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An MDP-Based Recommender System *

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Abstract

Typical recommender systems adopt a static view of the recommendation process and treat it as a prediction problem. We argue that it is more appropriate to view the problem of generating recommendations as a sequential optimization problem and, consequently, that Markov decision processes (MDPs) provide a more appropriate model for recommender systems. MDPs introduce two benefits: they take into account the long-term effects of each recommendation and the expected value of each recommendation. To succeed in practice, an MDP-based recommender system must employ a strong initial model, must be solvable quickly, and should not consume too much memory. In this paper, we describe our particular MDP model, its initialization using a predictive model, the solution and update algorithm, and its actual performance on a commercial site. We also describe the particular predictive model we used which outperforms previous models. Our system is one of a small number of commercially deployed recommender systems. As far as we know, it is the first to report experimental analysis conducted on a real commercial site. These results validate the commercial value of recommender systems, and in particular, of our MDP-based approach.

Keywords: recommender systems, Markov decision processes, learning, commercial applications

1. Introduction

In many markets, consumers are faced with a wealth of products and information from which they can choose. To alleviate this problem, many web sites attempt to help users by incorporating a *recommender system* (Resnick and Varian, 1997) that provides users with a list of items and/or web-pages that are likely to interest them. Once the user makes her choice, a new list of recommended items is presented. Thus, the recommendation process is a sequential process. Moreover, in many domains, user choices are sequential in nature – for example, we buy a book by the author of a recent book we liked.

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The sequential nature of the recommendation process was noticed in the past (Zimdars et al., 2001). Taking this idea one step farther, we suggest that recommendation is not simply a sequential prediction problem, but rather, a sequential decision problem. At each point the Recommender System makes a decision: which recommendation to issue. This decision should take into account the sequential process involved and the optimization criteria suitable for the recommender system, such as the profit generated from selling an item. Thus, we suggest the use of Markov decision processes (MDP) (Puterman, 1994), a well known stochastic model of sequential decisions.

With this view in mind, a more sophisticated approach to recommender systems emerges. First, one can take into account the utility of a particular recommendation – for example, we might want to recommend a product that has a slightly lower probability of being bought, but generates higher profits. Second, we might suggest an item whose immediate reward is lower, but leads to more likely or more profitable rewards in the future.

These considerations are taken into account automatically by any good or optimal policy generated for an MDP model of the recommendation process. In particular, an optimal policy will take into account the likelihood of a recommendation to be accepted by the user, the immediate value to the site of such an acceptance, and the long-term implications of this on the user's future choices. These considerations are taken with the appropriate balance to ensure the generation of the maximal expected reward stream.

For instance, consider a site selling electronic appliances faced with the option to suggest a video camera with a success probability of 0.5, or a VCR with a probability of 0.6. The site may choose the camera, which is less profitable, because the camera has accessories that are likely to be purchased, whereas the VCR does not. If a video-game console is another option with a smaller success probability, the large profit from the likely future event of selling game cartridges may tip the balance toward this latter choice. Similarly, when the products sold are books, by recommending a book for which there is a sequel, we may increase the likelihood that this sequel will be purchased later.

Indeed, in our implemented system, we observed less obvious instances of such sequential behavior: users who purchased novels by the well-known science fiction author, Roger Zelazny, who uses many mythological themes in his writing, often later purchase books on Greek or Hindu mythology. On the other hand, users who buy mythology books do not appear to buy Roger Zelazny novels afterwards.

The benefits of an MDP-based recommender system discussed above are offset by the fact that the model parameters are unknown. Standard reinforcement learning techniques that learn optimal behaviors will not do – they take considerable time to converge and their initial behavior is random. No commercial site will deploy a system with such behavior. Thus, we must find ways for generating good initial estimates for the MDP parameters. The approach we suggest initializes a predictive model of user behavior using data gathered on the site prior to the implementation of the recommender system. We then use the predictive model to provide initial parameters for the MDP.

Our initialization process can be performed using *any* predictive model. In this paper we suggest a particular model that outperforms previous approaches. The predictive model we describe is motivated by our sequential view of the recommendation process, but constitutes an independent contribution. The model can be thought of as an n-gram model (Chen and Goodman, 1996) or, equivalently, a (first-order) Markov chain in which states correspond to sequences of events. In this paper, we emphasize the latter interpretation due to its natural relationship with an MDP. We note that Su et al. (2000) have described the use of simple n-gram models for predicting web pages.

Their methods, however, yield poor performance on our data, probably because in our case, due to the relatively limited data set, the use of the enhancement techniques discussed below is needed.

Validating recommender system algorithms is not simple. Most recommender systems, such as dependency networks (Heckerman et al., 2000), are tested on historical data for their predictive accuracy. That is, the system is trained using historical data from sites that do not provide recommendations, and tested to see whether the recommendations conform to actual user behavior. We present the results of a similar test with our system showing it to perform better than the previous leading approach.

However, predictive accuracy is not an ideal measure, as it does not test how user behavior is influenced by the system's suggestions or what percentage of recommendations are accepted by users. To obtain this data, one must employ the system at a real site with real users, and compare the performance of this site with and without the system (or with this and other systems). The extent to which such experiments are possible is limited, as commercial site owners are unlikely to allow experiments which can degrade the performance or the "look-and-feel" of their systems. However, we were able to perform a certain set of experiments using our commercial system at the online bookstore Mitos (www.mitos.co.il) by running two models simultaneously on different users: one based on a predictive model and one based on an MDP model. We were also able, for a short period, to compare user behavior with and without recommendations. These results, which to the best of our knowledge are among the first reports of online performance in a commercial site, are reported in Section 6, providing very encouraging validation to recommender systems in general, and to our sequential optimization approach in particular.

The main contributions of this paper are: (1) A novel approach to recommender systems based on an MDP model together with appropriate initialization and solution techniques. (2) A novel predictive model that outperforms previous predictive models. (3) One of a small number of commercial applications based on MDPs. (4) The first (to the best of our knowledge) experimental analysis of a commercially deployed recommender system.

We note that the use of MDPs for recommender systems was previously suggested by Bohnenberger and Jameson (2001). They used an MDP to model the process of a consumer navigating within an airport. The state of this MDP was the consumer's position and rewards were obtained when the consumer entered a store or bought an item. Recommendations were issued on a palm-top, suggesting routes and stores to visit. However, the MDP model was hand-coded and experiments were conducted with students rather than real users.

The paper is structured as follows. In Section 2 we review the necessary background on recommender systems, MDPs, and reinforcement learning. In Section 3 we describe the predictive model we constructed whose goal is to accurately predict user behavior in an environment without recommendations. In Section 4 we present our empirical evaluation of the predictive model. In Section 5 we explain how we use this predictive model as a basis for a more sophisticated MDP-based model for the recommender system. In Section 6 we provide an empirical evaluation of the actual recommender system based on data gathered from our deployed system. We conclude the paper in Section 7 discussing our current and future work.

2. Background

In this section we provide the necessary background on recommender systems, N-gram models, and MDPs.

2.1 Recommender Systems

Early in the 1990s, when the Internet became widely used as a source of information, *information explosion* became an issue that needed addressing. Many web sites presenting a wide variety of content (such as articles, news stories, or items to purchase) discovered that users had difficulties finding the items that interested them out of the total selection. *Recommender Systems* (Resnick and Varian, 1997) help users limit their search by supplying a list of items that might interest a specific user. Different approaches were suggested for supplying meaningful recommendations to users and some were implemented in modern sites (Schafer et al., 2001). Traditional data mining techniques such as association rules were tried at the early stages of the development of recommender systems. Initially, they proved to be insufficient for the task, but more recent attempts have yielded some successful systems (Kitts et al., 2000).

Approaches originating from the field of *information retrieval (IR)* rely on the *content* of the items (such as description, category, title, author) and therefore are known as *content-based recommendations* (Mooney and Roy, 2000). These methods use some similarity score to match items based on their content. Based on this score, a list of items similar to the ones the user previously selected can be supplied. *Knowledge-based* recommender systems (Burke, 2000) go one step farther by using deeper knowledge about the user and the domain. In particular, the user is able to introduce explicit information about her preferences. Thus, for instance, the user could specify interest in Thai cuisine, and the system might suggest a restaurant serving some other south-Asian cuisine.

Another possibility is to avoid using information about the content, but rather use historical data gathered from other users in order to make a recommendation. These methods are widely known as *collaborative filtering (CF)* (Resnick et al., 1994), and we discuss them in more depth below. Finally, some systems try to create hybrid models that combine collaborative filtering and content-based recommendations (Balabanovic and Shoham, 1997; Burke, 2002).

2.2 Collaborative Filtering

The collaborative filtering approach originates in human behavior: people searching for an interesting item they know little of, such as a movie to rent at the video store, tend to rely on friends to recommend items they tried and liked. The person asking for advice is using a (small) community of friends that know her taste and can therefore make good predictions as to whether she will like a certain item. Over the net however, a larger community that can recommend items to our user is available, but the persons in this large community know little or nothing about each other. Conceptually, the goal of a collaborative filtering engine is to identify those users whose taste in items is predictive of the taste of a certain person (usually called a *neighborhood*), and use their recommendations to construct a list of items interesting for her.

To build a user's neighborhood, these methods rely on a database of past users interactions with the system. Early systems used *explicit ratings*. In such systems, users grade items (e.g., 5 stars to a great movie, 1 star to a horrible one) and then receive recommendations.[1] Later systems shifted toward *implicit ratings*. A common approach assumes that people like what they buy. A binary grading method is used when a value of 1 is given to items the user has bought and 0 to other items. Many modern recommender systems successfully implement this approach. Claypool et al. (2001) have suggested the use of other implicit grading methods through a special web browser that keeps track of user behavior such as the time spent looking at the web page, the scrolling of the page by

1. An example of such a system can be found at http://www.movielens.umn.edu/.

t	X_{t-2}	X_{t-1}	X_t
1	–	–	x_1
2	–	x_1	x_2
3	x_1	x_2	x_3
4	x_2	x_3	x_4

Table 1: An auto-regressive transformation of the sequence x_1, x_2, x_3, x_4 for $k = 2$.

the user, and movements of the mouse over the page. Their evaluation, however, failed to establish a method of rating that gave results consistently better than the binary method mentioned above.

As described in Breese et al. (1998), collaborative filtering systems are either memory based or model based. Memory-based systems work directly with user data. Given the selections of a given user, a memory-based system identifies similar users and makes recommendations based on the items selected by these users. Model-based systems compress such user data into a predictive model. Examples of model-based collaborative filtering systems are Bayesian networks (Breese et al., 1998) and dependency networks (Heckerman et al., 2000). In this paper, we consider model-based systems.

2.3 The Sequential Nature of the Recommendation Process

Most recommender systems work in a sequential manner: they suggest items to the user who can then accept one of the recommendations. At the next stage a new list of recommended items is calculated and presented to the user. This sequential nature of the recommendation process, where at each stage a new list is calculated based on the user's past ratings, will lead us naturally to our reformulation of the recommendation process as a sequential optimization process.

There is yet another sequential aspect to the recommendation process. Namely, optimal recommendations may depend not only on previous items pruchased, but also on the order in which those items are purchased. Zimdars et al. (2001) recognized this possible dependency and suggested the use of an auto-regressive model (a k-order Markov chain) to represent it. They divided a sequence of transactions $X_1, \ldots, X_T$ (for example, product purchases, web-page views) into cases $(X_{t-k}, \ldots, X_{t-1}, X_t)$ for $t = 1, \ldots, T$ as shown in Table 1. They then built a model (in particular, a dependency network) to predict the last column given the other columns, under the assumption that the cases were exchangeable. Our model will also incorporate this sequential view.

2.4 *N*-gram Models

N-gram models originate in the field of language modeling. They are used to predict the next word in a sentence given the last $n-1$ words. In the simplest form of the model, probabilities for the next word are estimated via maximum likelihood; and many methods exist for improving this simple approach including skipping, clustering, and smoothing. Skipping assumes that the probability of the next word x_i depends on words other than just the previous $n-1$. A separate model is built using skipping and then combined with the standard n-gram model. Clustering is an approach that groups some states together for purposes of predicting next states. For example, we can group items such a basketball, football, and volleyball into a "sports ball" class. Such grouping helps to address the problem of data sparsity. Smoothing is a general name for methods that modify the estimates of probabilities to achieve higher accuracy by adjusting zero or low probabilities upward. One type of smoothing is finite mixture modeling, which combines multiple models via a convex combination. In particular, given k component models for x_i given a prior sequence X—$p_{M_1}(x_i|X),\ldots,p_{M_k}(x_i|X)$—we can define the k-component mixture model $p(x_i|X) = \pi_1 \cdot p_{M_1}(x_i|X) + \cdots + \pi_k \cdot p_{M_k}(x_i|X)$, where $\sum_{i=1}^{k} \pi_i = 1$ are its mixture weights. Details of these and other methods are given in Chen and Goodman (1996).

2.5 MDPs

An MDP is a model for sequential stochastic decision problems. As such, it is widely used in applications where an autonomous agent is influencing its surrounding environment through actions (for example, a navigating robot). MDPs (Bellman, 1962) have been known in the literature for quite some time, but due to some fundamental problems discussed below, few commercial applications have been implemented.

An MDP is by definition a four-tuple: $\langle S, A, Rwd, tr\rangle$, where S is a set of states, A is a set of actions, Rwd is a reward function that assigns a real value to each state/action pair, and tr is the state-transition function, which provides the probability of a transition between every pair of states given each action.

In an MDP, the decision-maker's goal is to behave so that some function of its reward stream is maximized – typically the average reward or the sum of discounted reward. An optimal solution to the MDP is such a maximizing behavior. Formally, a stationary policy for an MDP π is a mapping from states to actions, specifying which action to perform in each state. Given such an optimal policy π, at each stage of the decision process, the agent need only establish what state s it is in and execute the action $a = \pi(s)$.

Various exact and approximate algorithms exist for computing an optimal policy. Below we briefly review the algorithm known as *policy-iteration* (Howard, 1960), which we use in our implementation. A basic concept in all approaches is that of the *value function*. The value function of a policy π, denoted V^π, assigns to each state s a value which corresponds to the expected infinite-horizon discounted sum of rewards obtained when using π starting from s. This function satisfies the following recursive equation:

$$V^\pi(s) = Rwd(s,\pi(s)) + \gamma \sum_{s_j \in S} tr(s,\pi(s),s_j)V^\pi(s_j) \tag{1}$$

where $0 < \gamma < 1$ is the discount factor.[2] An *optimal* value function, denoted V^*, assigns to each state s its value according to an optimal policy π^* and satisfies

$$V^*(s) = \max_{a\in A}[Rwd(s,a)) + \gamma \sum_{s_j \in S} tr(s,a,s_j)V^*(s_j)]. \tag{2}$$

To find a π^* and V^* using the policy-iteration algorithm, we search the space of possible policies. We start with an initial policy $\pi_0(s) = \operatorname*{argmax}_{a\in A} Rwd(s,a)$. At each step we compute the value

2. We use discounting mostly for mathematical convenience. True discounting of reward would have to take into account the actual time in which each book is purchased, which does not seem worth the extra effort involved.

function based on the former policy and update the policy given the new value function:

$$V_i(s) = Rwd(s,\pi_i(s)) + \gamma \sum_{s_j \in S} tr(s,\pi_i(s),s_j)V_i(s_j), \quad (3)$$

$$\pi_{i+1}(s) = \underset{a\in A}{\operatorname{argmax}}\,[Rwd(s,a) + \gamma \sum_{s_j \in S} tr(s,a,s_j)V_i(s_j)]. \quad (4)$$

These iterations will converge to an optimal policy (Howard, 1960).

Solving MDPs is known to be a polynomial problem in the number of states (via a reduction to linear programming (Puterman, 1994)). It is usually more natural to represent the problem in terms of states variables, where each state is a possible assignment to these variables and the number of states is hence exponential in the number of state variables. This well known "curse of dimensionality" makes algorithms based on an explicit representation of the state-space impractical. Thus, a major research effort in the area of MDPs during the last decade has been on computing an optimal policy in a tractable manner using factored representations of the state space and other techniques (for example Boutilier et al. (2000); Koller and Parr (2000)). Unfortunately, these recent methods do not seem applicable in our domain in which the structure of the state space is quite different – that is, each state can be viewed as an assignment to a very small number of variables (three in the typical case) each with very large domains. Moreover, the values of the variables (describing items bought recently) are correlated. However, we were able to exploit the special structure of our state and action spaces using different techniques. In addition, we introduce approximations that exploit the fact that most states – that is, most item sequences – are highly unlikely to occur (a detailed explanation will follow in Section 3).

MDPs extend the simpler Markov chain (MC) model – a well known model of dynamic systems. A Markov chain is simply an MDP without actions. It contains a set of states and a stochastic transition function between states. In both models the next state does not depend on any states other than the current state.

In the context of recommender systems, if we equate actions with recommendations, then an MDP can be used to model user behavior with recommendations – as we show below – whereas an MC can be used to model user behavior without recommendations. Markov chains are also closely related to n-gram models. In a bi-gram model, the choice of the next word depends probabilistically on the previous word only. Thus, a bi-gram is simply a first-order Markov chain whose states correspond to words. An n-gram is a $n-1$-order Markovian model in which the next state depends on the previous $n-1$ states. Such variants of MDP-models are well known. A non-first-order Markovian model can be converted into a first-order model by making each state include information related to the previous $n-1$ states. More general transformation techniques that attempt to reduce the size of the state space have been investigated in the literature (for example, see Bacchus et al. (1996); Thiébaux et al. (2002)).

3. The Predictive Model

Our first step is to construct a predictive model of user purchases, that is, a model that can predict what item the user will buy next. This model does not take into account its influence on the user, as it does not model the recommendation process and its effects. Nonetheless, we shall use a Markov chain, with an appropriate formulation of the state space, as our model. In Section 4 we shall show that our predictive model outperforms previous models, and in Section 5 we shall intialize our MDP-based recommender system using this predictive model.

3.1 The Basic Model

A Markov chain is a model of system dynamics – in our case, user "dynamics." To use it, we need to formulate an appropriate notion of a user state and to estimate the state-transition function.

States. The states in our MC model represent the relevant information that we have about the user. This information corresponds to previous choices made by users in the form of a set of ordered sequences of selections. We ignore data such as age or gender, although it could be beneficial.[3] Thus, the set of states contains all possible sequences of user selections. Of course, this formulation leads to an unmanageable state space with the usual associated problems—data sparsity and MDP solution complexity. To reduce the size of the state space, we consider only sequences of at most k items, for some relatively small value of k. We note that this approach is consistent with the intuition that the near history (for example, the current user session) often is more relevant than selections made less recently (for example, past user sessions). These sequences are represented as vectors of size k. In particular, we use $\langle x_1,\ldots,x_k\rangle$ to denote the state in which the user's last k selected items were $x_1, \ldots, x_k$. Selection sequences with $l < k$ items are transformed into a vector in which x_1 through x_{k-l} have the value *missing*. The initial state in the Markov chain is the state in which every entry has the value *missing*.[4] In our experiments, we used values of k ranging from 1 to 5.

The Transition Function. The transition function for our Markov chain describes the probability that a user whose k recent selections were $x_1,\ldots,x_k$ will select the item x' next, denoted $tr_{MC}(\langle x_1,x_2,\ldots,x_k\rangle,\langle x_2,\ldots,x_k,x'\rangle)$. Initially, this transition function is unknown to us; and we would like to estimate it based on user data. As mentioned, a maximum-likelihood estimate can be used:

$$tr_{MC}(\langle x_1,x_2,x_3\rangle,\langle x_2,x_3,x_4\rangle) = \frac{count(\langle x_1,x_2,x_3,x_4\rangle)}{count(\langle x_1,x_2,x_3\rangle)} \tag{5}$$

where $count(\langle x_1,x_2,...,x_k\rangle)$ is the number of times the sequence $x_1,x_2,...,x_k$ was observed in the data set. This model, however, still suffers from the problem of data sparsity (for example, see Sarwar et al. (2000a)) and performs poorly in practice. In the next section, we describe several techniques for improving the estimate.

3.2 Some Improvements

We experimented with several enhancements to the maximum-likelihood n-gram model on data different from that used in our formal evaluation. The improvements described and used here are those that were found to work well.

One enhancement is a form of *skipping* (Chen and Goodman, 1996), and is based on the observation that the occurrence of the sequence x_1,x_2,x_3 lends some likelihood to the sequence x_1,x_3. That is, if a person bought x_1,x_2,x_3, then it is likely that someone will buy x_3 after x_1. The particular

3. Those user attributes could be incorporated into our model by adding state variables. Attributes with large domains, such as age, can be joined into a (small) number of groups (for example, age groups) to avoid an explosion of the state space. Our similarity and clustering methods (see below) can be adapted to share training data between states with different, but related, attribute values (such as age group 25-30 and age group 30-40).

4. To accommodate systems that collect explicit rather than implicit ratings, each item x_i would be replaced by an item-rating element – for example, x_i =high.

skipping model that we found to work well is a simple additive model. First, the count for each state transition is initialized to the number of observed transitions in the data. Then, given a user sequence $x_1, x_2, \ldots, x_n$, we add the fractional count $1/2^{(j-(i+3))}$ to the transition from $\langle x_i, x_{i+1}, x_{i+2}\rangle$ to $\langle x_{i+1}, x_{i+2}, x_j\rangle$, for all $i+3 < j \leq n$. This fractional count corresponds to a diminishing probability of skipping a large number of transactions in the sequence. We then normalize the counts to obtain the transition probabilities:

$$tr_{MC}(s,s') = \frac{count(s,s')}{\sum_{s'} count(s,s')} \tag{6}$$

where $count(s,s')$ is the (fractional) count associated with the transition from s to s'.

A second enhancement is a form of clustering that we have not found in the literature. Motivated by properties of our domain, the approach exploits similarity of sequences. For example, the state $\langle x,y,z\rangle$ and the state $\langle w,y,z\rangle$ are similar because some of the items appearing in the former appear in the latter as well. The essence of our approach is that the likelihood of transition from s to s' can be predicted by occurrences from t to s', where s and t are similar. In particular, we define the similarity of states s_i and s_j to be

$$sim(s_i,s_j) = \sum_{m=1}^{k} \delta(s_i^m, s_j^m)\cdot(m+1) \tag{7}$$

where $\delta(\cdot,\cdot)$ is the Kronecker delta function and s_i^m is the mth item in state s_i. This similarity is arbitrary up to a constant. In addition, we define the *similarity count* from state s to s' to be

$$simcount(s,s') = \sum_{s_i} sim(s,s_i)\cdot tr_{MC}^{old}(s_i,s') \tag{8}$$

where $tr_{MC}^{old}(s_i,s')$ is the original transition function, with or without skipping (we shall compare the models created with and without the benefit of skipping). The new transition probability from s' to s is then given by[5]

$$tr_{MC}(s,s') = \frac{1}{2} tr_{MC}^{old}(s,s') + \frac{1}{2}\frac{simcount(s,s')}{\sum_{s''} simcount(s,s'')} \tag{9}$$

A third enhancement is the use of finite mixture modeling.[6] Similar methods are used in n-gram models, where—for example—a trigram, a bigram, and a unigram are combined into a single model. Our mixture model is motivated by the fact that larger values of k lead to states that are more informative whereas smaller values of k lead to states on which we have more statistics. To balance these conflicting properties, we mix k models, where the ith model looks at the last i transactions. Thus, for $k = 3$, we mix three models that predict the next transaction based on the last transaction, the last two transactions, and the last three transactions. In general, we can learn mixture weights from data. We can even allow the mixture weights to depend on the given case (and informal experiments on our data suggest that such context-specificity would improve predictive accuracy). Nonetheless, for simplicity, we use $\pi_1 = \cdots = \pi_k = 1/k$ in our experiments. Because our primary model is based on the k last items, the generation of the models for smaller values entails little computational overhead.

5. We examined several weighing techniques and the one described yielded the best results. The use of more complex techniques as well as attempts to learn the proper weights resulted in very minor changes.
6. Note that Equation 9 is also a simple mixture model.

4. Evaluation of the Predictive Model

Before incorporating our predictive model into an MDP-based recommender system, we evaluated the accuracy of the predictive model. Our evaluation used data corresponding to user behavior on a web site (without recommendation) and employed the evaluation metrics commonly used in the collaborative filtering literature. In Section 6 we evaluate the MDP-based approach using an experimental approach in which recommendations on an e-commerce site are manipulated by our algorithms.

4.1 Data Sets

We base our evaluations on real user transactions from the Israeli online bookstore *Mitos* (www.mitos.co.il). Two data sets were used: one containing user transactions (purchases) and one containing user browsing paths obtained from web logs. We filtered out items that were bought/visited less than 100 times and users who bought/browsed no more than one item as is commonly done when evaluating predictive models (for example, Zimdars et al. (2001)). We were left with 116 items and 10820 users in the transactions data set, and 65 items and 6678 users in the browsing data set.[7] In our browsing data, no cookies were used by the site. If the same user visited the site with a new IP address, then we would treat her as a new user. Also, activity on the same IP address was attributed to a new user whenever there were no requests for two hours. These data sets were randomly split into a training set (90% of the users) and a test set (10% of the users).

The rational for removing items that were rarely bought is that they cannot be reliably predicted. This is a conservative approach which implies, in practice, that a rarely visited item will not be recommended by the system, at least initially.

We evaluated predictions as follows. For every user sequence $t_1, t_2, .., t_n$ in the test set, we generated the following test cases:

$$\langle t_1 \rangle, \langle t_1, t_2 \rangle, ..., \langle t_{n-k}, t_{n-k+1}, ..., t_{n-1} \rangle \tag{10}$$

closely following tests done by Zimdars et al. (2001). For each case, we then used our various models to determine the probability distribution for t_i given $t_{i-k}, t_{i-k+1}, ..., t_{i-1}$ and ordered the items by this distribution. Finally, we used the t_i actually observed in conjunction with the list of recommended items to compute a score for the list.

4.2 Evaluation Metrics

We used two scores: Recommendation Score (RC) (Microsoft, 2002) and Exponential Decay Score (ED) (Breese et al., 1998) with slight modifications to fit into our sequential domain.

4.2.1 RECOMMENDATION SCORE

For this measure of accuracy, a recommendation is deemed successful if the observed item t_i is among the top m recommended items (m is varied in the experiments). The score RC is the percentage of cases in which the prediction is successful. A score of 100 means that the recommendation was successful in all cases. This score is meaningful for commerce sites that require a short list of recommendations and therefore care little about the ordering of the items in the list.

7. There are more items and users in the transaction data set since we used transactions over one year, whereas browsing data was collected only during one week.

4.2.2 EXPONENTIAL DECAY SCORE

This measure of accuracy is based on the position of the observed t_i on the recommendation list, thus evaluating not only the content of the list but also the order of items in it. The underlying assumption is that users are more likely to select a recommendation near the top of the list. In particular, it is assumed that a user will actually see the mth item in the list with probability

$$p(m) = 2^{-(m-1)/(\alpha-1)}, (m \geq 1) \tag{11}$$

where α is the half-life parameter—the index of the item in the list with probability 0.5 of being seen. The score is given by

$$100 \cdot \frac{\sum_{c \in C} p(m = pos(t_i|c))}{|C|} \tag{12}$$

where C is the set of all cases, $c = t_{i-k}, t_{i-k+1}, \ldots, t_{i-1}$ is a case, and $pos(t_i|c)$ is the position of the observed item t_i in the list of recommended items for c. We used $\alpha = 5$ in our experiments in order to be consistent with the experiments of Breese et al. (1998) and Zimdars et al. (2001). The relative performance of the models was not sensitive to α.

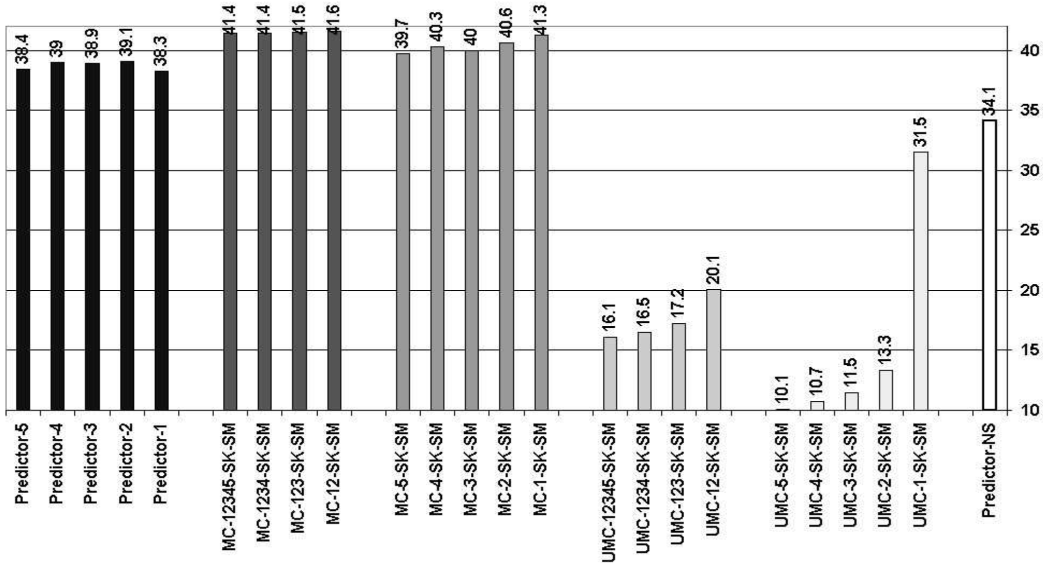
4.3 Comparison Models

4.3.1 COMMERCE SERVER 2000 PREDICTOR

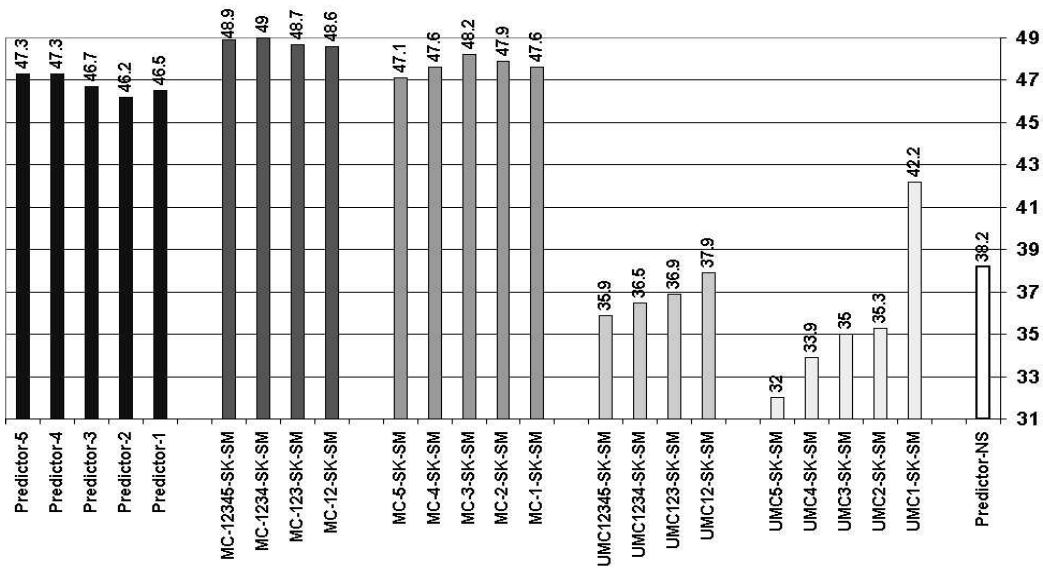
A model to which we compared our results is the *Predictor* tool developed by Microsoft as a part of Microsoft Commerce Server 2000, based on the models of Heckerman et al. (2000). This tool builds dependency-network models in which the local distributions are probabilistic decision trees. We used these models in both a non-sequential and sequential form. These two approaches are described in Heckerman et al. (2000) and Zimdars et al. (2001), respectively. In the non-sequential approach, for every item, a decision tree is built that predicts whether the item will be selected based on whether the remaining items were or were not selected. In the sequential approach, for every item, a decision tree is built that predicts whether the item will be selected next, based on the previous k items that were selected. The predictions are normalized to account for the fact that only one item can be predicted next. Zimdars et al. (2001) also use a "cache" variable, but preliminary experiments showed it to decrease predictive accuracy. Consequently, we did not use the cache variable in our formal evaluation.

These algorithms appear to be the most competitive among published work. The combined results of Breese et al. (1998) and Heckerman et al. (2000) show that (non-sequential) dependency networks are no less accurate than Bayesian-network or clustering models, and about as accurate as *Correlation*, the most accurate (but computationally expensive) memory-based method. Sarwar et al. (2000b) apply dimensionality reduction techniques to the user rating matrix, but their approach fails to be consistently more accurate than Correlation. Only the sequential algorithm of Zimdars et al. (2001) is more accurate than the non-sequential dependency network to our knowledge.

We built five sequential models $1 \leq k \leq 5$ for each of the data sets. We refer to the non-sequential Predictor models as Predictor-NS, and to the Predictor models built using the data expansion methods with a history of length k as Predictor-k.



(a) Transactions data set.

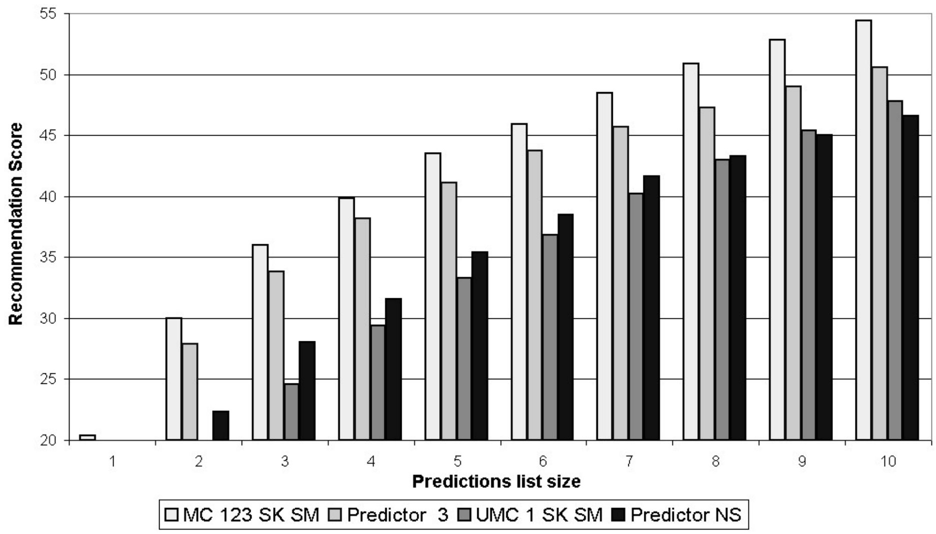


(b) Browsing data set.

Figure 1: Exponential decay score for different models.

4.3.2 UNORDERED MCS

We also evaluated a non-sequential version of our predictive model, where sequences such as $\langle x,y,z\rangle$ and $\langle y,z,x\rangle$ are mapped to the same state. If our assumption about the sequential nature of recommendations is incorrect, then we should expect this model to perform better than our MC model, as it learns the probabilities using more training data for each state, gathering all the ordered data into one unordered set. Skipping, clustering, and mixture modeling were included as described in section 2. We call this model UMC (Unordered Markov chain).



(a) Transactions data set.

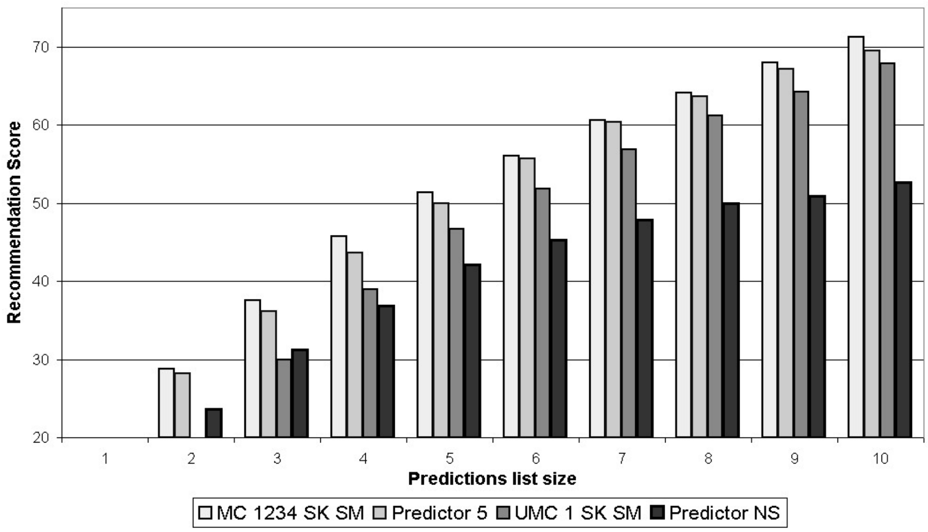
4.4 Variations of the MC Model

In order to measure how each *n*-gram enhancement influenced predictive accuracy, we also evaluated models that excluded some of the enhancements. In reporting our results, we refer to a model that uses skipping and similarity clustering with the terms SK and SM, respectively. In addition, we use numbers to denote which mixture components are used. Thus, for example, we use MC 123 SK to denote a Markov chain model learned with three mixture components—a bigram, trigram, and quadgram—where each component employs skipping but not clustering.

4.5 Experimental Results

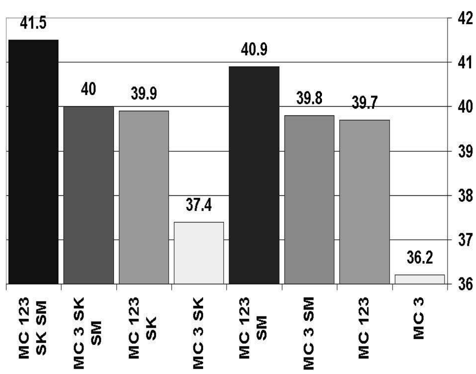
Figure 1(a) and figure 1(b) show the exponential decay score for the best models of each type (Markov chain, Unordered Markov chain, Non-Sequential Predictor model, and Sequential Predictor Model). It is important to note that *all* the MC models using skipping, clustering, and mixture modelling yielded better results than *every one of* the Predictor-k models and the non-sequential Predictor model. We see that the sequence-sensitive models are better predictors than those that ignore sequence information. Furthermore, the Markov chain predicts best for both data sets.

Figure 2(a) and Figure 2(b) show the recommendation score as a function of list length (m). Once again, sequential models are superior to non-sequential models, and the Markov chain models are superior to the Predictor models.

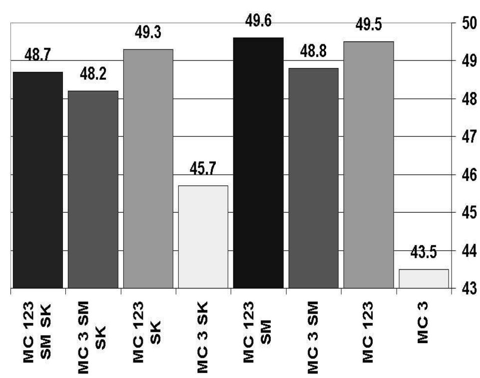


(b) Browsing data set.

Figure 2: Recommendation score for different models.



(a) Transactions data set.



(b) Browsing data set.

Figure 3: Exponential decay score for different Markov chain versions.

Figure 3(a) and Figure 3(b) show how different versions of the Markov chain performed under the exponential decay score in both data sets. We see that multi-component models out-perform single-component models, and that similarity clustering is beneficial. In contrast, we find that skipping is only beneficial for the transactions data set. Perhaps users tend to follow the same paths in a rather conservative manner, or site structure does not allow users to “jump ahead”. In either case, once recommendations are available in the site (thus changing the site structure), skipping may prove beneficial.

5. An MDP-Based Recommender Model

The predictive model we described above does not attempt to capture the short and long-term effect of recommendations on the user, nor does it try to optimize its behavior by taking into account such effects. We now move to an MDP model that explicitly models the recommendation process and attempts to optimize it. The predictive model plays an important role in the construction of this model.

We assume that we are given a set of cases describing user behavior within a site that does not provide recommendations, as well as a probabilistic predictive model of a user acting without recommendations generated from this data. The set of cases is needed to support some of the approximations we make, and in particular, the lazy initialization approach we take. The predictive model provides the probability the user will purchase a particular item x given that her sequence of past purchases is $x_1,\dots,x_k$. We denote this value by $Pr_{pred}(x|x_1,\dots,x_k)$, where $k=3$ in our case. It is important to stress that the approach presented here is independent of the particular technique by which the above predictive value is approximated. Naturally, in our implementation we used the predictive model developed in Section 3, but there are other ways of constructing such a model (for example, Zimdars et al. (2001); Kadie et al. (2002)).

5.1 Defining the MDP

Recall that to define an MDP, we need to provide a set of states, actions, transition function, and a reward function. We now describe each of these elements. The states of the MDP for our recommender system are k-tuples of items (for example, books, CDs), some prefix of which may contain null values corresponding to missing items. This allows us to model shorter sequences of purchases.

The actions of the MDP correspond to a recommendation of an item. One can consider multiple recommendations but, to keep our presentation simple, we start by discussing single recommendations.

Rewards in our MDP encode the utility of selling an item (or showing a web page) as defined by the site. Because the state encodes the list of items purchased, the reward depends on the last item defining the current state only. For example, the reward for state $\langle x_1,x_2,x_3\rangle$ is the reward generated by the site from the sale of item x_3. In this paper, we use net profit for reward.

The state following each recommendation is determined by the user's response to that recommendation. When we recommend an item x', the user has three options:

- Accept this recommendation, thus transferring from state $\langle x_1,x_2,x_3\rangle$ into $\langle x_2,x_3,x'\rangle$
- Select some non-recommended item x'', thus transferring the state $\langle x_1,x_2,x_3\rangle$ into $\langle x_2,x_3,x''\rangle$.
- Select nothing (for example, when the user terminates the session), in which case the system remains in the same state.

Thus, the stochastic element in our model is the user's actual choice. The transition function for the MDP model:

$$tr^1_{MDP}(\langle x_1,x_2,x_3\rangle, x', \langle x_2,x_3,x''\rangle) \tag{13}$$

is the probability that the user will select item x'' given that item x' is recommended in state $\langle x_1, x_2, x_3\rangle$. We write tr^1_{MDP} to denote that only single item recommendations are used.

5.1.1 INITIALIZING tr_{MDP}

Proper initialization of the transition function is an important implementation issue in our system. Unlike traditional model-based reinforcement learning algorithms that learn the proper values for the transition function and hence an optimal policy online, our system needs to be fairly accurate when it is first deployed. A for-profit e-commerce[8] site is unlikely to use a recommender system that generates irrelevant recommendations for a long period, while waiting for it to converge to an optimal policy. We therefore need to initialize the transition function carefully. We can do so based on any good predictive model, making the following assumptions:

- A recommendation increases the probability that a user will buy an item. This probability is proportional to the probability that the user will buy this item in the absence of recommendations. This assumption is made by most collaborative filtering models dealing with e-commerce sites.[9] We denote the proportionality constant for recommendation r in state s by $\alpha_{s,r}$, where $\alpha_{s,r} > 1$.
- The probability that a user will buy an item that was not recommended is lower than the probability that she will buy when the system issues no recommendations at all, but still proportional to it. We denote the proportionality constant for recommendation r in state s by $\beta_{s,r}$, where $\beta_{s,r} < 1$.

To allow for a simpler representation of the equations, for a state $s = \langle x_1, ..., x_k\rangle$ and a recommendation r let us use $s \cdot r$ to denote the state $s' = \langle x_2, ..., x_k, r\rangle$. We use $tr_{predict}(s, s\cdot r)$ to denote the probability that the user will choose r next, given that its current state is s according to the predictive model in which recommendations are not considered, that is, $Pr_{pred}(r|s)$. Thus, with $\alpha_{s,r}$ and $\beta_{s,r}$ constant over s and r and equal to α and β, respectively, we have

$$tr^1_{MDP}(s, r, s\cdot r) = \alpha \cdot tr_{predict}(s, s\cdot r), \tag{14}$$

the probability that a user will buy r next if it was recommended;

$$tr^1_{MDP}(s, r', s\cdot r) = \beta \cdot tr_{predict}(s, s\cdot r),\ r' \neq r, \tag{15}$$

the probability that a user will buy r if something else was recommended; and

$$tr^1_{MDP}(s, r, s) = 1 - tr^1_{MDP}(s, r, s\cdot r) - \sum_{r' \neq r} tr^1_{MDP}(s, r, s\cdot r'), \tag{16}$$

the probability that a user will not buy any new item after r was recommended. We do not see a reason to stipulate a particular relationship between α and β, although we must have

$$tr^1_{MDP}(s, r, s\cdot r) + \sum_{r' \neq r} tr^1_{MDP}(s, r', s\cdot r) < 1. \tag{17}$$

8. We use the term e-commerce, although our system, and recommender systems in general, can be used in content sites and other applications.
9. Actually CF models do not refer to the presence of recommendations, but using such systems to generate recommendations to users in commercial applications has the underlying assumption that the recommendation will increase the likelihood that a user will purchase an item.

The exact values of $\alpha_{s,r}$ and $\beta_{s,r}$ should be chosen carefully. Choosing $\alpha_{s,r}$ and $\beta_{s,r}$ to be constants over all states and recommendations (say $\alpha = 2$, $\beta = 0.5$) might cause the sum of transition probabilities in the MDP to exceed 1. The approach we took was motivated by Kitts et al. (2000), who showed that the *increase* in the probability of following a recommendation is large when one recommends items having high *lift*, defined to be $\frac{pr(x|h)}{pr(x)}$. Thus, it is not unreasonable to assume that this increase in probability is proportional to lift:

$$pr(r|s,r) - pr(r|s,r') \sim \gamma \frac{p(r|s)}{p(r)} \tag{18}$$

where $p(r)$ is the prior probability of buying r. Fixing $\alpha_{s,r}$ to be a little larger than 1 as follows:

$$\alpha_{s,r} = \frac{\gamma + p(r)}{p(r)} \tag{19}$$

where γ is a very small constant (we use $\gamma = \frac{1}{1000}$), and solving for $\beta_{s,r}$, we obtain

$$\beta_{s,r} = \frac{1 - \sum_{r'} \alpha_{s,r'} p(s \cdot r'|s)}{(n-1)\ p(s \cdot r|s)} + \alpha_{s,r}. \tag{20}$$

If $\beta_{s,r}$ is negative, we set it to a very small positive value and normalize the probabilities afterwards.

There are a few things to note about $tr^1_{MDP}(s,r',s\cdot r)$, the probability that a user will buy r if something else was recommended, and its representation. First, since $tr^1_{MDP}(s,r',s\cdot r) = \beta_{s,r} \cdot tr(s,s\cdot r)$, the MDP's initial transition probability does not depend on r' because our initialization is based on data that was collected without the benefit of recommendations. Of course, if one has access to data that reflects the effect of recommendations ($pr_{predict}(s\cdot r|s,r)$), one can use it to provide a more accurate initial model. Next, note that we can represent this transition function concisely using at most two values for every state-item pair: the probability that an item will be selected in a state when it is recommended (that is, $pr(s\cdot r|s,r)$) and the probability that an item will be selected when it is not recommended (that is, $pr(s\cdot r|s,r')$). Because the number of items is much smaller than the number of states, we obtain significant reduction in the space requirements of the model.

5.1.2 GENERATING MULTIPLE RECOMMENDATIONS

When moving to multiple recommendations, we make the assumption that recommendations are independent. Namely we assume that for every pair of sets of recommended items, R, R', we have that

$$(r \in R \wedge r \in R') \vee (r \notin R \wedge r \notin R') \implies tr_{MDP}(s,R,s\cdot r) = tr_{MDP}(s,R',s\cdot r) \tag{21}$$

This assumption might prove to be false. It seems reasonable that, as the list of recommendations grows, the probability of selecting any item decreases. Another more subtle example is the case where the system "thinks" that the user is interested in an inexpensive cooking book. It can then recommend a few very expensive cooking books and one is reasonably priced (but in no way cheap) cooking book. The reasonably priced book will seem like a bargain compared to the expensive ones, thus making the user more likely to buy it.

Nevertheless, we make this assumption so as not to be forced to create a larger action space where actions are ordered combinations of recommendations. Taking the simple approach for representing the transition function we defined above, we still keep only two values for every state–item pair:

$$tr_{MDP}(s, r \in R, s \cdot r) = tr^1_{MDP}(s, r, s \cdot r), \tag{22}$$

the probability that r will be bought if it appeared in the list of recommendations; and

$$tr_{MDP}(s, r \notin R, s \cdot r) = tr^1_{MDP}(s, r', s \cdot r) \text{ for all } r' \neq r, \tag{23}$$

the probability that r will be bought if it did not appear in the list.

As before, $tr_{MDP}(s, r \notin R, s \cdot r)$ does not depend on r, and will not depend on R in the discussion that follows. We note again, that these values are merely reasonable initial values and are adjusted by our system based on actual user behavior, as we shall discuss.

5.2 Solving the MDP

Having defined the MDP, we now consider how to solve it in order to obtain an optimal policy. Such a policy will, in effect, tell us what item to recommend given any sequence of user purchases. For the domains we studied, we found policy iteration (Howard, 1960)—with a few approximations to be described—to be a tractable solution method. In fact, on tests using real data, we found that policy iteration terminates after a few iterations. This stems from the special nature of our state space and the approximations we make, as we now explain.

Our state space enjoys a number of features that lead to fast convergence of the policy iteration algorithm:

Directionality. Transitions in our state space seem to have inherent directionality: First, a state representing a short sequence cannot follow a state representing a longer sequence. Second, the success of the sequential prediction model indicates that typically, if x is likely to follow y, y is less likely to follow x – otherwise, the sequence x, y and y, x would have similar probabilities, and we could simply use sets. Thus, loops, which in principle could occur in our MDP model because we maintain only a limited amount of history, are not very likely. Indeed, an examination of the loops in our state space graph reveals them to be small and scarce. Moreover, in the web site implementation, it is easy enough to filter out items that were already bought by the user from our list of recommendations. It is well-known that directionality can be used to reduce the running time of MDP solution algorithm (for example, Bonet and Geffner (2003)).

Insensitivity to k. We have also found that the computation of an optimal policy is not heavily sensitive to variations in k—the number of past transactions we encapsulate in a state. As k increases, so does the number of states, but the number of positive entries in our transition matrix remains similar. Note that, at most, a state can have as many successors as there are items. When k is small, the number of observed successors for a state can be large. When k grows, however, the number of successors decreases considerably. Table 2 demonstrates this relation in our implemented model.

Despite these properties of the state space, policy evaluation still requires much effort given the large state and action space we have to deal with. To alleviate this problem we resort to a number of approximations.

Ignoring Unobserved States. The vast majority of states in our models do not correspond to sequences that were observed in our training set because most combinations of items are extremely unlikely. For example, it is unlikely to find adjacent purchases of a science-fiction and a gardening book. We leverage this fact to save both space and computation time. First, we maintain transition probabilities only for states for which a transition occurred in our training data. These transitions

k	Number of states	Average number of successors
1	16,859	15.56
2	79,640	11.98
3	89,221	3.92

Table 2: The number of initialized states and the average number of state successors for different values of k.

correspond to pairs of states of the form s and $s \cdot r$. Thus, the number of transitions required per state is bounded by the number of items rather than by an amount exponential in k in the worst case. The non-zero transitions are stored explicitly, and as can be inferred from Table 2, their number is much smaller than the total number of entries in the explicit transition matrix. And while much memory is still required, in Section 6.2, we show that these requirements are not too large for modern computers to handle.

Moreover, we do not compute a policy choice for a state that was not encountered in our training data. When the value of such a state is needed for the computation of an optimal policy of some observed state, we simply use its immediate reward. That is, if the sequence $\langle x,y,z\rangle$ did not appear in the training data, we do not calculate a policy for it and assume its value to be $R(z)$—the reward for the last item in the sequence. Note that given the skipping and clustering methods we use, the probability of making a transition from some (observed) sequence $\langle w,x,y\rangle$ to $\langle w,x,y\rangle$ is not zero even though $\langle x,y,z\rangle$ was never observed. This approximation, although risky in general MDPs, is motivated by the fact that in our initial model, for each state there is a relatively small number of items that are likely to be selected; and the probability of making a transition into an un-encountered state is very low. Moreover, the reward (that is, profit) does not change significantly across different states, so, there are no "hidden treasures" in the future that we could miss.

When a recommendation must be generated for a state that was not encountered in the past, we compute the value of the policy for this state online. This requires us to estimate the transition probabilities for a state that did not appear in our training data. We handle such new states in the same manner that we handled states for which we had sparse data in the initial predictive model – that is, using the techniques of skipping, clustering, and finite mixture of unigram, bigram, and trigrams described in Section 3.2.

Using the Independence of Recommendations. One of the basic steps in policy iteration is policy determination. At each iteration, we compute the best action for each state s – that is, the action satisfying:

$$\begin{aligned}
&\operatorname*{argmax}_R \, [Rwd(s) + \gamma \textstyle\sum_{s' \in S} tr(s,R,s')V_i(s')] = \\
&\operatorname*{argmax}_R \, [Rwd(s) + \gamma(\textstyle\sum_{r \in R} tr_{MDP}(s, r \in R, s \cdot r)V_i(s \cdot r) + \\
&\quad \textstyle\sum_{r \notin R} tr_{MDP}(s, r \notin R, s \cdot r)V_i(s \cdot r))]
\end{aligned} \tag{24}$$

where $tr(s, r \in R, s \cdot r)$ and $tr(s, r \notin R, s \cdot r)$ follow the definitions above.

The above equation requires maximization over the set of possible recommendations for each state. The number of possible recommendations is n^{κ}, where n is the number of items and κ is the number of items we recommend each time. To handle this large action space, we make use of our independence assumption. Recall that we assumed that the probability that a user buys a particular item depends on her current state, the item, and whether or not this item is recommended. It does not depend on the identity of the other recommended items. The following method uses this fact to quickly generate an optimal set of recommendations for each state.

Let us define $\Delta(s,r)$ – the additional value of recommending r in state s:

$$\Delta(s,r) = (tr(s, r \in R, s \cdot r) - tr(s, r \notin R, s \cdot r))V(s \cdot r). \tag{25}$$

Now define

$$\begin{aligned} R^{s,\kappa}_{max\Delta} = \{r_1, \ldots, r_\kappa | \Delta(s,r_1) \geq \ldots \geq \Delta(s,r_\kappa) \text{ and} \\ \forall r \neq r_i (i = 1, \ldots, \kappa), \Delta(s,r_\kappa) \geq \Delta(s,r)\}. \end{aligned} \tag{26}$$

$R^{s,\kappa}_{max\Delta}$ is the set of κ items that have the maximal $\Delta(s,r)$ values.

Theorem 1 $R^{s,\kappa}_{max\Delta}$ *is the set that maximizes* $V_{i+1}(s)$ *– that is,*

$$\begin{aligned} &V_{i+1}(s) = \\ &\quad Rwd(s) + \gamma(\textstyle\sum_{r \in R^{s,\kappa}_{max\Delta}} tr(s, r \in R, s \cdot r)V_i(s \cdot r) + \\ &\quad \textstyle\sum_{r \notin R^{s,\kappa}_{max\Delta}} tr(s, r \notin R, s \cdot r)V_i(s \cdot r)). \end{aligned} \tag{27}$$

Proof Let us assume that there exists some other set of κ recommendations $R \neq R^{s,\kappa}_{max\Delta}$ that maximizes $V_{i+1}(s)$. For simplicity, we shall assume that all Δ values are different. If that is not the case, then R should be a set of recommendations not equivalent to $R^{s,\kappa}_{max\Delta}$. Let r be an item in R but not in $R^{s,\kappa}_{max\Delta}$, and r' be an item in $R^{s,\kappa}_{max\Delta}$ but not in R. Let R' be the set we get when we replace r with r' in R. We need only show that $V_{i+1}(s,R) < V_{i+1}(s,R')$:

$$\begin{aligned} &V_{i+1}(s,R') - V_{i+1}(s,R) = \\ &Rwd(s) + \textstyle\sum_{s'} tr(s,R,s')V_i(s') - (Rwd(s) + \textstyle\sum_{s'} tr(s,R',s')V_i(s')) = \\ &\textstyle\sum_{r'' \in R} tr(s, r'' \in R, s \cdot r'')V_i(s \cdot r) + \textstyle\sum_{r'' \notin R} tr(s, r'' \notin R, s \cdot r'')V_i(s \cdot r'') - \\ &\quad \textstyle\sum_{r'' \in R'} tr(s, r'' \in R', s \cdot r'')V_i(s \cdot r) - \textstyle\sum_{r'' \notin R'} tr(s, r'' \notin R', s \cdot r'')V_i(s \cdot r'') = \\ &tr(s, r \in R, s \cdot r)V_i(s \cdot r) - tr(s, r' \notin R, s \cdot r')V_i(s \cdot r') - \\ &\quad (tr(s, r' \in R', s \cdot r)V_i(s \cdot r) - tr(s, r \notin R', s \cdot r')V_i(s \cdot r)) = \\ &\Delta(s,r) - \Delta(s,r') > 0 \end{aligned} \tag{28}$$

■

To compute $V_{i+1}(s)$ we therefore need to compute all $\Delta(s,r)$ and find $R^{s,\kappa}_{max\Delta}$, making the computation of $V_{i+1}(s)$ independent of the number of subsets (or even worse—ordered subsets) of κ items. The complexity of finding an optimal policy when recommending multiple items at each stage under our assumptions remains the same as the complexity of computing an optimal policy for single item recommendations.

By construction, our MDP optimizes site profits. In particular, the system does not recommend items that are likely to be bought whether recommended or not, but rather recommends items whose likelihood of being purchased is *increased* when they are recommended. Nonetheless, when recommendations are based solely on lift, it is possible that many recommendations will be made for which the absolute probability of a purchase (or click) is small. In this case, if recommendations are seldom followed, users might start ignoring them altogether, making the overall benefit zero. Our model does not capture such effects. One way to remedy this possible problem is to alter the reward function so as to provide a certain immediate reward for the acceptance of a recommendation. Another way to handle this problem is to recommend a book with a large MDP score only if the probability of buying it passes some threshold. We did not find it necessary to introduce these modifications in our current system.

5.3 Updating the Model Online

Once the recommender system is deployed with its initial model, we need to update the model according to actual observations. One approach is to use some form of reinforcement learning—methods that improve the model after each recommendation is made. Although such models need little administration to improve, the implementation requires many calls and computations by the recommender system online, which will lead to slower responses—an undesirable result. A simpler approach is to perform off-line updates at fixed time intervals. The site need only keep track of the recommendations and the user selections and, say, once a week use those statistics to build a new model and replace it with the old one. This is the approach we used.

In order to re-estimate the transition function the following counts are obtained from the recently collected statistics:

- $c_{in}(s,r,s\cdot r)$—the number of times the r recommendation was accepted in state s.
- $c_{out}(s,r,s\cdot r)$—the number of times the user took item r in state s even though it was not recommended,
- $c_{total}(s,s\cdot r)$—the number of times a user took item r while being in state s, regardless of whether it was recommended or not.

We compute the new counts and the new approximation for the transition function at time $t+1$ based on the counts and probabilities at time t as follows:

$$
\begin{aligned}
c_{in}^{t+1}(s,r,s\cdot r) &= c_{in}^{t}(s,r,s\cdot r) + count(s,r,s\cdot r), && (29)\\
c_{total}^{t+1}(s,s\cdot r) &= c_{total}^{t}(s,r,s\cdot r) + count(s,s\cdot r), && (30)\\
c_{out}^{t+1}(s,r,s\cdot r) &= c_{out}^{t}(s,r,s\cdot r) + count(s,s\cdot r) - count(s,r,s\cdot r), && (31)\\
tr(s,r\in R,s\cdot r) &= \frac{c_{in}^{t+1}(s,r,s\cdot r)}{c_{total}^{t+1}(s,s\cdot r)}, && (32)\\
tr(s,r\notin R,s\cdot r) &= \frac{c_{out}^{t+1}(s,r,s\cdot r)}{c_{total}^{t+1}(s,s\cdot r)}. && (33)
\end{aligned}
$$

Note that at this stage the constants $\alpha_{s,r}$ and $\beta_{s,r}$ no longer play a role—they were used only to generate the initial model. We still need to define how the counts at time $t=0$ are initialized. We showed in section 5.1.1 how the transition function tr is initialized, and now we define:

$$c^0_{in}(s,r,s\cdot r) = \xi_s \cdot tr(s,r,s\cdot r), \quad (34)$$
$$c^0_{out}(s,r,s\cdot r) = \xi_s \cdot tr(s,r,s\cdot r), \quad (35)$$
$$c^0_{total}(s,s\cdot r) = \xi_s, \quad (36)$$

where ξ_s is proportional to the number of times the state s was observed in the training data (in our implementation we used $10 \cdot count(s)$). This initialization causes states that were observed infrequently to be updated faster than states that were observed frequently and in whose estimated transition probabilities we have more confidence.[10]

To ensure convergence to an optimal solution, the system must obtain accurate estimates of the transition probabilities. This, in turn, requires that for each state s and for every recommendation r, we observe the response of users to a recommendation of r in state s sufficiently many times. If at each state the system always returns the best recommendations only, then most values for $count(s,r,s\cdot r)$ would be 0, because most items will not appear among the best recommendations. Thus, the system needs to recommend non-optimal items occasionally in order to get counts for those items. This problem is widely known in computational learning as the *exploration versus exploitation tradeoff* (for some discussion of learning rate decay and exploration vs. exploitation in reinforcement learning, see, for example Kaelbling et al. (1996) and Sutton and Barto (1998)). The system balances the need to explore unobserved options in order to improve its model and the desire to exploit the data it has gathered so far in order to get rewards.

One possible solution is to select some constant ε, such that recommendations whose expected value is ε-close to optimal will be allowed—for example, by following a Boltzmann distribution:

$$Pr(choose(r_i)) = \frac{\exp\frac{V(s\cdot r_i)}{\tau}}{\sum_{j=1}^{n} exp\frac{V(s\cdot r_j)}{\tau}} \quad (37)$$

with an ε cutoff—meaning that only items whose value is within ε of the optimal value will be allowed. The exact value of ε can be determined by the site operators. The price of such a conservative exploration policy is that we are not guaranteed convergence to an optimal policy. Another possible solution is to show the best recommendation on the top of the list, but show items less likely to be purchased as the second and third items on the list. In our implementation we use a list of three recommendations where the first one is always the optimal one, but the second and third items are selected using the Boltzman distribution without a cutoff.

We also had to equip our system to change with frequent changes (for example, addition and removal of items). When new items are added, users will start buying them and positive counts for them will appear. At this stage, our system adds new states for these new items, and the transition function is expanded to express the transitions for these new states. Of course, prior to updating the model, the system is not able to recommend those new items (the well-known “cold start” problem (Good et al., 1999) in recommender systems). In our implementation, when the first transition to a state $s\cdot r$ is observed, its probability is initialized to 0.9 the probability of the most likely next item in state s with $\xi_s = 10$. This approach causes the new items to be recommended quite frequently.

One possible approach to handling removed items is to do nothing to our system, in which case the transition probabilities slowly decay to zero. Using this approach, however, we may still

10. This approach is similar to assigning an independent learning rate for each state and decaying it based on the amount of observed data.

insert deleted items into the list of recommended items – an undesirable feature. Consequently, in our Mitos implementation, items are programmatically removed from the model during offline updates. Another solution that we have implemented but not evaluated is to use weighted data and to exponentially decay the weights in time, thus placing more weight on more recently observed transitions.

6. Evaluation of the MDP Recommender Model

The main thesis of this work is that (1) recommendation should be viewed as a sequential optimization problem, and (2) MDPs provide an adequate model for this view. This is to be contrasted with previous systems which used predictive models for generating recommendations. In this section, we present an empirical validation of our thesis. We compare the performance of our MDP-based recommender system (denoted MDP) with the performance of a recommender system based on our predictive model (denoted MC) as well as other variants.

Our studies were performed on the online book store Mitos (www.mitos.co.il) from August, 2002 till April, 2004. During our evaluations, approximately $5000-6000$ different users visited the *Mitos* site daily. Of those, around 900 users inserted items into their basket, thus entering our data-set.[11] On average, each customer inserted 1.97 items into the shopping basket. Over $15,000$ items were available for purchase on the site.

Users received recommendations when adding items to the shopping cart.[12] The recommendations were based on the last k items added to the cart ordered by the time they were added. An example is shown in Figure 4 where the three book covers at the bottom are the recommended items. Every time a user was presented with a list of recommendations on either page, the system stored the recommendations that were presented and recorded whether the user purchased a recommended item. Cart deletions were rare and ignored. Once every two or three weeks, a process was run to update the model given the data that was collected over the latest time period.[13]

We compared the MDP and MC models both in terms of their value or utility to the site as well as their computational costs.

6.1 Utility Performance

Our first set of results is based on the assumption that the transition function we learn for our MDP using data collected *with* recommendations, provides the the best available model of user behavior under recommendation. Under this assumption, we can measure the effect of different recommendation policies. An important caveat is that the states in our MDP correspond to truncated (that is, last k) user sequences. Thus, the model does not exclude repeated purchases of the same item. Despite this shortcoming, we proceeded with the evaluation.

As discussed above, a predictive model can answer queries in the form $Pr(x|h)$—the probability that item x will be purchased given user history h. Recommender systems may employ different strategies when generating recommendations using such a predictive model. Assuming that an MDP formalizes the recommendation problem well, we may use the learned MDP model to evaluate these strategies. The evaluation of the quality of different possible policies for the MDP, each corre-

11. We do not supply accurate numbers for number of users and actual profits due to the request of the site owners.
12. Users also received recommendations when looking at the description of a book, but these recommendations where based only on the user's visit to the current page and not on her cart.
13. The update process was executed by the site administrator manually and therefore the update interval varies.

Figure 4: Recommendations in the shopping cart web page.

sponding to a popular approach to recommending, may shed light on the preferred recommendation strategy.

The MDP model was built using data gathered while the model was running in the site with incremental updates (as described above) for almost a year. We compared four policies, where the first policy uses information about the effect of recommendations, and the remaining policies are based on the predictive model solely:

- Optimal – recommends items based on optimal policy for the MDP.
- Greedy – recommends items that maximize $Pr(x|h) \cdot R(x)$ (where $Pr(x|h)$ is the probability of buying item x given user history h, and $R(x)$ is the value of x to the site – for example, net profit).
- Most likely – recommends items that maximize $Pr(x|h)$.
- Lift – recommends items that maximize $\frac{Pr(x|h)}{Pr(x)}$, where $Pr(x)$ is the prior probability of buying item x.

To evaluate the different policies we ran a simulation of the interaction of a user with the system. During the simulation the system generated a list of recommended items R, from which the simulated user selected the next item, using the distribution $tr(s,R,s \cdot x)$—the probability that the next selected item is x given the current state s and the recommendation list R, simulating the purchase of x by the user. The length of user session was taken from the learned distribution of user session length in the actual site. We ran the simulation for $10,000$ iterations for each policy, and calculated the average accumulated reward for user session.

Policy	Value
Optimal	118.5
Greedy	116.1
Most Likely	117.0
Lift	112.8

Table 3: Performance of different policies.

The results are presented in Table 3. The calculated value for each policy is the sum of discounted profit in (New Israeli Shekels) averaged over all states. We used a weighted average, where the weight of each state was the probability of observing it. Obviously, an optimal policy results in the highest value. However, the differences are small, and it appears that one can use the predictive model alone with very good results.

Next, we performed an experiment to compare the performance of the MDP-based system with that of the MC-based system. In this experiment, each user entering the site was assigned a randomly generated cart-id. Based on the last bit of this cart-id, the user was provided with recommendations by the MDP or MC. Reported mean profits were calculated for each user session (a single visit to the site). Data gathered in both cases was used to update both models.[14]

The deployed system was built using three mixture components, with history length ranging from one to three for both the MDP model and the MC model. Recommendations from the different mixture components were combined using an equal (0.33) weight. We used the policy-iteration procedure and approximations described in Section 5 to compute an optimal policy for the MDP. Our model encoded approximately $25,000$ states in the two top mixture components ($k = 2$, $k = 3$). The reported results were gathered after the model was running in the site with incremental updates (as described above) for almost a year.

During the testing period, 50.7% of the users who made at least one purchase were shown MDP-based recommendations and the other 49.3% of these users were shown MC-based recommendations. For each user, we computed the average site profit per session for that user, leaving out of consideration the first purchase made in each session. The first item was excluded as it was bought without the benefit of recommendations, and is therefore irrelevant to the comparison between the recommender systems.[15]

The average site profit generated by the users was 28% higher for the MDP group.[16] We used a permutation test (see, for example, Yeh (2000)) to see how likely it would be for a difference this large to emerge if there were in fact no systematic difference in the effectiveness of the two recommendation methods.[17] We randomly generated 10000 permutations of the assignments of

14. We update the MC model by recording the transition without considering the recommendation used.
15. This is not entirely accurate as the site also provides recommendations for items in the book description page. We do not present here any experimental results for those recommendations and do not model their effect on the user, but we note that a user that received MDP recommendations in the cart page, got MDP recommendations in the book description page; users who got MC recommendations in the basket got MC recommendations in the description page as well.
16. We are not at liberty to provide accurate numbers.
17. We used a permutation test to establish the validity of our results, as this test is non-parametric, and does not require any prior assumptions about the distribution of the data, and is quite robust to noise in the data. We used the one-tailed version of the test as the directional hypothesis that the MDP recommender is better than the MC recommender has been theoretically motivated above.

session profits to users, for each permutation computing the ratio of average session profits between the MDP and the MC groups. With only 8% of these random assignments was the ratio as large as (or larger than) 1.282. Therefore, the better performance of the MDP recommender is statistically significant with $p = 0.08$ by a one-tailed permutation test.

There are two possible sources for the observed improvement—the MDP may be generating more sales or sales of more expensive items. In our experiment, the average number of items bought per user session was 6.8% in favor of the MDP-based recommender ($p = 0.15$), whereas the average price of items was 4% higher in favor of the MDP-based recommender ($p = 0.04$). Thus, both effects may have played a role.

In our second and last experiment, we compared site performance with and without a recommender system. Ideally, we would have liked to assign users randomly to an experience with and without recommendations. This option was ruled-out by the site owner because it would have led to a non-uniform user experience. Fortunately, the site owner was willing to remove the recommender system from the site for one week. Thus, we were able to compare average profits per user session during two consecutive weeks – one with recommendations and one without recommendations.[18] We found that, when the recommender system was not in use, average site profit dropped 17% ($p = 0.0$). Although, we cannot rule out the possibility that this difference is due to other factors (for example, seasonal effects or special events), these result are quite encouraging.

Overall, our experiments support the claims concerning the added value of using recommendations in commercial web sites and the validity of the MDP-based model for recommender systems.

6.2 Computational Analysis

In this section, we compare computational costs of the MDP-based and the Predictor recommender system.

Our comparison uses the transaction data set and corresponding models described in Section 4. In addition to using the full data set, we measured costs associated with smaller versions of the data in which transactions among only the the top N items were considered, in order to demonstrate the effect of the size of the data-set on performance.

	$N = 15231$	$N = 2661$	$N = 1142$	$N = 354$	$N = 86$
MDP	112	63	58	41	16
Predictor-NS	3504	631	177	80	25

Table 4: Required time (seconds) for model building.

First, let us consider the time it takes to make a recommendation. Recommendation time is typically the most critical of computational costs. If recommendation latency is noticeable, no reasonable site administrator will use the recommender system. Table 5 shows the number of recommendations generated per second by the recommender system. The results show that the MDP model is faster. This result is due to the fact that, with the MDP model, we do almost no computations online. While predicting, the model simply finds the proper state and returns the state's pre-calculated list of recommendations.

18. We display recommendations between 3/27/2003 and 4/3/2003, and without recommendations from 3/19/2003 to 3/26/2003.

	$N = 15231$	$N = 2661$	$N = 1142$	$N = 354$	$N = 86$
MDP	250	277	322	384	1030
Predictor-NS	23	74	175	322	1000

Table 5: Recommendations per second.

The price paid for faster recommendation is a larger memory footprint. Table 6 shows the amount of memory needed to build and store a model in megabytes. The MDP model requires more memory to store than the Predictor model, due to the structured representation of the Predictor model using a collection of decision trees.

Finally, we consider the time needed to build a new model. This computational cost is perhaps the least important parameter when selecting a recommender system, as model building is an off-line task executed at long time intervals (say once a week at most) on a machine that does not affect the performance of the site. That being said, as we see in Table 4, the MDP model has the smallest build times.

	$N = 15231$	$N = 2661$	$N = 1142$	$N = 354$	$N = 86$
MDP	138	74	55.7	33.3	11.4
Predictor-NS	50.1	26	25	22.3	18

Table 6: Required memory (megabytes) for building a model and generating recommendations.

Overall the MDP-based model is quite competitive with the Predictor model. It provides the fastest recommendations at the price of more memory use, and builds models more quickly.

7. Discussion

This paper describes a new model for recommender systems based on an MDP. Our work presents one of a few examples of commercial systems that use MDPs, and one of the first reports of the performance of commercially deployed recommender system. Our experimental results validate both the utility of recommender systems and the utility of the MDP-based approach to recommender systems.

To provide the kind of performance required by an online commercial site, we used various approximations and, in particular, made heavy use of the special properties of our state space and its sequential origin. Whereas the applicability of these techniques beyond recommender systems is not clear, it represents an interesting case study of a successful real system. Moreover, the sequential nature of our system stems from the fact that we need to maintain history of past purchases in order to obtain a Markovian state space. The need to record facts about the past in the current state arises in various domains, and has been discussed in a number of papers on handling non-first-order Markov reward functions (see, for example, Bacchus et al. (1996) or Thiébaux et al. (2002)).

Another interesting technique is our use of off-line data to initialize a model that can provide adequate initial performance.

In the future, we hope to improve our transition function on those states that are seldom encountered using generalization techniques, such as skipping and clustering, that are similar to the ones we employed in the predictive Markov chain model. Other potential improvements are the use of a partially observable MDP to model the user. As a model, this is more appropriate than an MDP, as it allows us to explicitly model our uncertainty about the true state of the user (Boutilier, 2002).

In fact, our current model can be viewed as approximating a particular POMDP by using a finite – rather than an unbounded – window of past history to define the current state. Of course, the computational and representational overhead of POMDPs are significant, and appropriate techniques for overcoming these problems must be developed.

Weaknesses of our predictive (Markov chain) model include the use of *ad hoc* weighting functions for skipping and similarity functions and the use of fixed mixture weights. Although the recommendations that result from our current model are (empirically) useful for ranking items, we have noticed that the model probability distributions are not calibrated. Learning the weighting functions and mixture weights from data should improve calibration. In addition, in informal experiments, we have seen evidence that learning case-dependent mixture weights should improve predictive accuracy.

Our predictive model should also make use of relations between items that can be explicitly specified. For example, most sites that sell items have a large catalogue with hierarchical structure such as categories or subjects, a carefully constructed web structure, and item properties such as author name. Finally, our models should incorporate information about users such as age and gender.

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