2: classical homomorphic encryption (as black box)

[AMTW00] A. Ambainis, M. Mosca, A. Tapp, and R. De Wolf. Private quantum channels. FOCS'00
[Gentry 09] C. Gentry: Fully homomorphic encryption using ideal lattices. STOC'09

CLIFFORD SCHEME







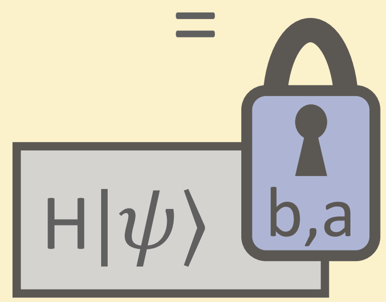
$$=$$

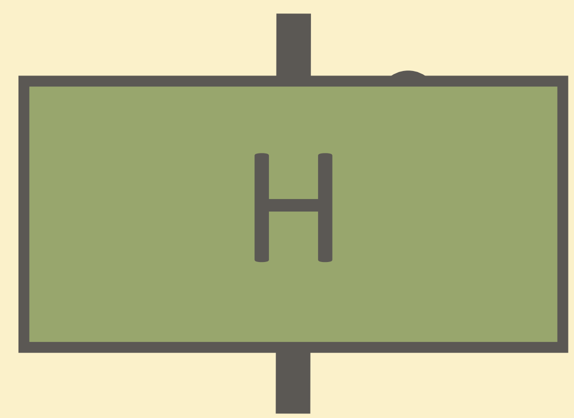
$$HX^aZ^b|\psi\rangle$$

$$=$$

$$X^bZ^aH|\psi\rangle$$

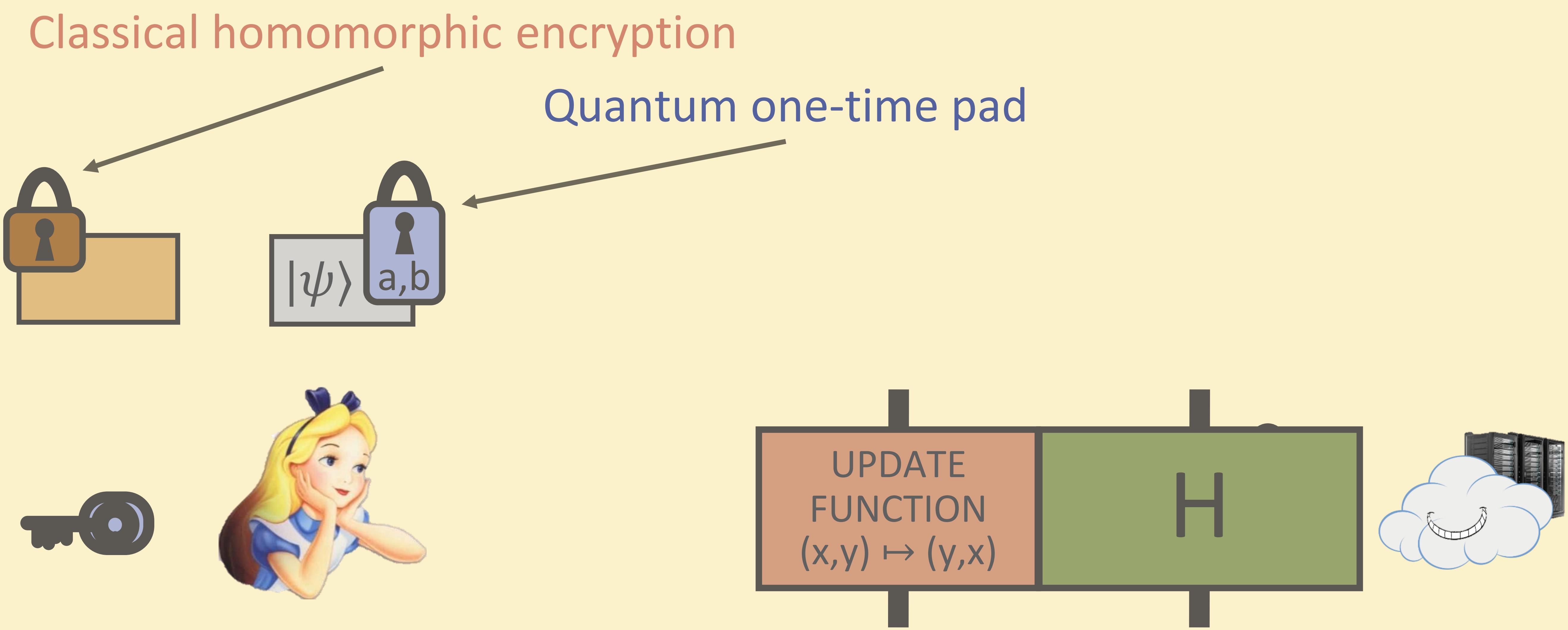
$$=$$





Folklore, last formalized by [BJ15] A. Broadbent, S. Jeffery. Quantum Homomorphic Encryption for Circuits of Low T-gate Complexity. CRYPTO 2015

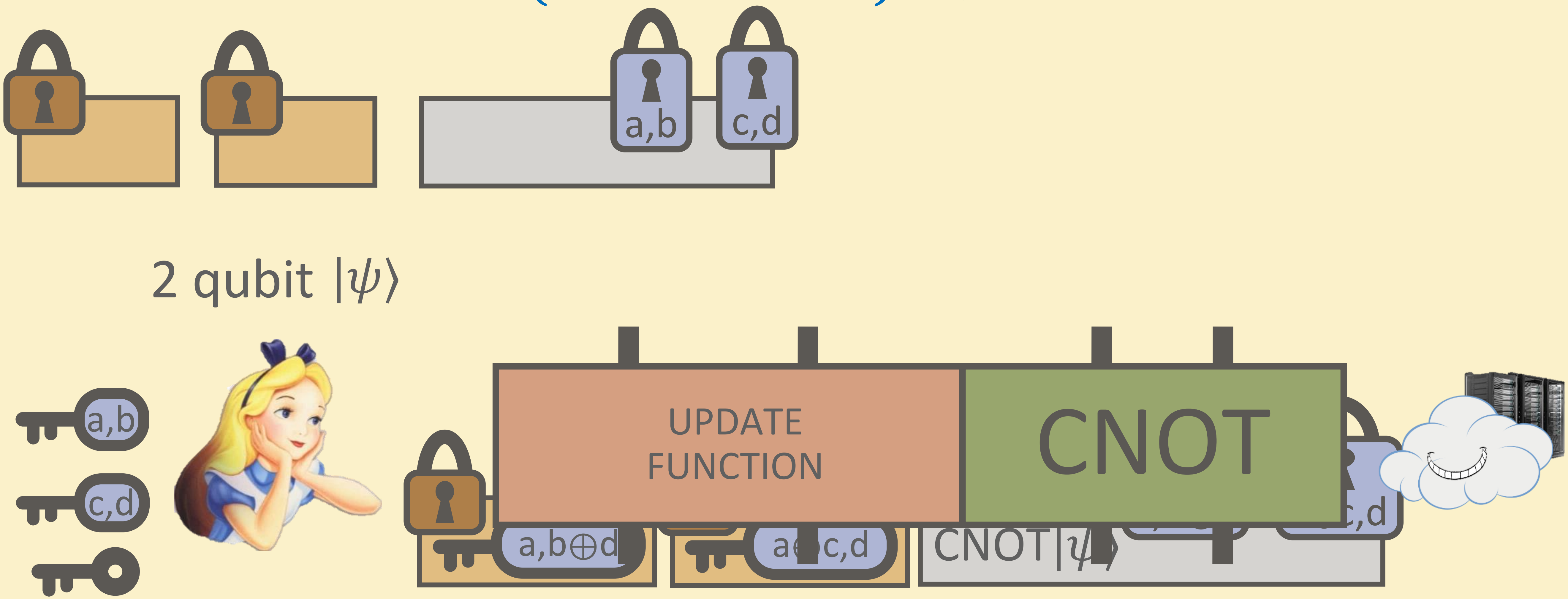
CLIFFORD SCHEME



Folklore, last formalized by [BJ15] A. Broadbent, S. Jeffery. Quantum Homomorphic Encryption for Circuits of Low T-gate Complexity. CRYPTO 2015

CLIFFORD SCHEME: CNOT

$$(X^aZ^b \otimes X^cZ^d)|\psi\rangle$$



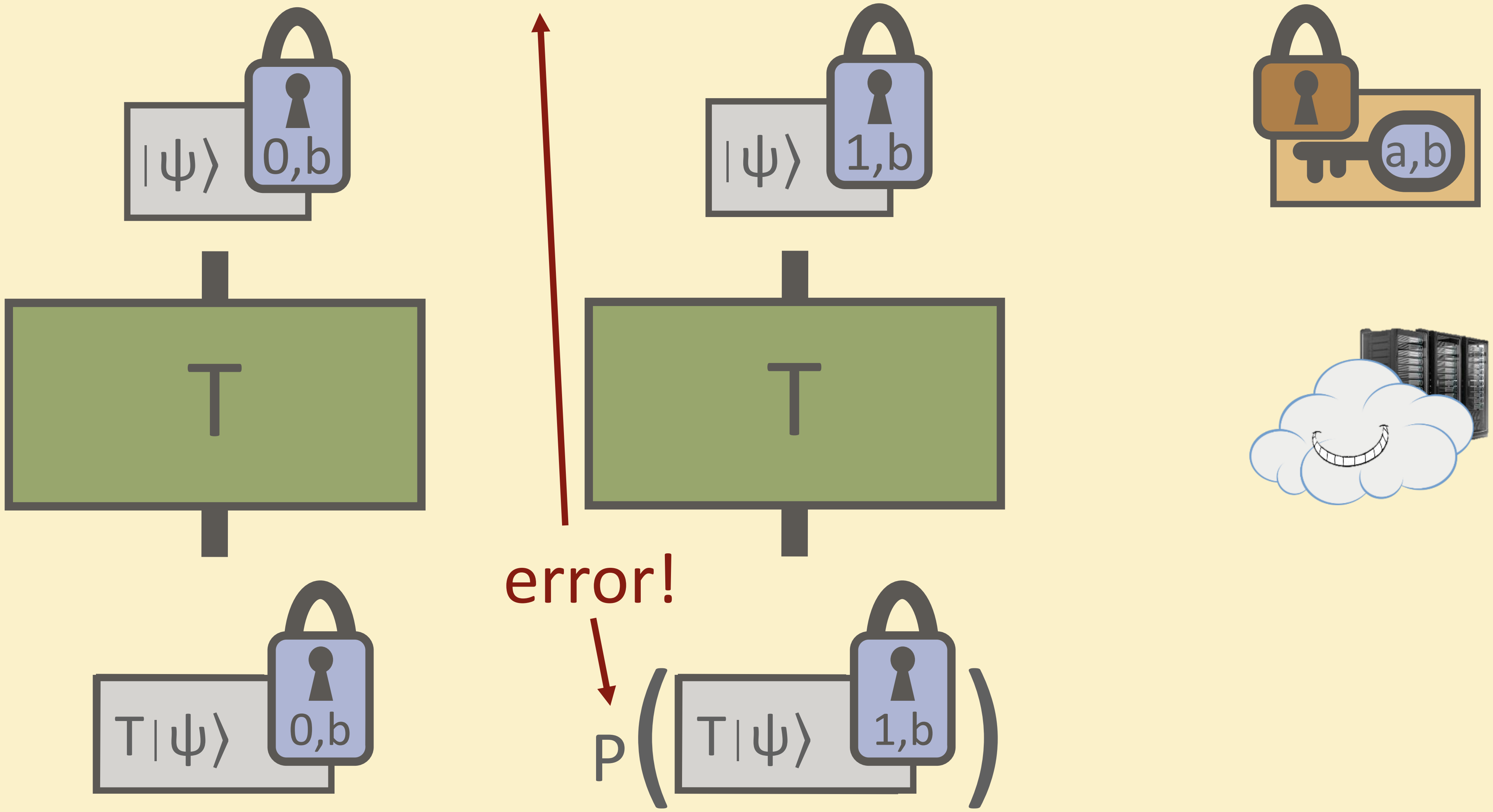
Folklore, last formalized by [BJ15] A. Broadbent, S. Jeffery. Quantum Homomorphic Encryption for Circuits of Low T-gate Complexity. CRYPTO 2015

THE CHALLENGE: T GATE

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

TZ=ZT

TX=PXT



how to apply correction P^{-1} iff a = 1?

PREVIOUS RESULTS: OVERVIEW

	homomorphic for	compactness	security
Not encrypting	Quantum circuits	yes	no
Quantum OTP	No	yes	inf theoretic
append evaluation description	Quantum circuits	Complexity of Dec prop to (# gates)	yes
Clifford Scheme	Clifford circuits	yes	computational
[BJ15]: AUX	QCircuits with constant T-depth	yes	computational
[BJ15]: EPR	Quantum circuits	Comp of Dec is prop to (#T-gates)^2	computational
[OTF15]	QCircuits with constant #T-gates	yes	inf theoretic
Our result	QCircuits of polynomial size (levelled FHE)	yes	computational

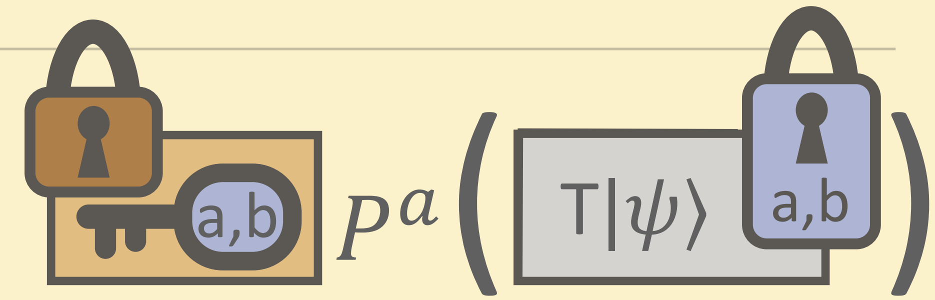
(comparison based on Stacey Jeffery's slides)

[BJ15] A. Broadbent, S. Jeffery. Quantum Homomorphic Encryption for Circuits of Low T-gate Complexity. CRYPTO 2015
[OTF15] Y. Ouyang, S-H. Tan, J. Fitzsimons. Quantum homomorphic encryption from quantum codes. arxiv:1508.00938

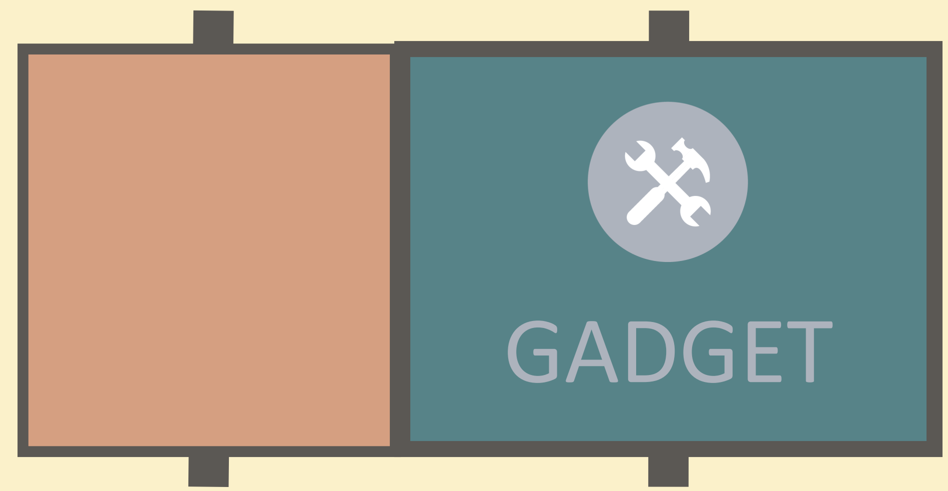
- ✓ HOMOMORPHIC ENCRYPTION
- ✓ PREVIOUS RESULTS: CLIFFORD SCHEME

3. NEW SCHEME

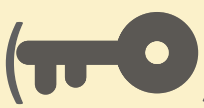
ERROR-CORRECTION 'GADGET'



- Build a 'gadget' that applies P^{-1} iff a = 1



- Apply correction iff :

  a=decrypt(, ) = 1

Properties:

- Efficiently constructable
- Destroyed after single use

EXCURSION

Theoretical Computer Science: Barrington's Theorem

PERMUTATION BRANCHING PROGRAM

$f : \{0,1\}^n \times \{0,1\}^m \to \{0,1\}$

- computes some Boolean function f(x,y)
- list of instructions: permutations of {1,2,3,4,5}

x_i	0: $\pi \in S_5$
	1: $\sigma \in S_5$
y_j	0: $\pi' \in S_5$
	1: $\sigma' \in S_5$
x_k	0: $\pi'' \in S_5$
	1: $\sigma'' \in S_5$

⋮

output: $\ldots \circ \sigma'' \circ \sigma' \circ \pi$

- id $\Rightarrow f(x,y) = 0$
- (fixed) cycle $\Rightarrow f(x,y) = 1$

length: # of instructions

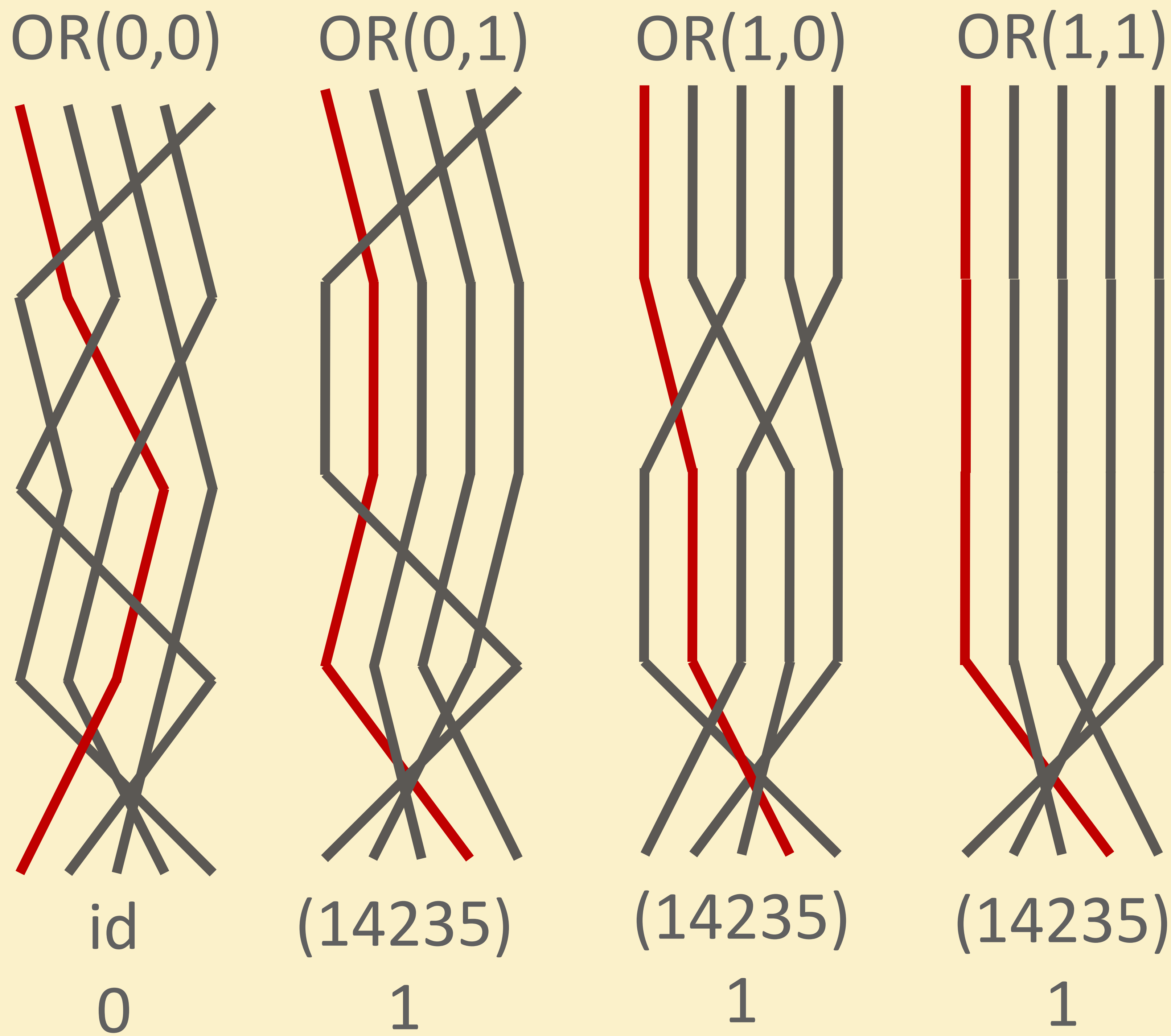
EXAMPLE PBP OR(x,y)

x	y	OR(x,y)
0	0	0
0	1	1
1	0	1
1	1	1

length 4:

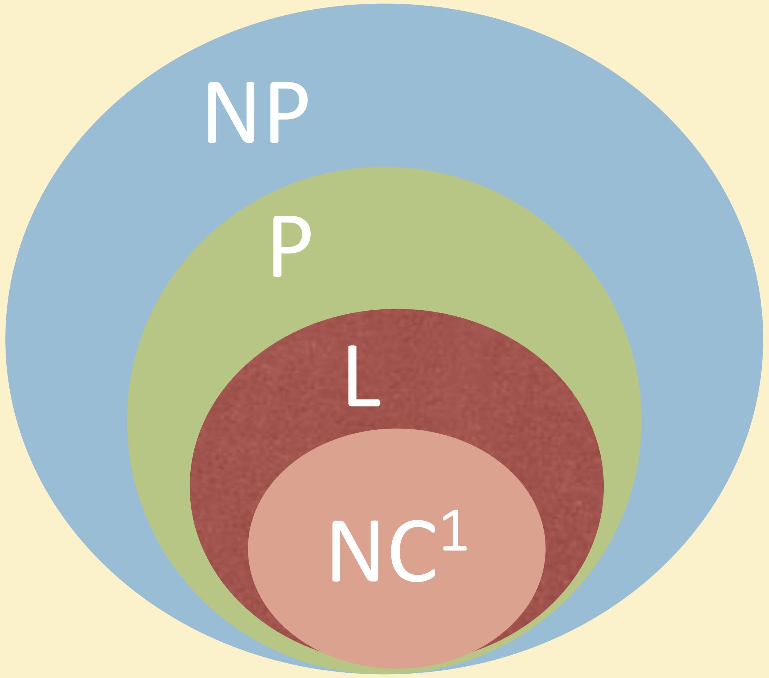
x	0: (12345)
	1: id
y	0: (12453)
	1: id
x	0: (54321)
	1: id
y	0: (15243)
	1: (14235)

output:



BARRINGTON'S THEOREM (1989)

Theorem (variation): if $f : \{0,1\}^n \times \{0,1\}^m \rightarrow \{0,1\}$ is in NC^1, then there exists a width-5 permutation branching program for f with length polynomial in (n+m)

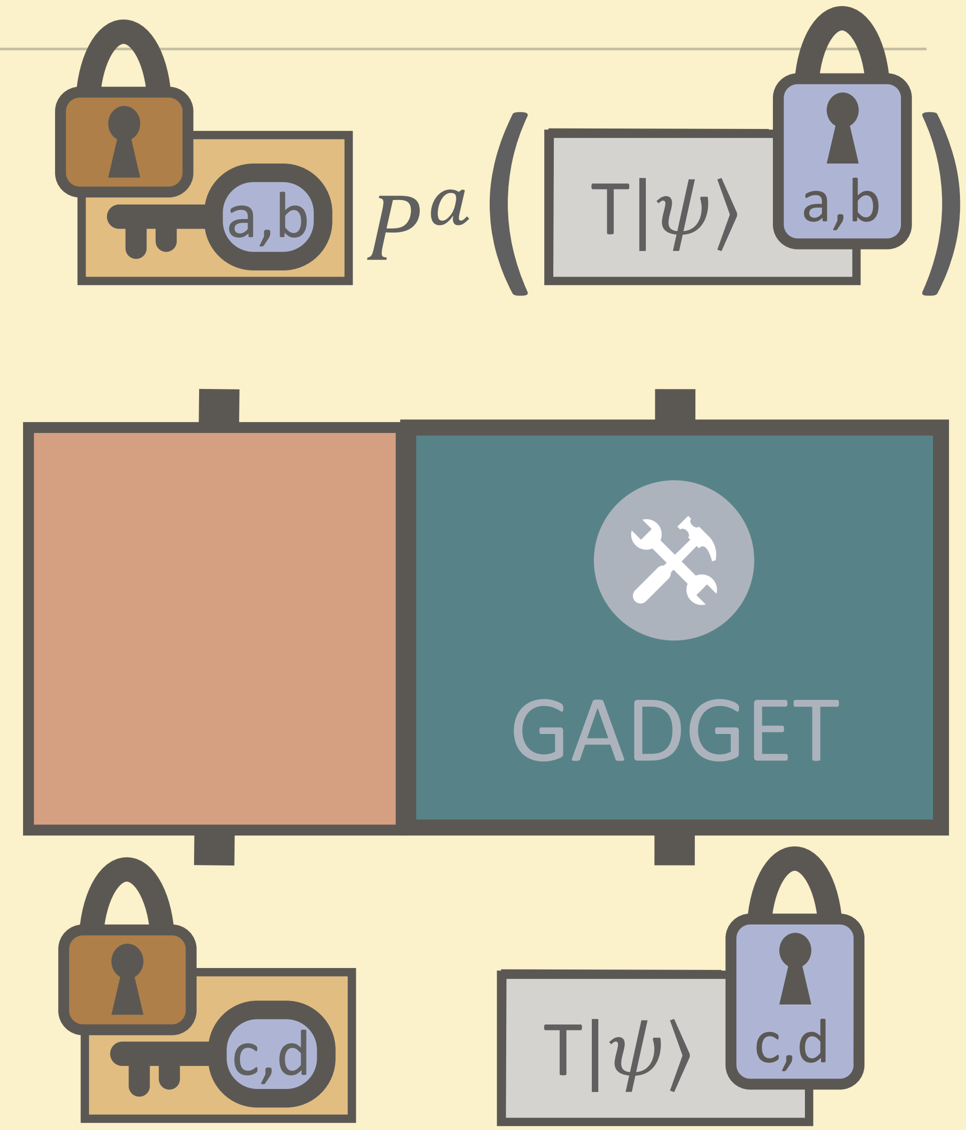


Classical homomorphic decryption functions happen to be in NC^1… [BV11]

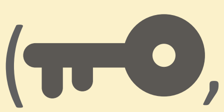
[Barrington 89] Bounded-Width Polynomial-Size Branching Programs Recognize Exactly Those Languages in NC1, J. Comput. Syst. Sci. 38 (1): 150–164, 1989
[BV11] Z. Brakerski, V. Vaikuntanathan. Efficient fully homomorphic encryption from (standard) LWE. FOCS 2011

ERROR-CORRECTION GADGET

- Build a 'gadget' that applies P^{-1} iff a = 1

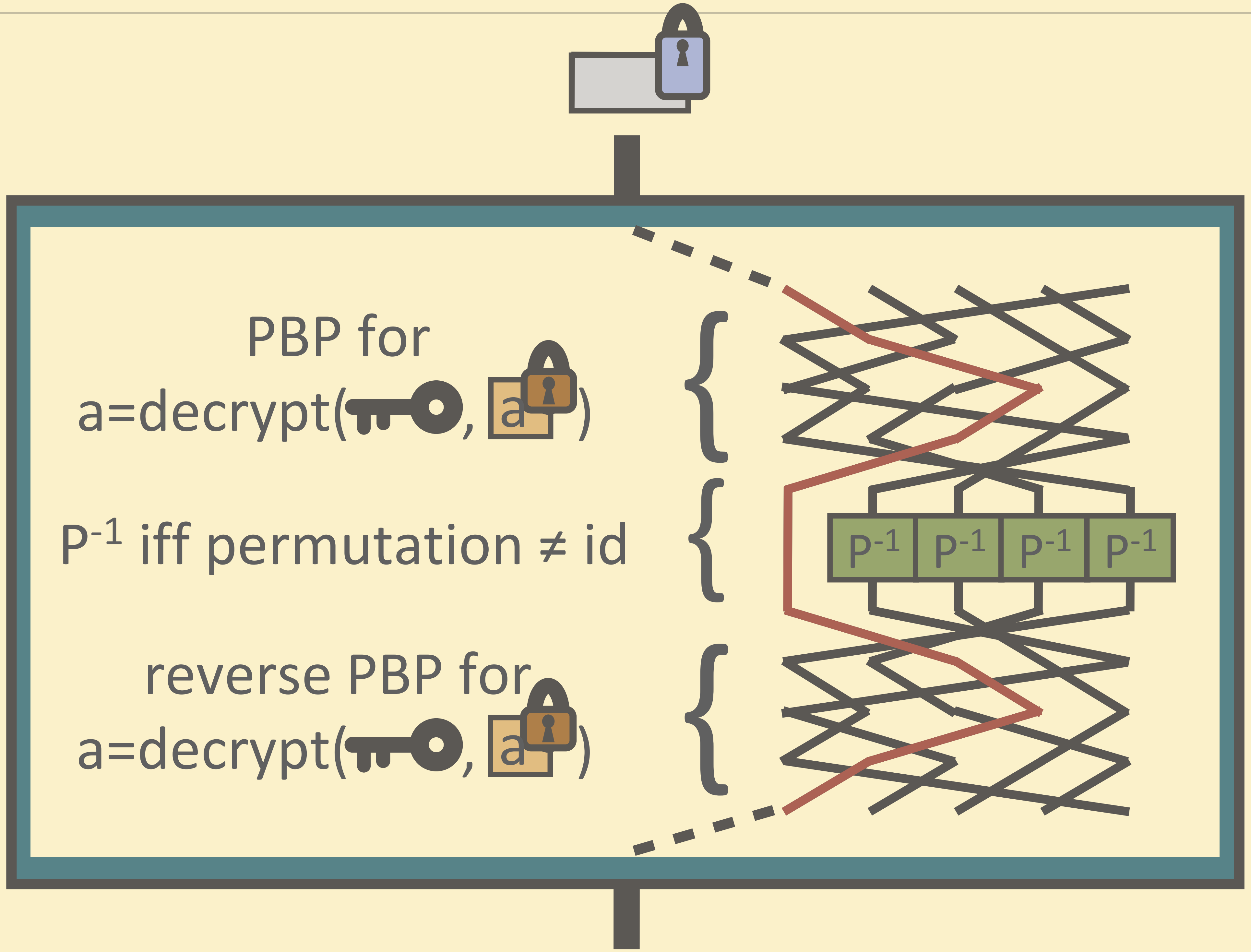


- :Apply correction iff

a = decrypt(, ) = 1

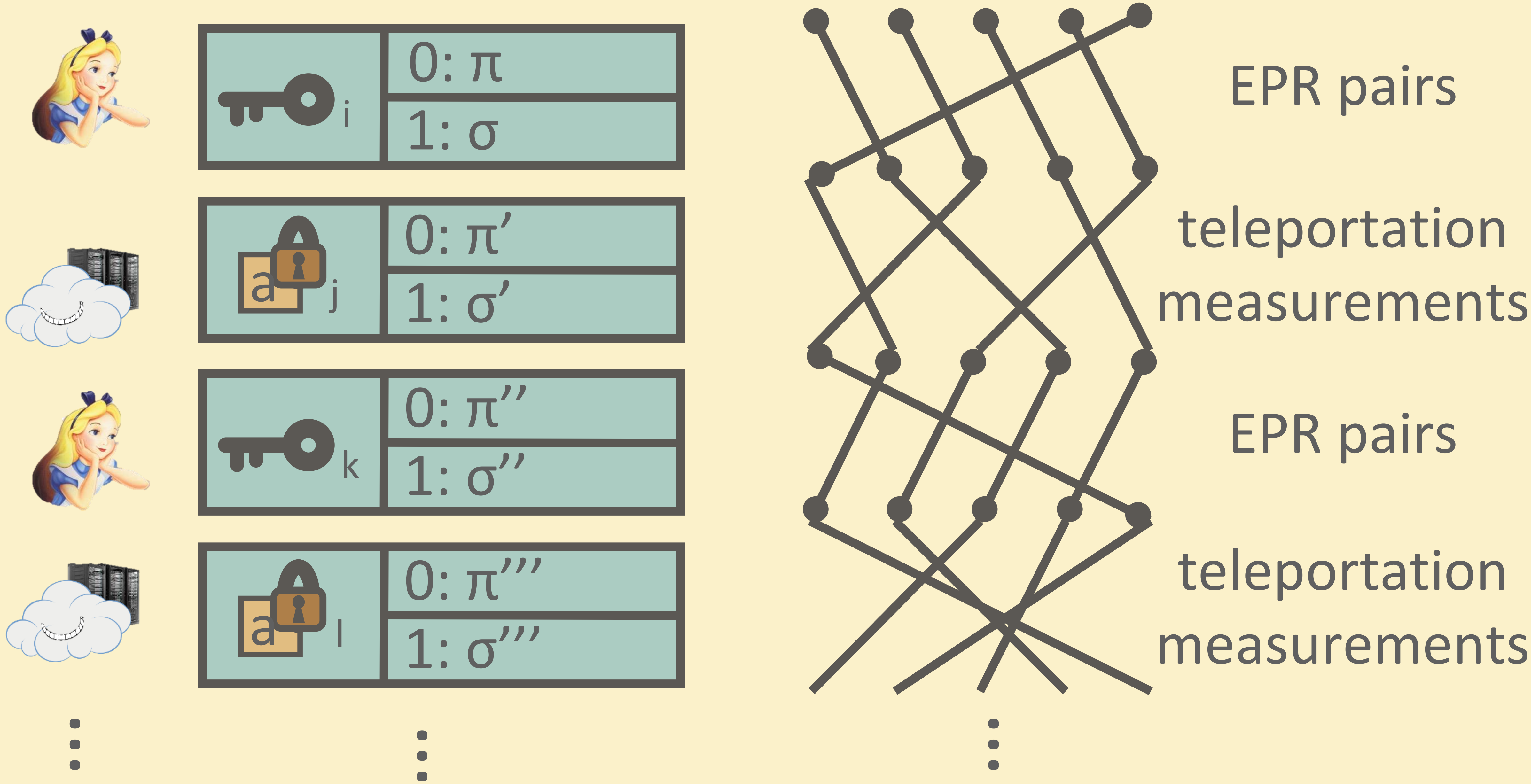
has a poly-size PBP

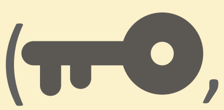
ERROR CORRECTION GADGET



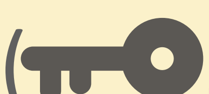
ERROR CORRECTION GADGET

Branching program for decrypt(, a)

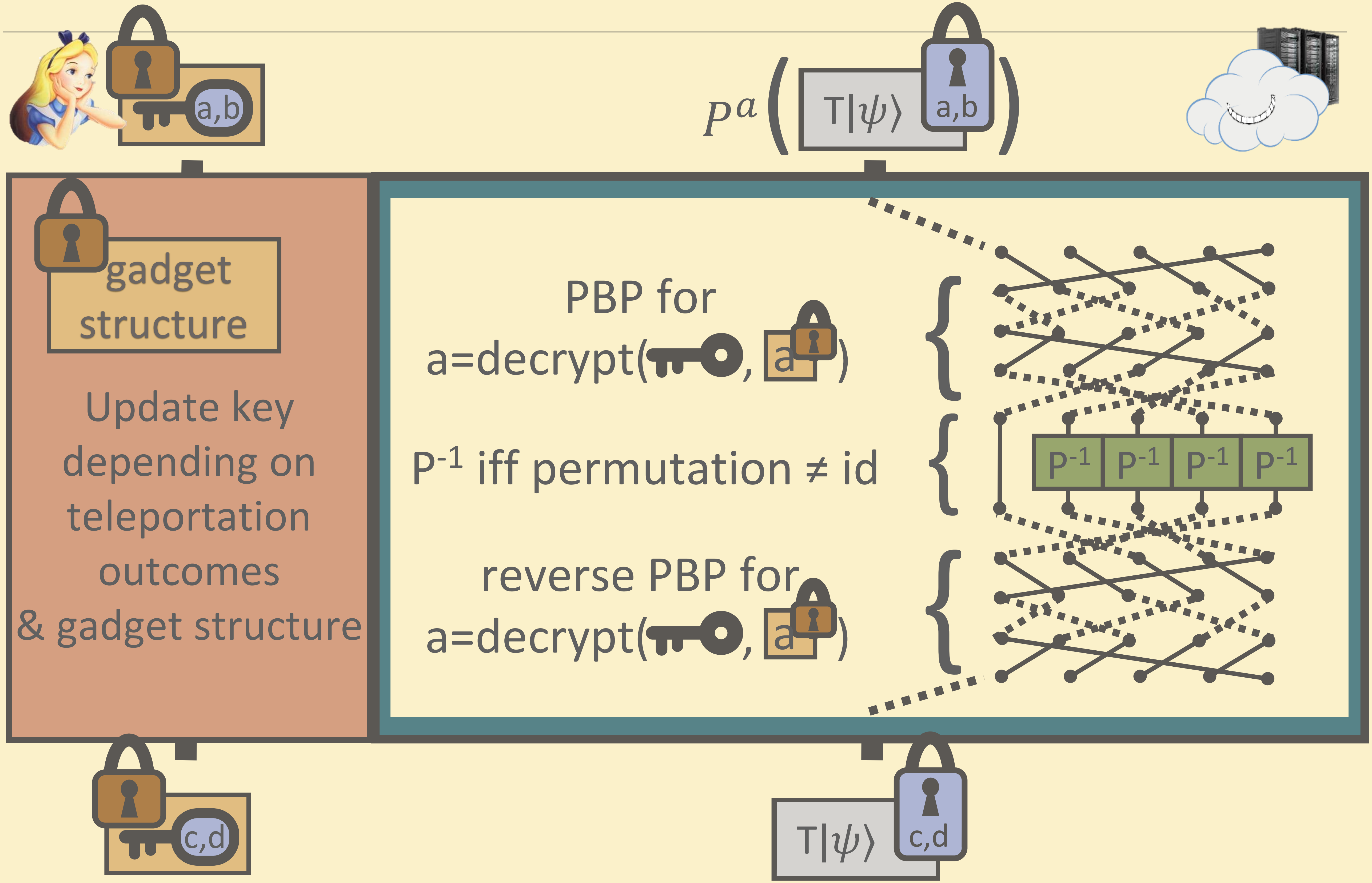


PBP for
a=decrypt(, )

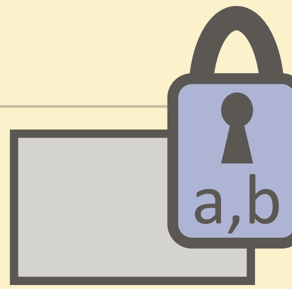
P^{-1} iff permutation ≠ id

reverse PBP for
a=decrypt(, )

ERROR CORRECTION GADGET

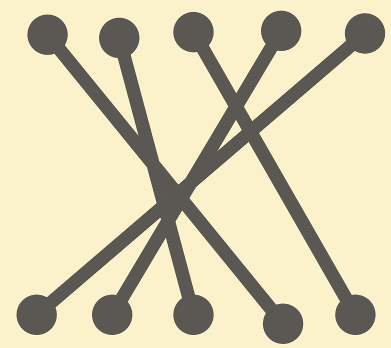


SECURITY



All quantum information: quantum one-time pad

(perfectly secure if classical info is hidden)



Gadget structure, each ‘connection’:

Random choice out of 4 Bell states

(perfectly secure if classical info is hidden)

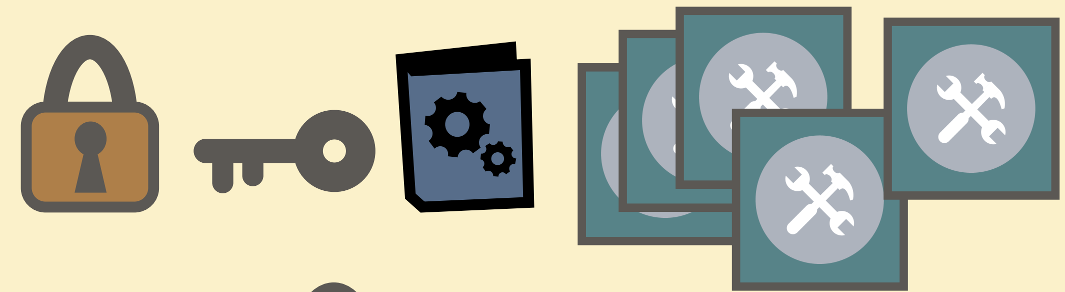
All classical information: classical homomorphic scheme

Security of classical scheme is the only assumption

NEW SCHEME: OVERVIEW

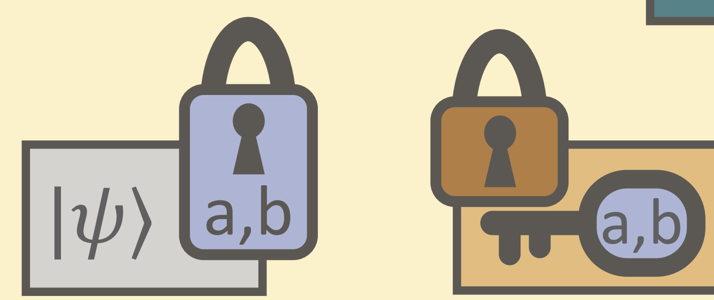
KEY GENERATION

- classical keys
- gadgets



ENCRYPTION

- apply quantum one-time pad
- classically encrypt pad keys

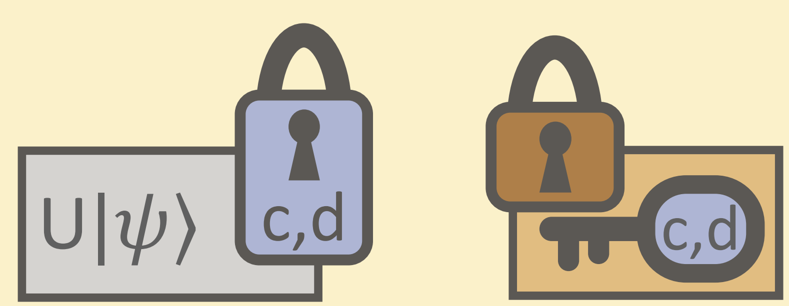


EVALUATION

- after H / P / CNOT : classically update keys
- after T : use 

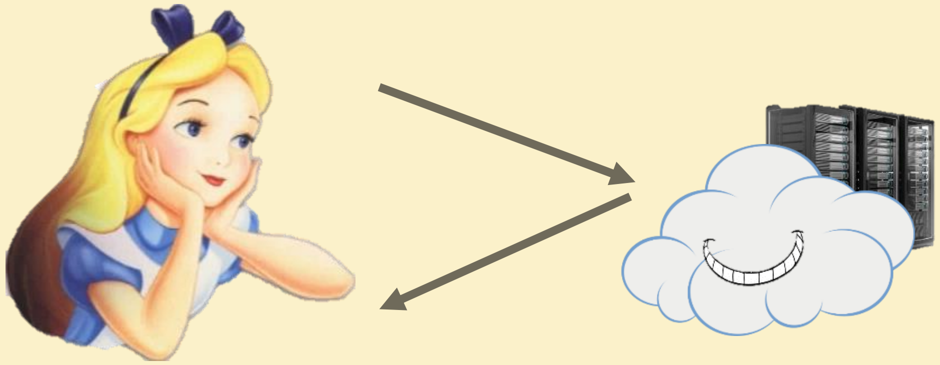
DECRYPTION

- classically decrypt pad keys
- remove quantum one-time pad



APPLICATIONS

- Delegated quantum computation in two rounds
  - No memory needed on Alice's side

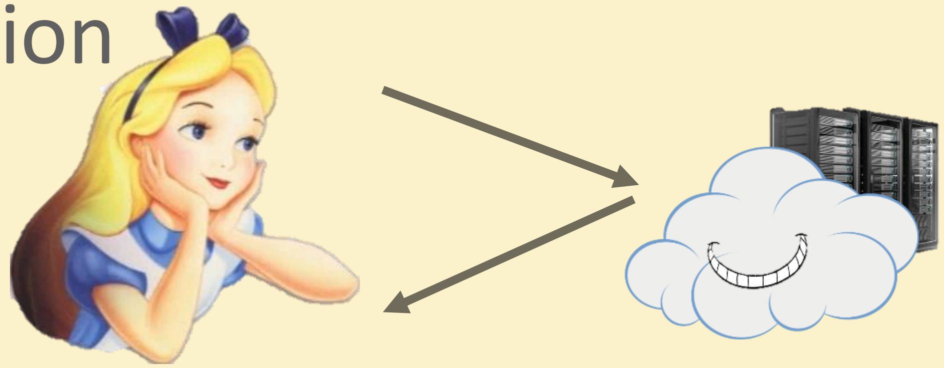


- Gadget generation on demand
- Circuit privacy

FUTURE WORK

- non-leveled QFHE?

- verifiable delegated quantum computation



- quantum obfuscation?

- …

THANK YOU!