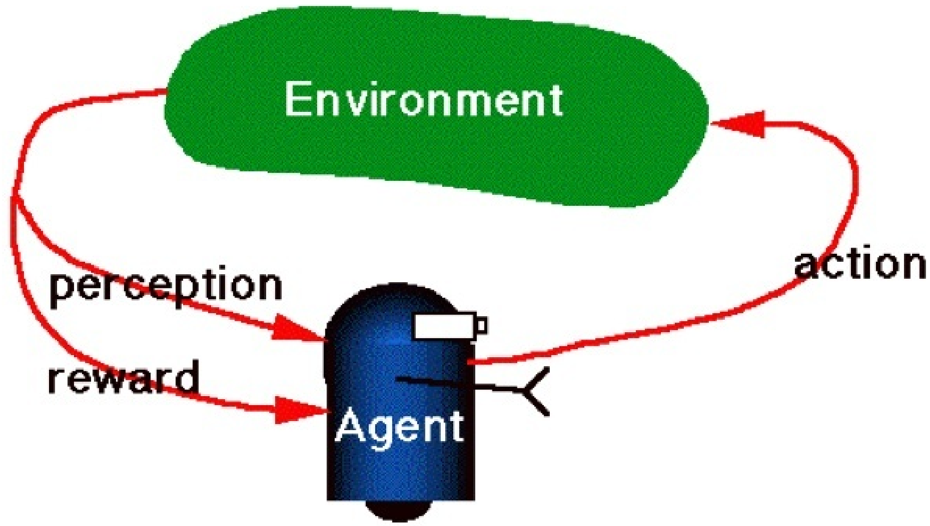
Representations for Reinforcement Learning

Doina Precup
Reasoning & Learning Lab
McGill University

With thanks to Rich Sutton, Satinder Singh, Pierre-Luc Bacon, Jean Harb

MSR Cambridge AI Summer School, July 2017

Reinforcement learning



- Learning by *trial-and-error*
- Learning is driven by a (numerical) *reward signal*, which may be *delayed*
- Goal: maximize a cumulative measure of reward (eg discounted sum)
- Draws ideas from animal learning/psychology, control, operations research

A big success story: AlphaGo


ARTICLE

doi:10.1038/nature16961

Mastering the game of Go with deep neural networks and tree search

David Silver[1*], Aja Huang[1*], Chris J. Maddison[1], Arthur Guez[1], Laurent Sifre[1], George van den Driessche[1], Julian Schrittwieser[1], Ioannis Antonoglou[1], Veda Panneershelvam[1], Marc Lanctot[1], Sander Dieleman[1], Dominik Grewe[1], John Nham[2], Nal Kalchbrenner[1], Ilya Sutskever[2], Timothy Lillicrap[1], Madeleine Leach[1], Koray Kavukcuoglu[1], Thore Graepel[1] & Demis Hassabis[1]

The first AI Go player to defeat a human (9 dan) champion

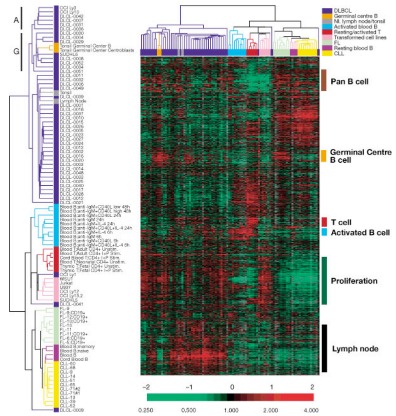
Contrast: Supervised learning

- Training experience: a set of *labeled examples* of the form

$$\langle x_1\, x_2\, \ldots x_n, y \rangle,$$

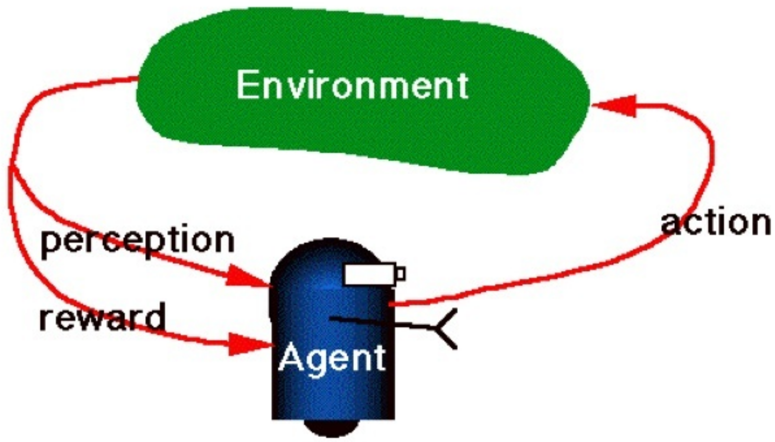
  where x_j are values for *input variables* and y is the *output*
- This implies the existence of a "teacher" who knows the right answers
- Goal: *minimize the prediction error (loss) function*

Contrast: Unsupervised learning



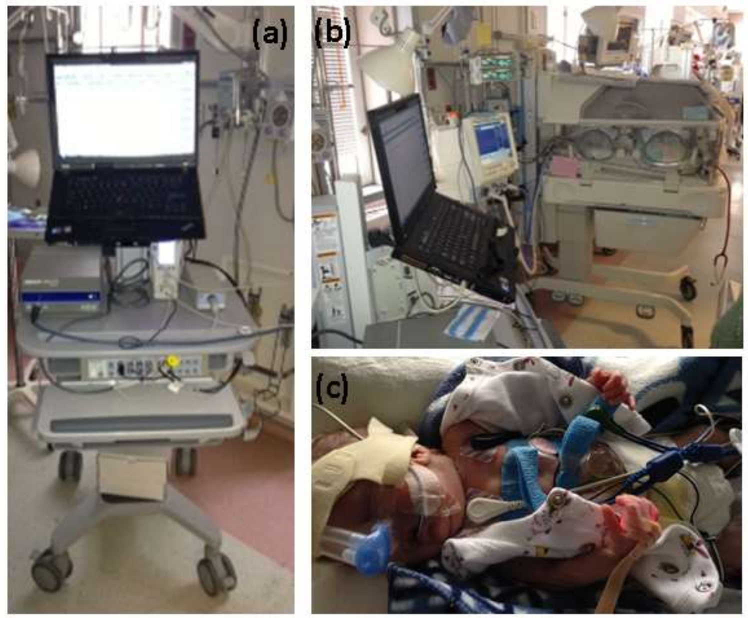
- Training experience: unlabelled data (eg gene level activity)
- What to learn: interesting associations in the data (often no single correct answer)
- E.g., clustering, dimensionality reduction
- Typical goal: produce a model that maximizes data likelihood

Reinforcement Learning Framework



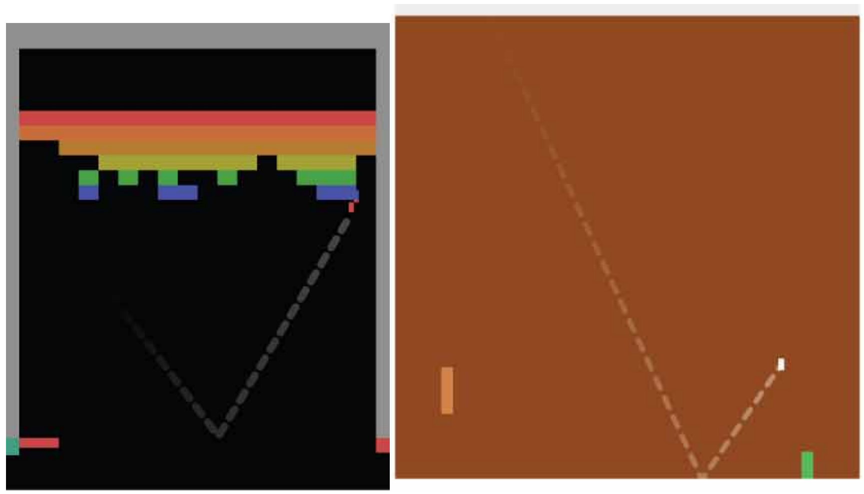
- At every time step t, the agent perceives the *state* of the environment
- Based on this perception, it chooses an *action*
- The action causes the agent to receive a *numerical reward*
- *Prediction:* Learn the expected cumulated future reward given the current state and current way of behaving
- *Control:* Find a way of choosing actions, called a *policy* which *maximizes the agent's long-term expected return*

Prediction Example: Medical Time Series (Apex Project)



- The states are cardio-respiratory measurements
- Reward is the patient outcome at the end of the procedure (delayed)
- Policy is unknown (hospital practice)

Control Example: Atari Games (Mnih et al, 2015)



- The states are board positions in which the agent can move
- The actions are the possible joystick moves allowed by the game
- Reward is given by the points achieved in the game

Key Features of RL Control

- The learner is not told what actions to take, instead it find finds out what to do by *trial-and-error search*

  Eg. Players trained by playing thousands of simulated games, with no expert input on what are good or bad moves
- The environment is *stochastic*
- The *reward may be delayed*, so the learner may need to sacrifice short-term gains for greater long-term gains

  Eg. Player might get reward only at the end of the game, and needs to assign credit to moves along the way
- The learner has to balance the need to *explore* its environment and the need to *exploit* its current knowledge

  Eg. One has to try new strategies but also to win games

Implementing reinforcement learning

- A *policy* $\pi : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$ is a way of choosing actions
- The *value of a state* is the *expected value of a long-term return* (cumulative function of the rewards)
  - E.g. average reward per time step over a long horizon
  - E.g. Discounted return:

$$V^*(s) = \max_{\pi} \mathbb{E}_{\pi}[r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \ldots | s_t = s]$$

  where $\gamma \in [0, 1]$ is a discount factor (probability of the task finishing at each step, or inflation rate) and π dictates the choices of action
- One can also *condition on actions as well as states:* $Q(s, a)$
- General approach: approximate the value of the current policy from data, then use these values to guide policy change
- If an action leads to an *improved state of affairs*, the tendency to pick it is strengthened (i.e., the *action is reinforced*)

The Curse of Dimensionality

- Values are governed by nice recursive equations:

$$V_{k+1}(s) \leftarrow \max_{a \in \mathcal{A}} \left(r_a(s) + \gamma \sum_{s' \in \mathcal{S}} P_a(s'|s) V_k(s') \right), \forall s \in \mathcal{S}$$

- The number of states grows *exponentially* with the number of state variables (the dimensionality of the problem)
  E.g. in Go, there are 10^{170} states
- The *action set* may also be very large or continuous
  E.g. in Go, branching factor is ≈ 100 actions
- The solution may require *chaining many steps* to find any information
  E.g. in Go games take ≈ 200 actions

How to Handle RL Big Data

- *Approximate the iterations* (using sampling, cf. asynchronous dynamic programming, temporal-difference learning)
- *Generalize* the value function to unseen states using *function approximation*
- *Shape the time scale* and nature of the actions using *temporal abstraction*

Simplifying the iterations
Temporal-difference (TD) learning (Sutton, 1988)

- Instead of looping over all states as in a Bellman backup target:

$$\left(r_a(s) + \gamma \sum_{s' \in \mathcal{S}} P_a(s'|s) V_k(s') \right), \forall s \in \mathcal{S}$$

  we will *sample transition* and use the samples
- Estimated value at time t: $V(s_t)$
- Estimated value at time $t+1$: $r_{t+1} + \gamma V(s_{t+1})$
- *Temporal-difference error*:

$$\delta = [r_{t+1} + \gamma V(s_{t+1})] - V(s_t)$$

  This is the *surprise* based on the new information at time step $t+1$
- Main idea: use TD-error to drive the learning of the correct values

Representing Value Functions

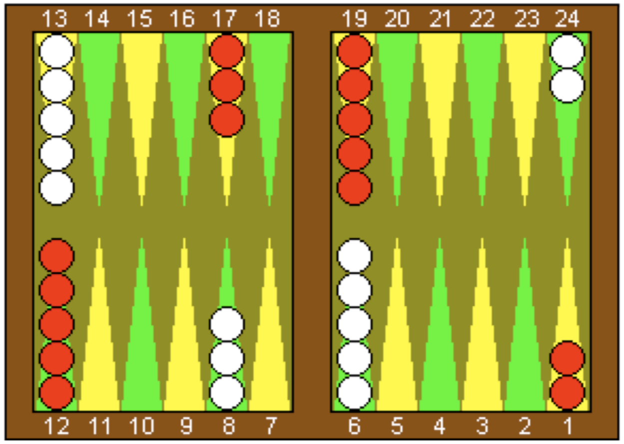
- Instead of using vectors with one entry per state, suppose that V is represented by some *function approximator* taking as input a description of the state, or *feature vector* ϕ_s
- E.g. Fitted Value Iteration:

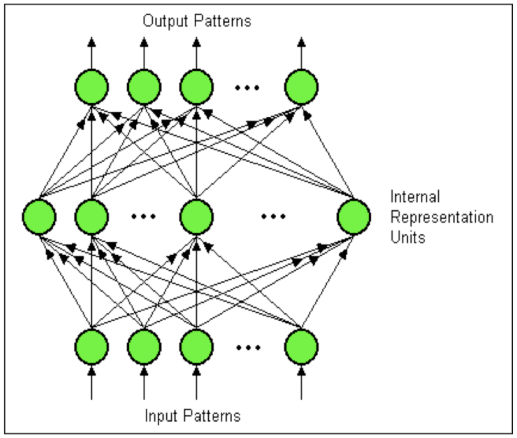
  Given $\langle s, a, s', r \rangle$ tuples and a current estimate $Q(s, a)$, form a data set of inputs ϕ_s and outputs $r + \gamma \max_{a'} Q(\phi_{s'}, a')$ and train a new approximation for Q
- We gain both in terms of space, and in terms of ability to *generalize* data to new situations
- Note that unlike in supervised learning, *target values depend on the current approximator* which causes interesting theoretical issues

What kind of function approximators?

- Linear (e.g. Sutton, 1998; Silver et al, 2010; Keller et al, 2006)
- Random projections (Fard et al, 2012)
- Nearest-neighbor
- Kernels (e.g. Barreto et al, 2012, 2013)
- Neural networks / deep architectures (e.g. Mnih et al, 2015)
- Randomized trees (e.g. Ernst et al, 2006)
- ...

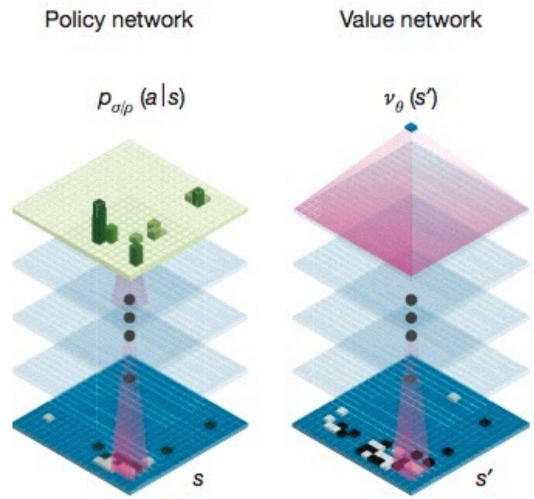
Example: TD-Gammon (Tesauro, 1990-1995)





- Early predecessor of AlphaGo
- Learning from self-play, using TD-learning
- Became the best player in the world
- Discovered new ways of opening not used by people before

Example: AlphaGo (Silver et al, 2015-present)

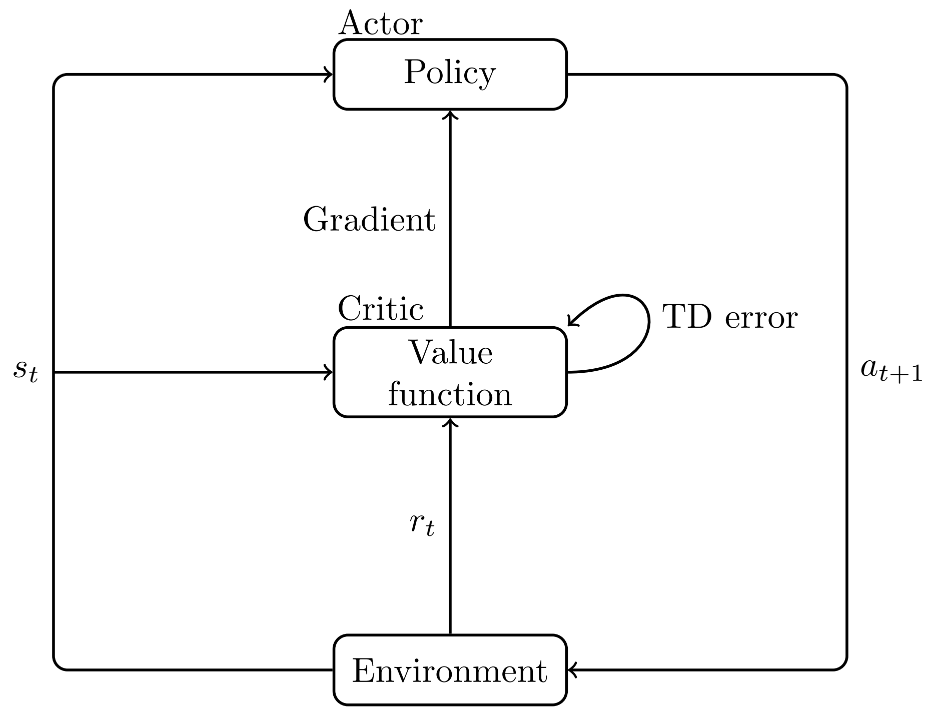


- Perceptions: state of the board
- Actions: legal moves
- Reward: +1 or -1 at the end of the game
- Trained by playing games against itself
- Invented new ways of playing which seem superior

Policy Search

- Sometimes, the value function might be complex but the policy itself may be simple (Farahmand et al, 2015)
- Instead of relying on the value function, one can search through a space of parametrized policies π_θ
- Outline:
  1. Initialize candidate policy
  2. Repeat
     - Estimate a new direction in which to move the parameters (using Monte Carlo, value-based methods etc)
     - Adjust the policy

Actor-critic architecture



- Clear optimization objective: average or discounted return
- Continual learning
- Handles both discrete and continuous states and actions

What is temporal abstraction?

- Consider an activity such as cooking dinner

  - High-level steps: choose a recipe, make a grocery list, get groceries, cook,...
  - Medium-level steps: get a pot, put ingredients in the pot, stir until smooth, check the recipe ...
  - Low-level steps: wrist and arm movement while driving the car, stirring, ...
- All have to be seamlessly integrated!
- Cf. macro actions in classical AI, controllers in robotics

Formalization of temporal abstraction

- Hierarchical abstract machines (Parr, 1998)
- MAXQ (Dietterich, 1998)
- Dynamic motion primitives (Schaal et al. 2004)
- Skills (Konidaris et al, 2009)
- Feudal RL (Dayan, 1994)
- *Options* (Sutton, Precup & Singh, 1999; Precup, 2000)

Options framework

- Suppose we have an MDP $\langle \mathcal{S}, \mathcal{A}, r, P, \gamma \rangle$
- An *option* ω consists of 3 components
  - An *initiation set* of states $I_\omega \subseteq \mathcal{S}$ (aka precondition)
  - A *policy* $\pi_\omega : \mathcal{S} \times \mathcal{A} \to [0, 1]$
    $\pi_\omega(a|s)$ is the probability of taking a in s when following option ω
  - A termination condition $\beta_\omega : \mathcal{S} \to [0, 1]$:
    $\beta_\omega(s)$ is the probability of terminating the option ω upon entering s
- Eg., robot navigation: if there is no obstacle in front (I_ω), go forward (π_ω) until you get too close to another object (β_ω)

Cf. Sutton, Precup & Singh, 1999; Precup, 2000

Options as behavioral programs

- *Call-and-return execution*
  - Option is a subroutine which gets called by a policy over options π_Ω
  - When called, ω is pushed onto the execution stack
  - During the option execution, the program looks at certain *variables (aka state)* and executes an *instruction (aka action)* until a termination condition is reached
  - The option can keep track of additional *local variables*, eg counting number of steps, saturation in certain features (e.g. Comanici, 2010)
  - *Options can invoke other options*
- *Interruption*
  - At each step, one can check if a better alternative has become available
  - If so, the option currently executing is *interrupted* (special form of concurrency)
- *The option identity is also a form of memory: what is the agent currently trying to achieve?* Cf. Shaul et al, 2014, Kulkarni et al, 2016

Option models

- *Option model* has two parts:
  1. *Expected reward* $r_\omega(s)$: the expected return during ω's execution from s
     - Needed because it is used to update the agent's internal representations
  2. *Transition model* $P_\omega(s'|s)$: a sub-probability distribution over next states (reflecting the discount factor γ and the option duration) given that ω executes from s
     - P specifies *where* the agent will end up after the option/program execution and *when* termination will happen
- Models are *predictions* about the future, conditioned on the option being executed

Option models provide semantics

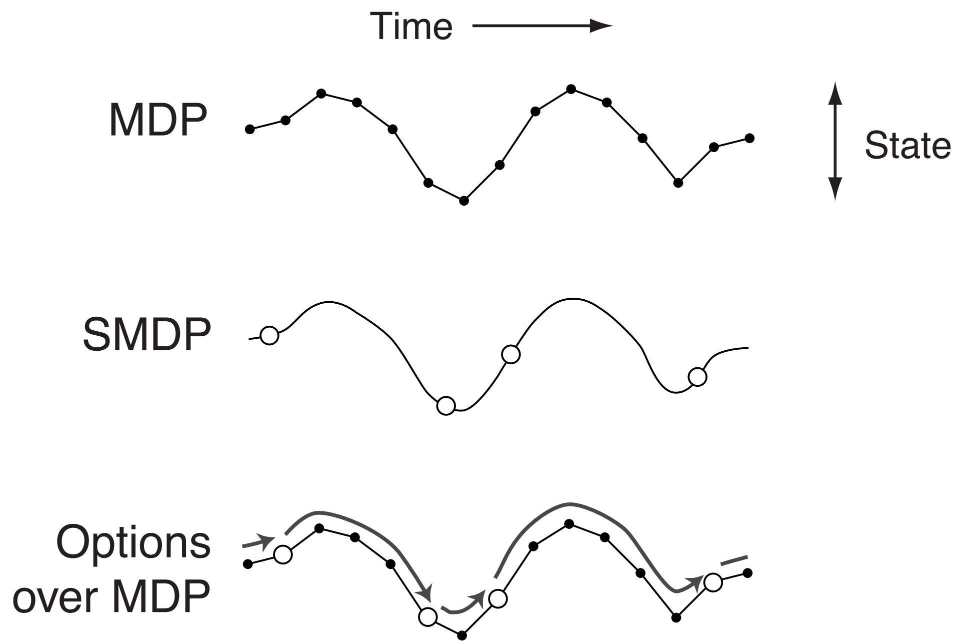
- Programming languages: preconditions (initiation set) and postconditions
- *Models of options represent (probabilistic) post-conditions*
- *Models that are compositional*, can be used to reason about the policy over options
- *Sequencing*

$$\begin{aligned} \mathbf{r}_{\omega_1\omega_2} &= \mathbf{r}_{\omega_1} + P_{\omega_1}\mathbf{r}_{o_2} \\ P_{\omega_1\omega_2} &= P_{\omega_1}P_{\omega_2} \end{aligned}$$

  Cf. Sutton et al, 1999, Sorg & Singh, 2010
- *Stochastic choice*: can take expectations of reward and transition models
- These are sufficient conditions to allow Bellman equations to hold
- Silver & Ciosek (2012): re-write model in one matrix, compose models to construct programs

  Eg. good generalization in Towers of Hanoi

MDP + Options = Semi-Markov Decision Precess



- Introducing options in an MDP induces a related semi-MDP
- Hence *all planning and learning algorithms* from classical MDPs transfer directly to options (Cf. Sutton, Precup & Singh, 1999; Precup, 2000)
- But planning and learning with options can be much faster!

Illustration: Navigation

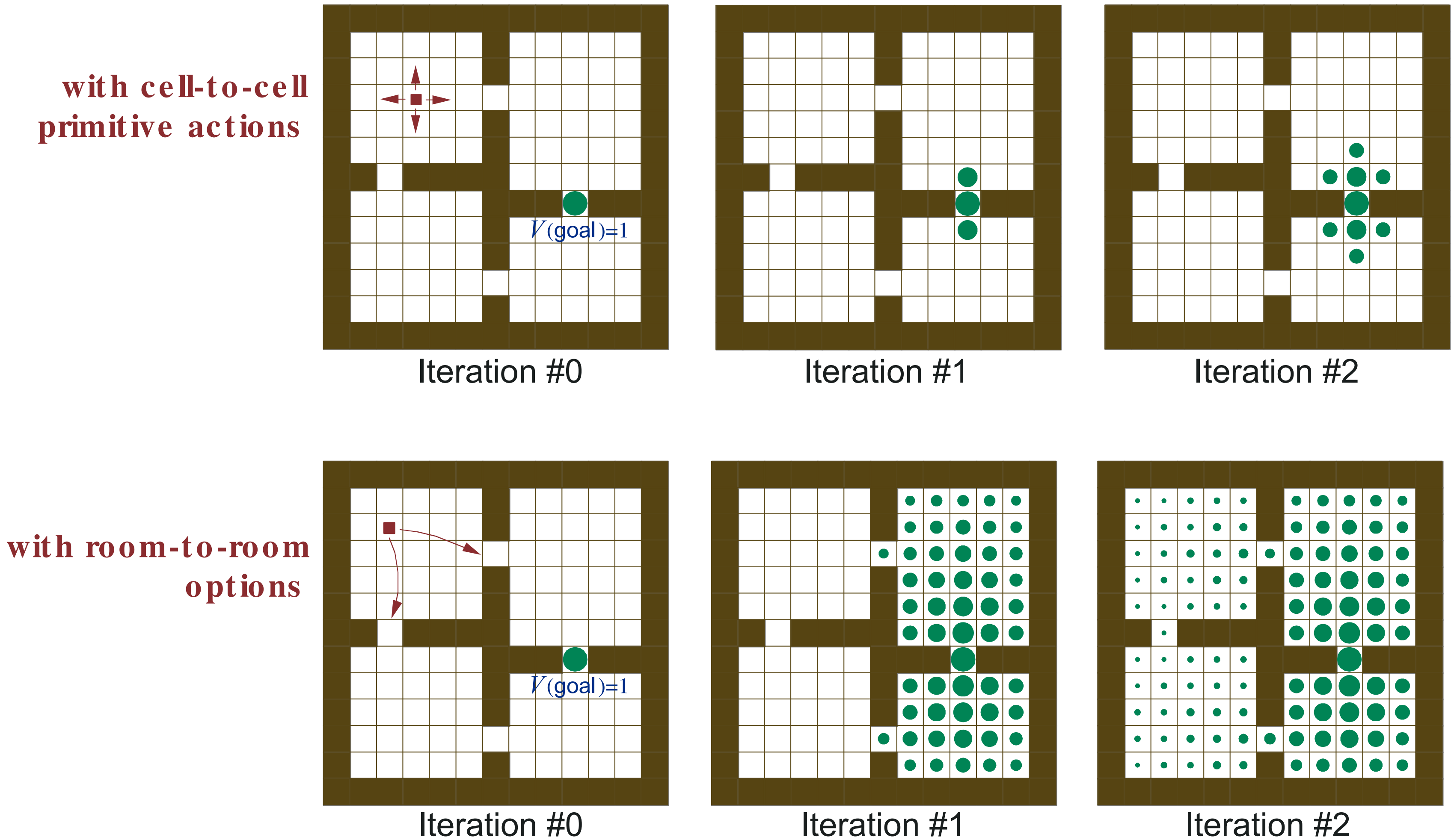
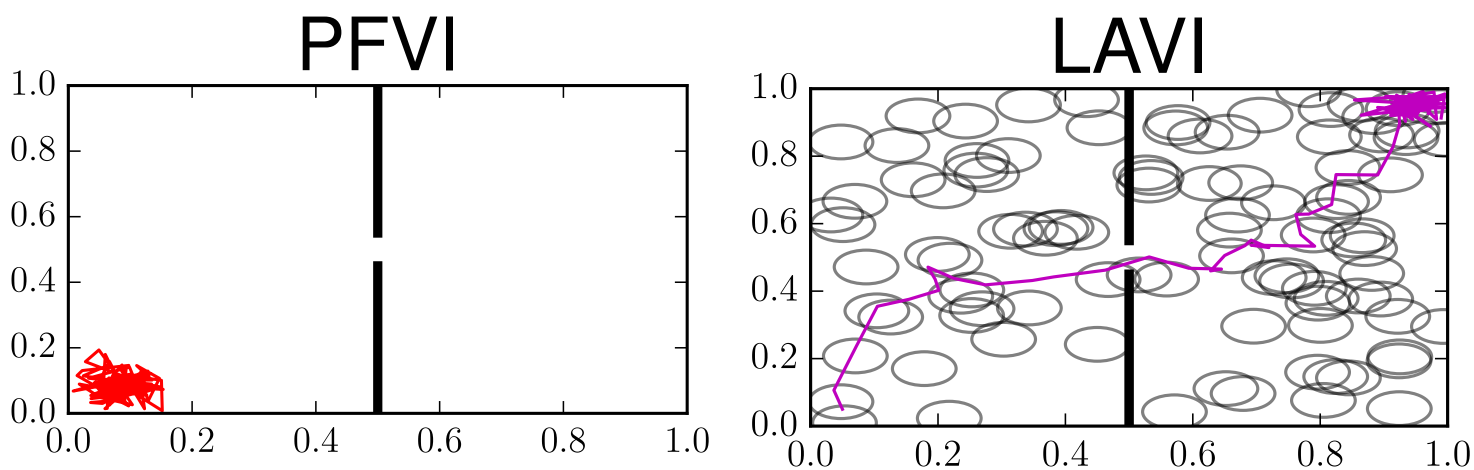


Illustration: Random landmarks

- *Generate a lot of options, then worry about which are useful!*
- Large set of *landmarks*, i.e. states in the environment, chosen at random (Mann, Mannor & Precup, 2015)
- Rough planner which can get to a landmark from its vicinity, by solving a *deterministic relaxation* of the MDP



Landmark-based approximate value iteration gets a good solution much faster!

The anatomy of the reward option model

- Primitive action model: $r_a(s) = \mathbb{E}[r_t | s_t = s, a_t = a]$
- Option model:

$$r_\omega(s) = \mathbb{E}[r_t + \gamma r_{t+1} + \ldots | s_t = s, \omega_t = \omega]$$

- This expectation indicates a Markov-style property, as it depends only on the identity of the state and the option, not on the time step
- Notice the *model is basically a value function* so we can write Bellman equations for the model:

$$r_\omega(s) = \sum_a \pi_\omega(a|s)[r_a(s) + \sum_{s'} \gamma(1 - \beta_\omega(s'))r_\omega(s')]$$

- This means that we can use RL methods to learn the models of options!
- Very similar equations hold for the transition model

Intra-option algorithms

- Learning about one option at a time is very inefficient
- In fact, we may not want to execute options at all!
- Instead, *learn about all options consistent with the behaviour*
- In some sense, a form of *attention*
- E.g. action-value function, tabular case
  On single-step transition $\langle s, a, r, s' \rangle$, for all ω that could have been executing in s and taken a:

$$\begin{aligned} Q_\Omega(s,\omega) &= Q_\Omega(s,\omega) + \alpha[r_a(s) + \gamma(1 - \beta_\omega(s'))Q_\Omega(s',\omega) + \\ &+ \gamma\beta_\omega(s') \sum_{s'} \max_{\omega'} Q_\Omega(s',\omega') - Q_\Omega(s,\omega)] \end{aligned}$$

  Red: continuation. Blue: termination
- In general function approximation, importance sampling will need to be used (several papers on this)

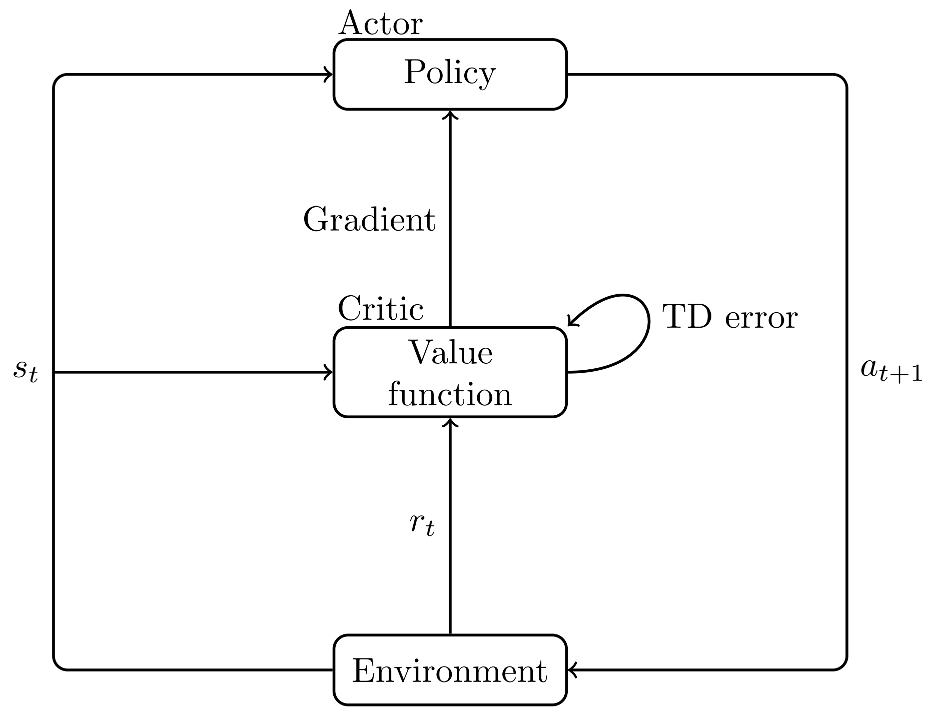
Frontier: Option Discovery

- Options can be given by a system designer
- If subgoals / secondary reward structure is given, the option policy can be obtained, by solving a smaller planning or learning problem (cf. Precup, 2000)
- *What is a good set of subgoals / options?*
- This is a *representation discovery* problem
- Studied a lot over the last 15 years
- Bottleneck states and change point detection currently the most successful methods

Goals of our current work

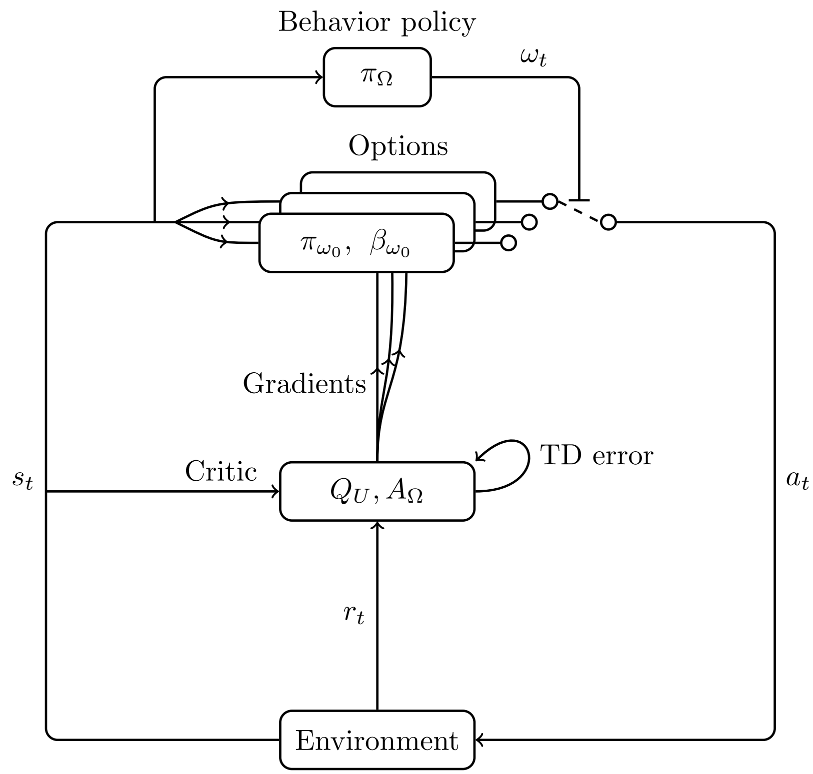
- Explicitly state an *optimization objective* and then solve it to find a set of options
- Handle both *discrete and continuous* set of state and actions
- Learning options should be *continual* (avoid combinatorially-flavored computations)
- Options should provide *improvement within one task* (or at least not cause slow-down...)

Actor-critic architecture



- Clear optimization objective: average or discounted return
- Continual learning
- Handles both discrete and continuous states and actions

Option-critic architecture (Bacon et al, 2017)



- Parameterize internal policies and termination conditions
- Policy over options is computed by a separate process (planning, RL, ...)

Formulation

- The option-value function of a policy over options π_Ω is given by

$$Q_{\pi_\Omega}(s,\omega) = \sum_a \pi_\omega(a|s) Q_U(s,\omega,a)$$

where

$$Q_U(s,\omega,a) = r_a(s) + \gamma \sum_{s'} P_a(s'|s) U(\omega, s')$$

- The last quantity is the utility from s' onwards, *given that we arrive in s' using ω*

$$U(\omega, s') = (1 - \beta_\omega(s')) Q_{\pi_\Omega}(s', \omega) + \beta_\omega(s') V_{\pi_\Omega}(s')$$

- We parameterize the internal policies by θ, as $\pi_{\omega,\theta}$, and the termination conditions by ν, as $\beta_{\omega,\nu}$
- *Note that θ and ν can be shared over the options!*

Main result: Gradient updates

- Suppose we want to optimize the expected return: $\mathbb{E}\left\{Q_{\pi_\Omega}(s,\omega)\right\}$
- The *gradient wrt the internal policy parameters* θ is given by:

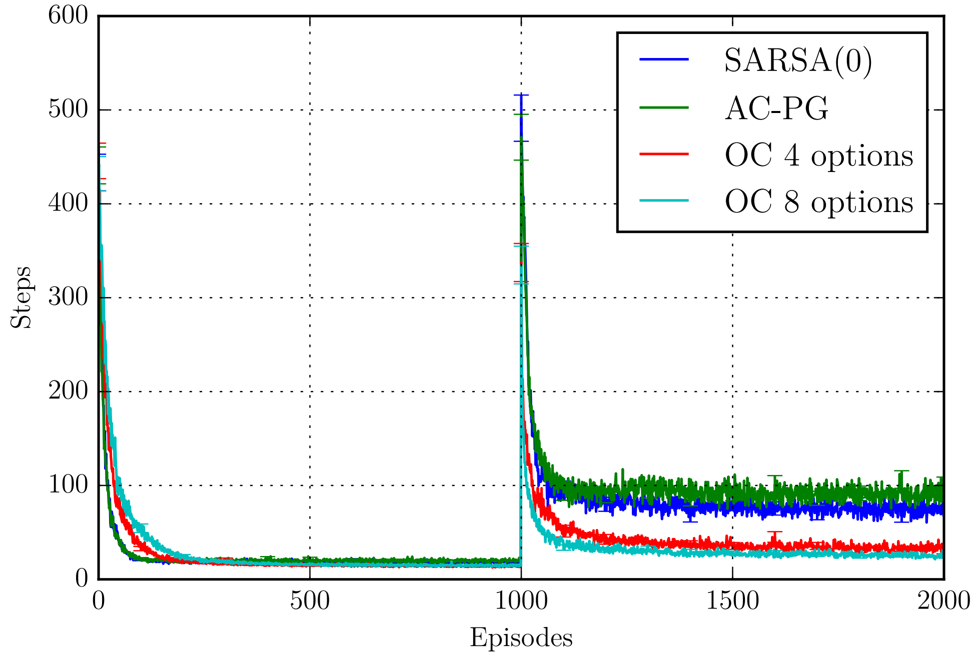
$$\mathbb{E}\left\{\frac{\partial \log \pi_{\omega,\theta}(a|s)}{\partial \theta} Q_U(s,\omega,a)\right\}$$

  This has the usual interpretation: *take better primitives more often* inside the option
- The *gradient wrt the termination parameters* ν is given by:

$$\mathbb{E}\left\{-\frac{\partial \beta_{\omega,\nu}(s')}{\partial \nu} A_{\pi_\Omega}(s',\omega)\right\}$$

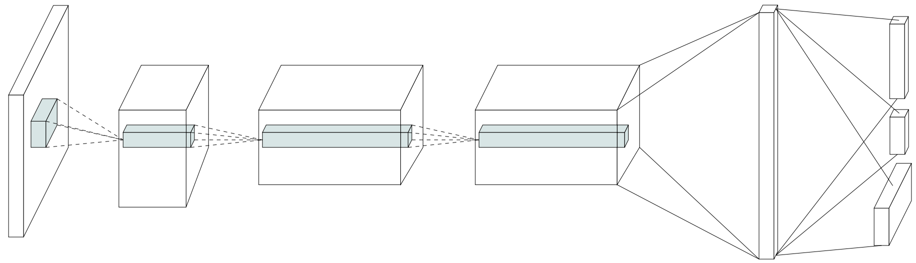
  where $A_{\pi_\Omega} = Q_{\pi_\Omega} - V_{\pi_\Omega}$ is the advantage function
  This means that we want to *lengthen options that have a large advantage*

Results: Options transfer

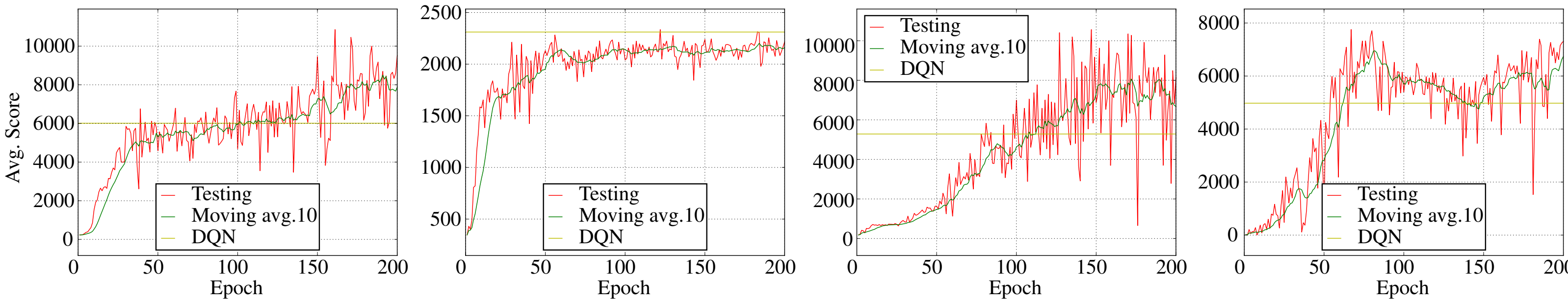


- 4-rooms domain, tabular representations, value functions learned by Sarsa
- Learning in the first task no slower than using primitives
- Learning once the goal is moved faster with the options

Results: Nonlinear function approximation



- Atari simulator, DQN to learn value function over options, actor as above

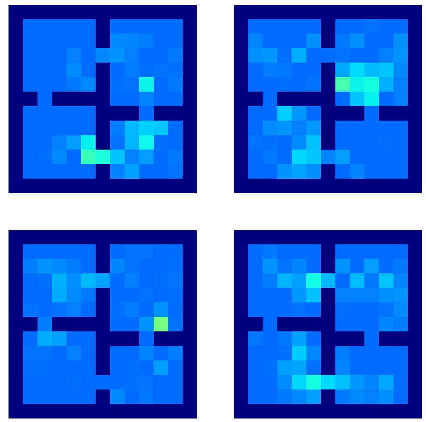


(a) Asterix (b) Ms. Pacman (c) Seaquest (d) Zaxxon

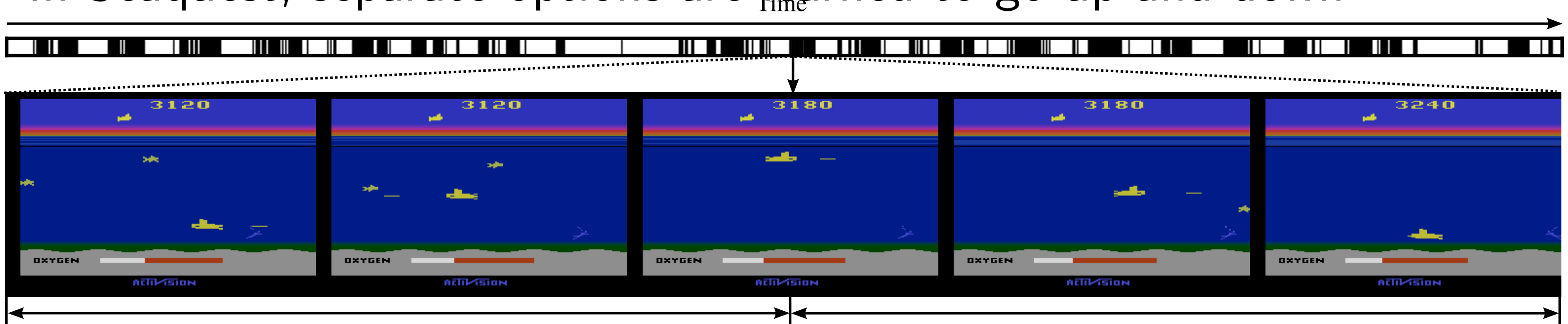
- Performance matching or better than DQN

Results: Learned options are intuitive

- In rooms environment, terminations are more likely near hallways (although there are no pseudo-rewards provided)



- In Seaquest, separate options are learned to go up and down



What are beneficial options

- Successful simultaneous learning of terminations and option policies
- But, as expected, *options shrink over time* unless a margin is required for the advantage

  Cf. time-regularized options, Mann et al, (2014)
- Intuitively, using longer options increase the speed of learning and planning (but may lead to a worse result in call-and-return execution)
- What is the right tool to formalize this intuition?

A proposal: Deliberation cost

- Assumption: *executing a policy is cheap, deciding what to do is expensive*
  - Many choices may need to be evaluated (branching factor over actions)
  - In planning, many next states may need to be considered (branching factor over states)
  - Evaluating the function approximator might be expensive (e.g. if it is a deep net)
- Deliberation is also expensive in animals:
  - Energy consumption (to engage higher-level brain function)
  - Missed opportunity cost: thinking too long means action is delayed

Problem formulation

- Let $c(s, \omega)$ be the immediate cost of deliberating to choose ω in s
- In the call-and-return model, it is easy to see that we have a *value function that expresses total deliberation cost* given by the following Bellman equation:

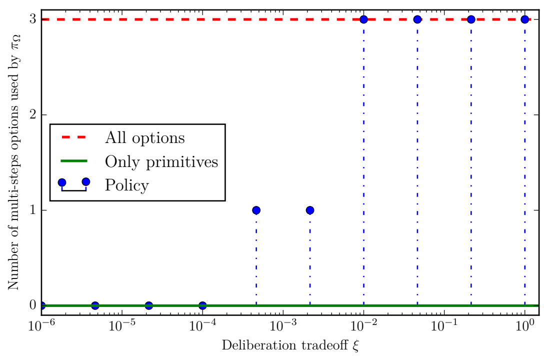
$$Q_c(s, \omega) = -c(s, \omega) + \sum_{s'} P_\omega(s'|s) \sum_{\omega'} \pi_\Omega(\omega'|s') Q_c(s', \omega')$$

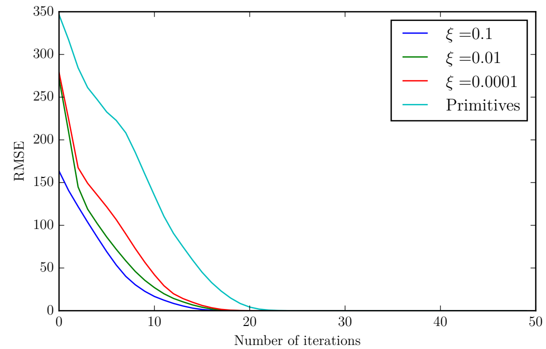
- We can obtain Q_c using learning, value iteration etc
- *New objective: maximize reward with reasonable effort*

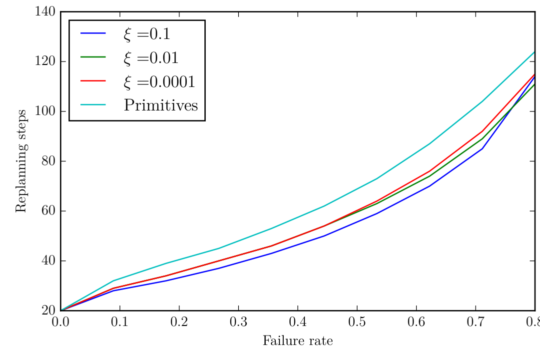
$$\max_\Omega \mathbb{E}\left[Q_\Omega(s, \omega) + \xi Q_c(s, \omega)\right]$$

- $\xi \geq 0$ controls the trade-off between value and computation effort ($\xi = 0$ means optimizing original reward)

Illustration: 4 rooms, option-critic







- Emphasizing deliberation cost, shifts the policy towards using options
- Number of iterations of planning is smaller for higher deliberation cost penalties
- When options are learned in one task and then used to plan in a different task, options obtained with deliberation costs are more robust

Conclusions

- Reinforcement learning is useful for temporal prediction under uncertainty as well as stochastic control
- *Good representations exist to re-shape the state and action space to handle larger problems, and increase efficiency*
- Temporal abstraction methods developed in reinforcement learning provide syntax and semantics of behavioral programs
- Option-critic allows using policy gradient ideas for *continual learning of temporal abstractions*, but there are lots of things to do:
  - More empirical work in option construction
  - Tighter integration with Neural Turing Machines and similar models
  - Improved reward shaping, eg see new Ms Pacman results from van Seijn et al, Maluuba/Microsoft
  - Other execution models