Error Probabilities of Synchronous DS/CDMA Systems with Random and Deterministic Signature Sequences for Ideal and Fading Channels

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Abstract - We present a software tool which efficiently computes the error probability of a direct sequence, code division multiple access (DS/CDMA) system. Several different system models are analyzed including random signature sequences with ideal channels, deterministic signature sequences with ideal channels, random signature sequences with slowly fading channels, and deterministic signature sequences with slowly fading channels. We assume that the chips received from each of the multiple-access interference (MAI) sources are synchronous with the chips from a reference source. This provides an upper bound for an asynchronous DS/CDMA system. Numerical contour integration of the test statistic's moment generating function (MGF) in the complex plane is used to efficiently evaluate the error probabilities.

1 Introduction

The recent introduction of inexpensive chip sets designed to implement DS/CDMA communication systems is allowing this technology to be implemented in many commercial products such as digital cellular telephones, wireless local area networks, and remote data collection devices. One of the main parameters to be considered during the system design phase is the probability of a bit error. Several results [1], [2], [3] have derived the average probability of error by analyzing the noise introduced by MAI sources which are asynchronous to the reference receiver. Due to the asynchronous nature of the models used in the previous works, the results are quite involved. The primary goal of this work is to provide a comprehensive software tool which quickly computes the probability of error for systems with either random or deterministic signature sequences and either ideal or slowly fading channels. To achieve fast execution, we assume that the chips received from each of the MAI sources are synchronous with the chips received from the reference source. Under this assumption, much of the complexity of the previous results can be minimized.

The technique used to efficiently evaluate the error probability is based on numerical contour integration of the test statistic's MGF in the complex plane which we refer to as numerical contour integration (NCI). This method was used to compute the probability of error resulting from intersymbol and cochannel interference [4] as well as radar detection probabilities [5]. NCI is superior to the characteristic function method proposed in [2] because it is less susceptible to oscillations which can cause the trapezoidal integration to converge slowly for a given error tolerance.

Another contribution of this work is to model the effects of a slowly fading channel using a Padé approximation. The integral which results from the expectation due to the random nature of a slowly fading channel with random signature sequences is replaced with a Padé approximation which matches the moments of the distribution of the channel's amplitude attenuation. In this paper, results are derived and evaluated for a slowly fading channel modeled by the Rayleigh distribution. The method is easily extended to other fading distributions.

2 System Model

The baseband signal transmitted by the l-th DS/CDMA source is

$$s_l(t) = a_l \sum_{k=-\infty}^{\infty} i_l[k] h_l(t-kT) \quad (1)$$

where a_l is the signal amplitude, $i_l[k] \in \{+1,-1\}$ is the equally likely, transmitted information bit, and $h_l(t)$ is the chip sequence defined to be

$$h_l(t) = \sum_{n=0}^{N-1} b_l[n] p(t-nT_c). \quad (2)$$

The signature sequence, $\mathbf{b_l} = [b_l[0], ..., b_l[N-1]]^T$, $b_l[k] \in \{+1,-1\}$, specifies the pseudo noise (PN) sequence employed by the l-th transmitter to spread the bandwidth of the transmitted signal. In this paper, the signal is transmitted using BPSK modulation. The chip waveform, p(t), is assumed to be zero outside of the interval $[0...T_c]$ and have normalized energy $\int_0^{T_c} p^2(t)dt = 1$.

Each of the sources is transmitted through a channel modeled by $\Re\{\alpha_l e^{j\theta_l}\}$. The amplitude attenuation introduced by the channel from the l-th transmitter to the reference receiver is given by α_l, while θ_l represents the phase offset. Since $\theta_l = 0$ for a synchronous system, the received baseband signal is

$$\begin{aligned} r(t) &= a_0\alpha_0 \sum_{k=-\infty}^{\infty} i_0[k] h_0(t-kT) \\ &+ \sum_{l=1}^{L} a_l\alpha_l \sum_{k=-\infty}^{\infty} i_l[k] h_l(t-kT-j_lT_c) + n(t). \end{aligned} \quad (3)$$

The first term represents the desired signal, the second term represents the MAI noise produced by the remaining L DS/CDMA transmitters in the area, and the last term represents additive white Gaussian noise with zero mean and variance σ_n^2. The code offset, j_l, indicates the relative offset between the start of the l-th MAI transmitter chip sequence and that of the reference transmitter. Although the synchronous assumption requires the individual chips to be aligned in time, the code offset is considered to be a discrete random variable uniformly distributed between 0 and N-1. The reference receiver is modeled by a continuous-time matched filter, which is matched to the transmitter's chip pulse, followed by a discrete-time correlator which correlates the output of the matched filter with the reference receiver's signature sequence. Given chip synchronization to the reference receiver, the test statistic for the l-th source and is

$$z_l = a_l i_l \alpha_l \beta_l \quad (4)$$

β_l is the periodic cross-correlation between the signature sequences of the l-th MAI source and the reference source

$$\beta_l = \sum_{n=0}^{N-1} b_l[n - j_l] b_0[n] \tag{5}$$

where $b_l[n - j_l] = b_l[N + n - j_l]$ for $n - j_l < 0$.

3 Random Signature Sequences, Ideal Channels

In this section, the probability of error, P_e, is derived for signature sequences with each of the equally likely bits $b_l[n] \in \{+1, -1\}$. We model ideal channels without fading where the only effect introduced by the channel is a deterministic amplitude attenuation given by α_l. Without loss of generality, we also assume that the information bit transmitted by the reference transmitter is $+1$. For this system model with L MAI sources, the probability of error is given by

$$\begin{aligned} P_e &= Pr(r = z_0 + \sum_{l=1}^{L} z_l + n < 0 \mid i_0 = +1) \\ &= \int_{-\infty}^{0} p(r) dr. \end{aligned} \tag{6}$$

where $p(r)$ is the probability density function (PDF) of the test statistic of the received signal. In order to avoid numerical inaccuracies incurred by the direct integration of (6), NCI calculates the P_e by integrating the MGF, $h(u) = E\{\exp(-ru)\}$, of the received signal's test statistic, r, along a path in the complex plane. Since the MGF is equivalent to the Laplace transform of the PDF, $p(r)$ can be recovered from the inverse Laplace transform of the MGF of the test statistic

$$p(r) = \frac{1}{2\pi j} \oint h(u) \exp(ur) du. \tag{7}$$

Substituting (7) into (6), the probability of error is given by

$$P_e = \frac{1}{2\pi j} \oint \frac{h(u)}{u} du \tag{8}$$

given that the real part of the contour is positive. Since z_0, z_l, and n in (6) are independent, the MGF of the test statistic r is

$$h(u) = \gamma(u) \eta(u) h_n(u) \tag{9}$$

where $\gamma(u)$, $\eta(u)$, and $h_n(u)$ represent the MGF of the reference signal, the multiple access interference and the Gaussian noise, respectively.

Recalling that $i_0 = +1$ was transmitted by the reference source, the MGF of the reference signal term is

$$\gamma(u) = E\{\exp(-uz_0)\} = \exp(-u a_0 \alpha_0 N). \tag{10}$$

For random signature sequences, the MGF due to the MAI term is

$$\begin{aligned} \eta(u) &= E\{\exp(-u \sum_{l=1}^{L} z_l)\} \\ &= \prod_{l=1}^{L} \prod_{n=0}^{N-1} \frac{1}{2} \exp(u a_l \alpha_l) + \frac{1}{2} \exp(-u a_l \alpha_l) \\ &= \prod_{l=1}^{L} (\cosh(u a_l \alpha_l))^N. \end{aligned} \tag{11}$$

In addition, the MGF of the white Gaussian noise is

$$h_n(u) = \exp(\frac{1}{2} \sigma_n^2 u^2). \tag{12}$$

In order to minimize round off error, Helstrom [4] rewrites the integral in (8) as

$$P_e = \frac{1}{2\pi j} \oint \exp(\Phi(u)) du \tag{13}$$

where the phase of the integrand, $\Phi(u)$, is

$$\Phi(u) = \ln(h(u)/u). \tag{14}$$

From (9), (10), (11), (12), and (14), the phase of the P_e for random signature sequences with ideal channels is given by

$$\Phi(u) = N \sum_{l=1}^{L} \ln(\cosh(u a_l \alpha_l)) + \frac{1}{2} \sigma_n^2 u^2 - a_0 \alpha_0 N u - \ln u. \tag{15}$$

Helstrom and Ritcey [5] showed the optimal contour to be the path of steepest descent where $\Im(\Phi(u)) = 0$. Hence, the contour in (13) is chosen to cross the real axis in the complex plane at the saddlepoint, u_0, of the phase where the saddlepoint is defined to be the location on the real axis where the first derivative of the phase equals zero, $\Phi'(u_0) = 0$. Typically, the Newton-Raphson method is used to numerically determine the saddlepoint. The first and second derivatives of the phase, which are required for the Newton-Raphson calculation, are

$$\Phi'(u) = N \sum_{l=1}^{L} a_l \alpha_l \tanh(u a_l \alpha_l) + \sigma_n^2 u - a_0 \alpha_0 N - \frac{1}{u} \tag{16}$$

and

$$\Phi''(u) = N \sum_{l=1}^{L} (a_l \alpha_l)^2 \text{sech}(u a_l \alpha_l) + \sigma_n^2 + \frac{1}{u^2}. \tag{17}$$

In order to simplify the calculations, the contour can be approximated as a straight line parallel to the imaginary axis intercepting the real axis at the saddlepoint. Thus, the P_e from (13) can be rewritten as

$$P_e = \frac{1}{2\pi j} \int_{u_0 - j\infty}^{u_0 + j\infty} \exp(\Phi(u)) du. \tag{18}$$

Rice [7] showed that trapezoidal integration should be used to approximate the contour integral in (18). The initial step size in the trapezoidal integration is chosen to be

$$\Delta v = \sqrt{2/\Phi''(u_0)} \tag{19}$$

and the step size should be halved until the difference between the last two P_e estimates is less than the desired accuracy.

4 Deterministic Signature Sequences, Ideal Channels

In the previous section, the P_e was computed under the assumption of random signature sequences. In practice, the signature sequences in a system are designed to minimize the cross-correlation between any two transmitters. The P_e in this section is developed for a known set of signature sequences. The MGF of the MAI term for a set of deterministic signature sequences with elements $b_l[n] \in \{1, -1\}$ is

$$\eta(u) = E\{\exp(-u \sum_{l=1}^{L} a_l i_l \alpha_l \beta_l)\}. \quad (20)$$

For the signature sequences of an interference source, $\mathbf{b_l}$, and the reference source, $\mathbf{b_0}$, the periodic cross-correlation is given by the discrete random variable β_l in (5) and has a probability mass function randomized by the code offset j_l. Since the random variables i_l and β_l, for $l \neq 0$, are independent

$$\eta(u) = \prod_{l=1}^{L} E_{i_l,\beta_l}\{\exp(-u a_l i_l \alpha_l \beta_l)\} = \prod_{l=1}^{L} \xi_l \quad (21)$$

where

$$\xi_l = \sum_{b=-N}^{N} \exp(-u a_l \alpha_l b) p_{\beta_l}(b). \quad (22)$$

To simplify the equation, we have accounted for the four possible combinations of information bits, $i_l[k-1] \in \{+1, -1\}$ and $i_l[k] \in \{+1, -1\}$, in determining $p_{\beta_l}(\beta_l)$ when averaging over all possible code offsets j_l. For the deterministic signature sequences, the phase and its first two derivatives are

$$\Phi(u) = \sum_{l=1}^{L} \ln(\xi_l) + \frac{1}{2}\sigma_n^2 u^2 - a_0 \alpha_0 N u - \ln u. \quad (23)$$

$$\Phi'(u) = \sum_{l=1}^{L} \frac{\psi_l}{\xi_l} + \sigma_n^2 u - a_0 \alpha_0 N - \frac{1}{u} \quad (24)$$

$$\Phi''(u) = \sum_{l=1}^{L} \frac{\zeta_l \xi_l - \psi_l^2}{\xi_l^2} + \sigma_n^2 + \frac{1}{u^2}. \quad (25)$$

where

$$\psi_l = -\sum_{b=-N}^{N} (a_l \alpha_l b) \exp(-u a_l \alpha_l b) p_{\beta_l}(b). \quad (26)$$

$$\zeta_l = \sum_{b=-N}^{N} (a_l \alpha_l b)^2 \exp(-u a_l \alpha_l b) p_{\beta_l}(b). \quad (27)$$

5 Random Signature Sequences, Slowly Fading Channels

Returning to random signature sequences, let us now consider the effects of frequency-nonselective, slowly fading channels on the P_e. To model a slowly fading channel, the deterministic amplitude attenuation of the ideal channel, α_l, is replaced by a random variable which we choose to represent with a Rayleigh distribution. For the test statistic given in (6), the expected value of the MGF is

$$E_\alpha\{h(u)\} = E_{\alpha_0}\{\gamma(u)\} E_{\alpha_l}\{\eta(u)\} h_n(u). \quad (28)$$

From Appendix A, the expected value of the MGF of the reference signal transmitted through a Rayleigh fading channel is

$$E_{\alpha_0}\{\gamma(u)\} = 1 - \exp(\frac{(u a_0 N \sigma_0)^2}{2})(u a_0 N \sigma_0) \times \sqrt{\frac{\pi}{2}} \mathrm{erfc}(\frac{u a_0 N \sigma_0}{2}) \quad (29)$$

where the erfc() is the complex complementary error function. An algorithm for computing the complex erfc() is provided in [9].

Like the reference source, all of the MAI sources are also transmitted through independent Rayleigh fading channels. With respect to the channel attenuation, the expected value of the MGF of the MAI term given in (11) is

$$E_{\alpha_l}\{\eta(u)\} = \prod_{l=1}^{L} (\int_0^\infty (\cosh(u a_l \alpha_l))^N p_{\alpha_l}(\alpha_l) d\alpha_l). \quad (30)$$

Each of the integrals in (30) cannot be solved in closed form due to the Rayleigh probability density function. Hence, we seek an approximation that can be rapidly computed for various values of complex u by expanding the cosh() as a Maclaurin series and substituting a Padé approximation for the infinite summation. The Miller algorithm, given in Appendix B, can be used to raise the cosh() to a power

$$(\cosh(u \alpha_l a_l))^N = \sum_{k=0}^{\infty} v_{2k} (u a_l \alpha_l)^{2k}. \quad (31)$$

Substituting (31) into (30) and reversing the integral and the summation, we get

$$E_{\alpha_l}\{(\cosh(u a_l \alpha_l))^N\} = \sum_{k=0}^{\infty} v_{2k} \mu_l^{(2k)} u^{2k} \quad (32)$$

where $\mu_l^{(2k)}$ are the even moments of α_l. For the Rayleigh distribution, the moments are

$$\mu_l^{(k)} = (2\sigma_l^2)^{\frac{k}{2}} \Gamma(1 + \frac{k}{2}). \quad (33)$$

Therefore, the expected value of the MGF of the MAI term is

$$E_{\alpha_l}\{\eta(u)\} = \prod_{l=1}^{L} (\sum_{k=0}^{\infty} v_{2k} \mu_l^{(2k)} u^{2k}). \quad (34)$$

A Padé approximation produces a rational approximation of the infinite series by the moment matching approach [8]. For each of the interference sources, the Padé approximation, $P_l(u)$, is

$$\begin{aligned} \sum_{k=0}^{\infty} v_{2k} \mu_l^{(2k)} u^{2k} &= \frac{g_l \prod_{i=1}^{M_N} (u - z_{l,i})}{\prod_{j=1}^{M_D} (u - p_{l,j})} + O(u^{M_N + M_D + 1}) \\ &= P_l(u) + O(u^{M_N + M_D + 1}) \end{aligned} \quad (35)$$

Replacing the cosh term in (15) with the Padé approximation for the MGF of each MAI source, the phase and its first two derivatives for this system model are

$$\Phi(u) = \sum_{l=1}^{L} \ln(P_l(u)) + \frac{1}{2}\sigma_n^2 u^2 + \ln(E_{\alpha_0}\{\gamma(u)\}) - \ln u. \quad (36)$$

$$\begin{aligned} \Phi'(u) &= \sum_{l=1}^{L} (\sum_{i=1}^{M_N} \frac{1}{(u - z_{l,i})} - \sum_{j=1}^{M_D} \frac{1}{(u - p_{l,j})}) \\ &+ \sigma_n^2 u + \frac{E'_{\alpha_0}\{\gamma(u)\}}{E_{\alpha_0}\{\gamma(u)\}} - \frac{1}{u}. \end{aligned} \quad (37)$$

$$\begin{aligned} \Phi''(u) &= \sum_{l=1}^{L} (-\sum_{i=1}^{M_N} \frac{1}{(u - z_{l,i})^2} + \sum_{j=1}^{M_D} \frac{1}{(u - p_{l,j})^2}) + \sigma_n^2 \\ &+ \frac{E_{\alpha_0}\{\gamma(u)\} E''_{\alpha_0}\{\gamma(u)\} - (E'_{\alpha_0}\{\gamma(u)\})^2}{(E_{\alpha_0}\{\gamma(u)\})^2} + \frac{1}{u^2}. \end{aligned} \quad (38)$$

6 Deterministic Signature Sequences, Slowly Fading Channels

The final DS/CDMA system model considered in this paper includes deterministic signature sequences with slowly fading channels. Again, the channel attenuation from each of the sources to the reference receiver is modeled as a random variable with a Rayleigh probability density function. The expected value of the MGF of the reference signal with Rayleigh fading is given in (29). With respect to α_l, the expected value of the MGF of the MAI term for deterministic signature sequences in (21) is

$$E_{\alpha_l}\{\eta(u)\} = \prod_{l=1}^{L} E_{\alpha_l,i_l,\beta_l}\{\exp(-ua_l i_l \alpha_l \beta_l)\} = \prod_{l=1}^{L} \nu_l(u) \quad (39)$$

where

$$\nu_l(u) = \sum_{b=-N}^{N} E_{\alpha_l}\{\exp(-ua_l\alpha_l b)\} p_{\beta_l}(b). \quad (40)$$

The phase and its first two derivatives for this system model are

$$\Phi(u) = \sum_{l=1}^{L} \ln(\nu_l(u)) + \frac{1}{2}\sigma_n^2 u^2 + \ln(E_{\alpha_0}\{\gamma(u)\}) - \ln u. \quad (41)$$

$$\Phi'(u) = \sum_{l=1}^{L} \frac{\chi_l(u)}{\nu_l(u)} + \sigma_n^2 u + \frac{E'_{\alpha_0}\{\gamma(u)\}}{E_{\alpha_0}\{\gamma(u)\}} - \frac{1}{u}. \quad (42)$$

$$\begin{aligned}\Phi''(u) &= \sum_{l=1}^{L} \frac{\nu_l(u)\upsilon_l(u) - \chi_l^2(u)}{\nu_l^2(u)} + \sigma_n^2 \\ &+ \frac{E_{\alpha_0}\{\gamma(u)\}E''_{\alpha_0}\{\gamma(u)\} - (E'_{\alpha_0}\{\gamma(u)\})^2}{(E_{\alpha_0}\{\gamma(u)\})^2} + \frac{1}{u^2}. \quad (43)\end{aligned}$$

where

$$\chi_l(u) = \frac{1}{2}\sum_{b=-N}^{N} E'_{\alpha_l}\{\exp(-ua_l\alpha_l b)\} p_{\beta_l}(b). \quad (44)$$

$$\upsilon_l(u) = \frac{1}{2}\sum_{b=-N}^{N} E''_{\alpha_l}\{\exp(-ua_l\alpha_l b)\} p_{\beta_l}(b). \quad (45)$$

7 Numerical Results

In this section, the P_e for the systems presented in the previous sections are evaluated using the DS/CDMA software tool based on NCI. Figure 1 shows how the probability of error varies according to the different system models for a range of values of E_b/N_0. In each model, L = 7 and N = 31 for each source. Gold codes are used for the deterministic chip sequences. For the random chip sequence with Rayleigh fading model, $M_n = 16$ and $M_d = 18$. The value of E_b/N_0 refers to the reference receiver and is defined to be $E_b/N_0 = 10\log_{10}(NE\{\alpha_0^2\}a_0^2)/(2\sigma_n^2))$ dB for the case of rectangular chips. In this example, the signal-to-interferer ratio (SIR), which provides the ratio of the power of the reference source to the power of the interference source, is 5 dB for the first MAI source and 10 dB for the remaining MAI sources where $SIR = 10\log_{10}((E\{\alpha_0^2\}a_0^2)/(E\{\alpha_l^2\}a_l^2))$ dB. In all of the examples, $\alpha_l = 1$ for each of the ideal channels and $E\{\alpha_l^2\} = 1$ for each of the Rayleigh fading channels. Figure 1 shows that, for the ideal channel cases, the difference between random chip sequences and deterministic chip sequences can be significant for smaller values of P_e. This is to be expected since the gold codes are designed to minimize the cross-correlation between the codes. However, the P_e for the models involving Rayleigh fading channels are indistinguishable since the effects of the Rayleigh fading completely mask the improvements introduced by specifying the signature sequences.

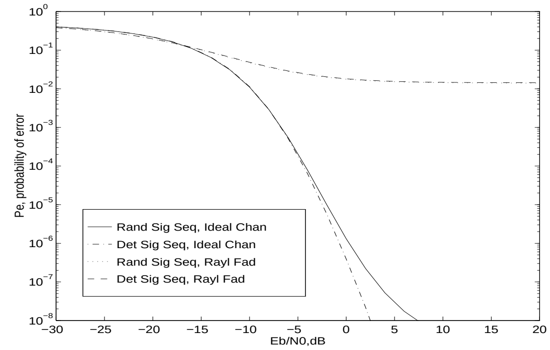

Figure 1: P_e vs. E_b/N_0 for all system models.

In the next example, we use the software tool to evaluate the how the number of MAI sources affects the P_e. Figure 2 plots the P_e for L = 2, 4, and 6 across a range of values of E_b/N_0 for the case of deterministic gold codes of length N = 31 with ideal channels. The SIR is 0 dB for the first MAI source and 5 dB for the remaining MAI sources.

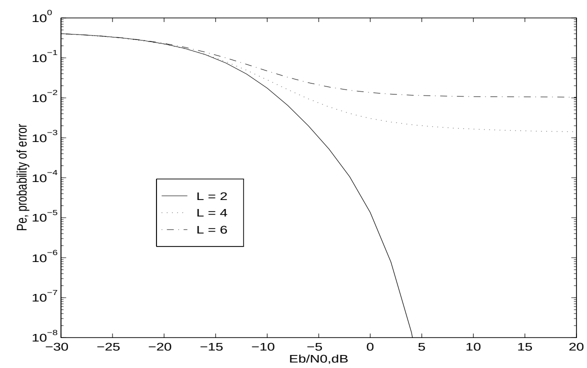

Figure 2: P_e vs. E_b/N_0 for various number of sources.

In figure 3, the effects of varying the SIR is studied for the case of deterministic gold codes of length N = 31 with Rayleigh fading channels. The P_e are shown for L = 4, and SIR = 0 dB, 5 dB, and 10 dB. In this example, we show how the tool is able to model different power levels for the MAI sources. Although we have chosen to make all of the power level identical for the each of the MAI sources, we can vary each of the source's power individually.

8 Conclusion

Under assumption that all of the MAI sources are chip synchronized to the reference transmitter and receiver, the error probabilities for all cases of random and deterministic signature sequences and ideal and Rayleigh fading channels were derived. Numerical

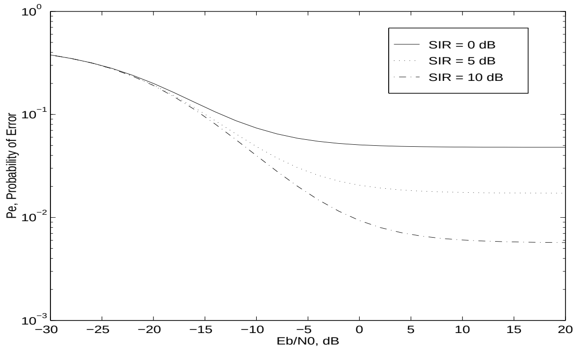

Figure 3: P_e vs. E_b/N_0 for various values of SIR.

contour integration was used as the computational engine of a software tool to efficiently compute the P_e. The computation time required to evaluate the P_e for the case of random signature sequences with slowly fading channels was significantly reduced by modeling the noise introduced by each MAI source as a Padé approximation. Results show that the difference in the P_e for the models based on random and deterministic signature sequences with ideal channels can be significant, but the P_e for the models involving Rayleigh fading channels are indistinguishable.

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A MGF of the Reference Signal in a Rayleigh Fading Channel

This appendix calculates the expected value, with respect to the channel attenuation α, of the moment generating function of the reference signal for the DS/CDMA system models with Rayleigh fading. The expected value is

$$E_\alpha\{\exp(-s\alpha)\} = \int_0^\infty \exp(-s\alpha)(\frac{\alpha}{\sigma^2}\exp(\frac{-\alpha^2}{2\sigma^2}))d\alpha. \tag{46}$$

Completing the square and substituting $t = \alpha + \sigma^2 s/\sigma$,

$$E_\alpha\{\exp(-s\alpha)\} = 1 - (\sigma s)\exp(\frac{(\sigma s)^2}{2})\sqrt{\frac{\pi}{2}}\mathrm{erfc}(\frac{\sigma s}{\sqrt{2}}). \tag{47}$$

The first derivative, which is needed for the determination of the saddlepoint, can be calculated using Leibnitz's Rule.

$$\begin{aligned} E'_\alpha\{\exp(-wu\alpha)\} &= \sigma^2 w^2 u \\ - \quad & (\sigma w + \sigma^3 w^3 u^2)\exp(\frac{(\sigma wu)^2}{2})\sqrt{\frac{\pi}{2}}\mathrm{erfc}(\frac{\sigma wu}{\sqrt{2}}). \end{aligned} \tag{48}$$

Likewise, the second derivative is

$$\begin{aligned} E''_\alpha\{\exp(-wu\alpha)\} = \sigma^4 w^4 u^2 + 2\sigma^2 w^2 - \exp(\frac{(\sigma wu)^2}{2}) \\ \times(3\sigma^3 w^3 u + \sigma^5 w^5 u^3)\sqrt{\frac{\pi}{2}}\mathrm{erfc}(\frac{\sigma wu}{\sqrt{2}}). \end{aligned} \tag{49}$$

B Miller's Algorithm

Miller's algorithm [8] provides an efficient method for raising a polynomial, f(z), to a power provided f(0) = 1.

$$f(z) = \sum_{n=0}^{\infty} c_n z^n \tag{50}$$

$$W(z) = [f(z)]^k = \sum_{n=0}^{\infty} v_n z^n \tag{51}$$

$$v_n = \frac{1}{n}\sum_{m=1}^{n}[(k+1)m - n]c_m v_{n-m} \qquad n = 1, 2, \cdots \tag{52}$$

$$v_0 = c_0 = 1 \tag{53}$$