Thinking Above the Code

Leslie Lamport
Microsoft Research

Why Think?

Why Think?

It helps us do most things:

Why Think?

It helps us do most things:

Hunting a sabre-toothed tiger.

Why Think?

It helps us do most things:

Hunting a sabre-toothed tiger.

Building a house.

Why Think?

It helps us do most things:

- Hunting a sabre-toothed tiger.
- Building a house.
- Writing a program.

When to Think

When to Think

Hunting a sabre-toothed tiger.

When to Think

Hunting a sabre-toothed tiger.

Before leaving the cave.

When to Think

Hunting a sabre-toothed tiger.

Before leaving the cave.

Building a house.

When to Think

Hunting a sabre-toothed tiger.

- Before leaving the cave.

Building a house.

- Before beginning construction.

When to Think

Hunting a sabre-toothed tiger.

Before leaving the cave.

Building a house.

Before beginning construction.

Writing a program.

When to Think

Hunting a sabre-toothed tiger.

Before leaving the cave.

Building a house.

Before beginning construction.

Writing a program.

Before writing any code.

How to Think

How to Think

"Writing is nature's way of letting you know how sloppy your thinking is."

Guindon

How to Think

"Writing is nature's way of letting you know how sloppy your thinking is."

Guindon

To think, you have to write.

How to Think

"Writing is nature's way of letting you know how sloppy your thinking is."

Guindon

To think, you have to write.

If you're thinking without writing, you only think you're thinking.

What to Write

What to Write

Hunting a sabre-toothed tiger.

What to Write

Hunting a sabre-toothed tiger.

Writing not invented, dangerous activity.

What to Write

Hunting a sabre-toothed tiger.

Writing not invented, dangerous activity.

Building a house.

What to Write

Hunting a sabre-toothed tiger.

Writing not invented, dangerous activity.

Building a house.

Draw blueprints.

What to Write

Hunting a sabre-toothed tiger.

  Writing not invented, dangerous activity.

Building a house.

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Writing a program.

What to Write

Hunting a sabre-toothed tiger.

- Writing not invented, dangerous activity.

Building a house.

- Draw blueprints.

Writing a program.

- Write a blueprint

What to Write

Hunting a sabre-toothed tiger.

Writing not invented, dangerous activity.

Building a house.

Draw blueprints.

Writing a program.

Write a ~~blueprint~~ specification.

Specifications

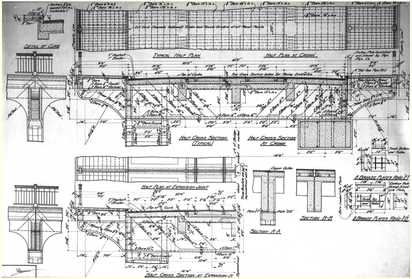
Specifications

Don’t Panic!

Specifications

Don’t Panic!

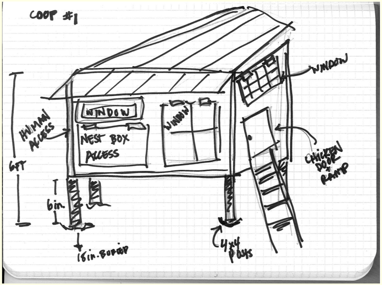
This is a blueprint:



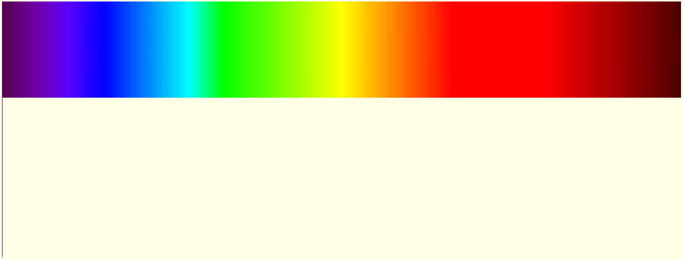
Specifications

Don’t Panic!

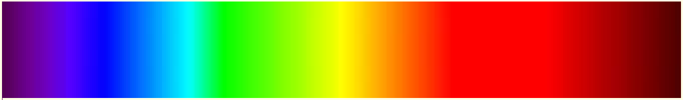
This is also a blueprint:



A spectrum of blueprints

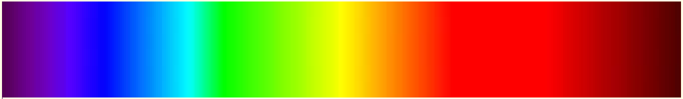


A spectrum of blueprints



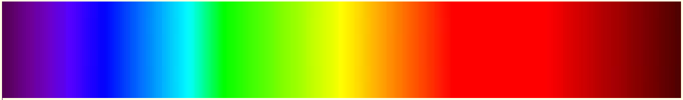
A spectrum of blueprints

Very Detailed



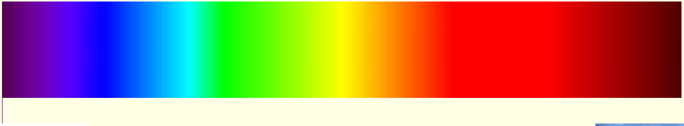
A spectrum of blueprints

Very Detailed



A spectrum of blueprints

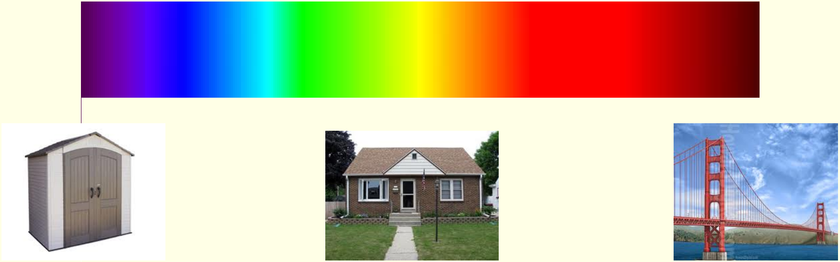
Rough Sketch Very Detailed



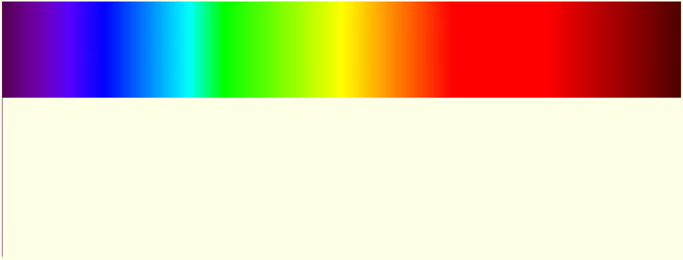
A spectrum of blueprints

A spectrum of blueprints

Rough Sketch Ordinary Very Detailed

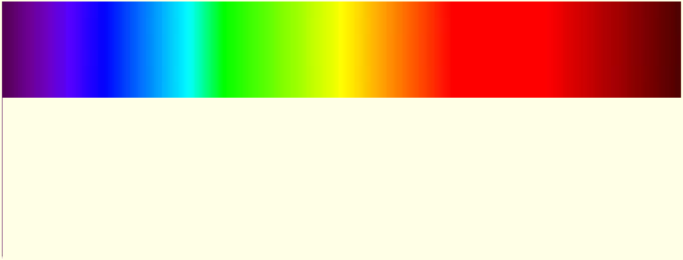


A spectrum of specifications



A spectrum of specifications

Formal



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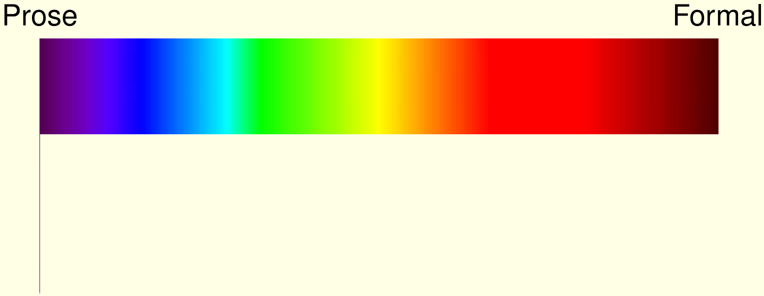

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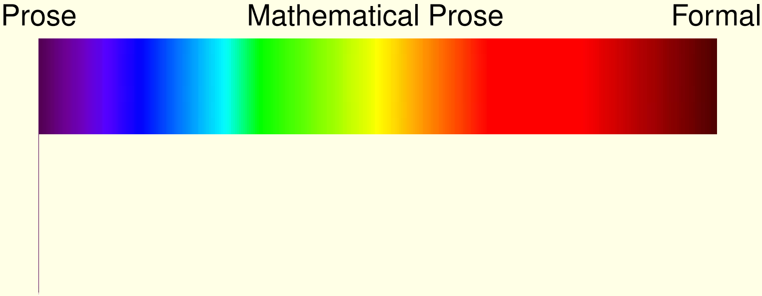
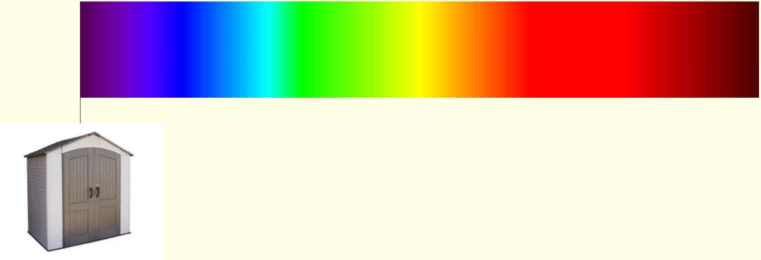


A spectrum of specifications

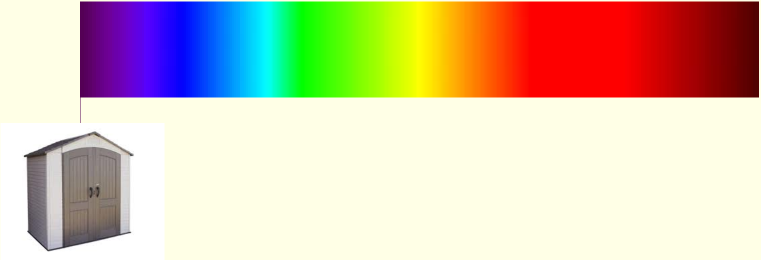
Prose



Most code is really simple.

A spectrum of specifications

Prose

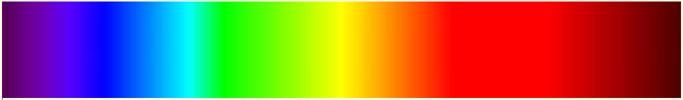


Most code is really simple.

It can be specified with a couple of lines of prose.

A spectrum of specifications

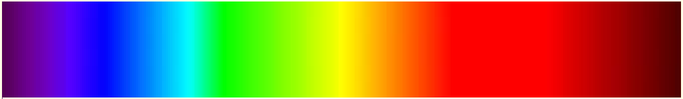
Mathematical Prose



Some code is subtle.

A spectrum of specifications

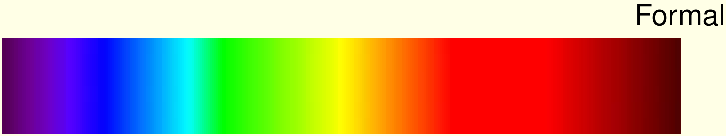
Mathematical Prose



Some code is subtle.

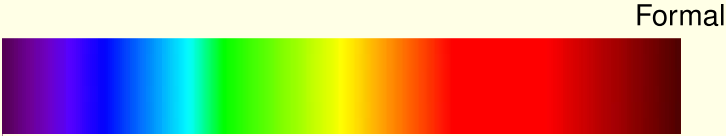
It requires more thought.

A spectrum of specifications



Some code is complex or very subtle or critical.

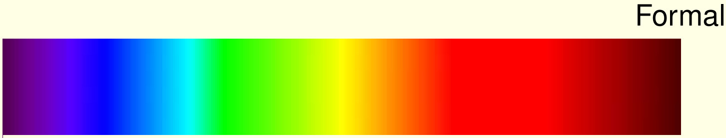
A spectrum of specifications



Some code is complex or very subtle or critical.

Especially in concurrent/distributed systems.

A spectrum of specifications



Some code is complex or very subtle or critical.

We should use tools to check it.

How to Write a Spec

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Writing requires thinking.

How to Think About Programs

How to Think About Programs

Like a Scientist

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A very successful way of thinking.

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Science makes mathematical models of reality.

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Astronomy:

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Reality: planets have mountains, oceans, tides, weather, ...

How to Think About Programs

Like a Scientist

A very successful way of thinking.

Science makes mathematical models of reality.

Astronomy:

Reality: planets have mountains, oceans, tides, weather, . . .

Model: planet a point mass with position & momentum.

Computer Science

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Reality: Digital systems

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a processor chip

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a computer executing a program

⋮

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Models:

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Models: Turing machines

Partially ordered sets of events

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The Two Most Useful Models

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Functions

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Functions

Sequences of States

Functions

Functions

Model a program as a function mapping input(s) to output(s).

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Example: the $square$ function on natural numbers

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$$square(2) = 4$$

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Domain of $square$ is $\{0, 1, 2, 3, \ldots\}$ a.k.a. Nat

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To define a function, specify:

Domain of $square$ = Nat

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$$\{\langle 0,0\rangle, \langle 1,1\rangle, \langle 2,4\rangle, \langle 3,9\rangle, \ldots\}$$

To define a function, specify:

Domain of $square$ = Nat

$square(x) = x^2$ for all x in its domain.

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Example: the $square$ function on natural numbers

$$\{\langle 0,0\rangle,\ \langle 1,1\rangle,\ \langle 2,4\rangle,\ \langle 3,9\rangle,\ \ldots\}$$

Functions in math $\neq$ functions in programming languages.

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Example: the $square$ function on natural numbers

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Functions in math $\neq$ functions in programming languages.

Math is much simpler.

Limitations of the Function Model

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Specifies what a program does, but not how.

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– Operating systems

The Standard Behavioral Model

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A program execution is represented by a **behavior.**

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A behavior is a sequence of **states**.

A state is an assignment of values to variables.

A program is modeled by a set of behaviors:

the behaviors representing possible executions.

An Example: Euclid's Algorithm

An Example: Euclid’s Algorithm

An algorithm is an abstract program.

An Example: Euclid's Algorithm

Computes GCD of M and N by:

– Initialize x to M and y to N.

An Example: Euclid's Algorithm

Computes GCD of M and N by:

– Initialize x to M and y to N.

– Keep subtracting the smaller of x and y from the larger.

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– Stop when $x = y$.

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Computes GCD of M and N by:

– Initialize x to M and y to N.

– Keep subtracting the smaller of x and y from the larger.

– Stop when $x = y$.

For $M = 12$ and $N = 18$, one behavior:

$$[x = 12,\ y = 18] \ \rightarrow\ [x = 12,\ y = 6] \ \rightarrow\ [x = 6,\ y = 6]$$

How to Describe a Set of Behaviors

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Theorem

Any set $\mathcal{B}$ of behaviors $=$

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Any set $\mathcal{B}$ of behaviors =

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Safety Property:

False iff violated at some point in behavior.

How to Describe a Set of Behaviors

Theorem

Any set $\mathcal{B}$ of behaviors =

all behaviors satisfying a safety property S

$\cap$ all behaviors satisfying a liveness property L

Safety Property:

False iff violated at some point in behavior.

Example: partial correctness.

How to Describe a Set of Behaviors

Theorem

Any set $\mathcal{B}$ of behaviors =

all behaviors satisfying a safety property S

$\cap$ all behaviors satisfying a liveness property L

Liveness Property:

Need to see complete behavior to know if it's false.

How to Describe a Set of Behaviors

Theorem

Any set $\mathcal{B}$ of behaviors =

all behaviors satisfying a safety property S

$\cap$ all behaviors satisfying a liveness property L

Liveness Property:

Need to see complete behavior to know if it's false.

Example: termination.

How to Describe a Set of Behaviors

Theorem

Any set $\mathcal{B}$ of behaviors =

all behaviors satisfying a safety property S

$\cap$ all behaviors satisfying a liveness property L

Specify a set of behaviors with

– a safety property

– a liveness property

In practice, specifying safety is more important.

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That’s where errors are most likely to occur.

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That's where errors are most likely to occur.

To save time, I'll ignore liveness.

How to Specify a Safety Property

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With two things:

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With two things:

– The set of possible initial states.

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– A next-state relation,
describing all possible successor states of any state.

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What language should we use?

Let's act like scientists.

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describing all possible successor states of any state.

What language should we use?

Let's act like scientists.

Let's use math.

The Set of Initial States

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Described by a formula.

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For Euclid's Algorithm: $(x = M) \land (y = N)$

The Set of Initial States

Described by a formula.

For Euclid's Algorithm: $(x = M) \wedge (y = N)$

Only possible initial state: $[x = M, \ y = N]$

The Next-State Relation

Described by a formula.

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Unprimed variables for current state,

Primed variables for next state.

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For Euclid's Algorithm:

$$
\begin{array}{ll}
 & (\quad x > y \\
 & \quad\quad\quad\quad) \\
\vee & (\quad y > x \\
 & \quad\quad\quad\quad)
\end{array}
$$

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For Euclid's Algorithm:

$$
\begin{array}{ll}
 & (\quad x > y \\
 & \quad \wedge\ x' = x - y \\
 & \quad \wedge\ y' = y \quad) \\
\vee & (\quad y > x \\
 & \\
 & \qquad\qquad\qquad)
\end{array}
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 & \quad \wedge\ x' = x \ )
\end{array}
$$

For Euclid's Algorithm

For Euclid's Algorithm

Take $M = 12$, $N = 18$

Behavior:

For Euclid's Algorithm

$$Init: \quad (x = M) \wedge (y = N)$$

Behavior:

For Euclid's Algorithm

$Init$: $(x = 12) \wedge (y = 18)$

Behavior:

$[x = 12,\ y = 18]$

For Euclid's Algorithm

$Init: \quad (x = M) \land (y = N)$

$$
\begin{aligned}
Next: \quad & \quad (\quad x > y \\
& \quad\ \ \land\ x' = x - y \\
& \quad\ \ \land\ y' = y \quad) \\
& \lor\ (\quad y > x \\
& \quad\ \ \land\ y' = y - x \\
& \quad\ \ \land\ x' = x \quad)
\end{aligned}
$$

Behavior:

$[x = 12,\ y = 18]$

For Euclid's Algorithm

$Init: \quad (x = M) \land (y = N)$

$$
\begin{array}{lll}
Next: & & (\quad 12 > 18 \\
& & \;\; \land \;\; x' = 12 - 18 \\
& & \;\; \land \;\; y' = 18 \;\;) \\
& \lor & (\quad 18 > 12 \\
& & \;\; \land \;\; y' = 18 - 12 \\
& & \;\; \land \;\; x' = 12 \;\;)
\end{array}
$$

Behavior:

$[x = 12, \; y = 18]$

For Euclid's Algorithm

$Init: \quad (x = M) \land (y = N)$

$$
\begin{aligned}
Next: \quad & (\ \boxed{12 > 18} \quad \textbf{FALSE} \\
& \ \land\ x' = 12 - 18 \\
& \ \land\ y' = 18\ ) \\
\lor\ & (\ \boxed{18 > 12} \quad \textbf{TRUE} \\
& \ \land\ y' = 18 - 12 \\
& \ \land\ x' = 12\ )
\end{aligned}
$$

Behavior:

$[x = 12,\ y = 18]$

For Euclid's Algorithm

$Init: \quad (x = M) \wedge (y = N)$

$$
\begin{aligned}
Next: \quad & (\quad 12 > 18 \\
& \;\wedge\; x' = 12 - 18 \\
& \;\wedge\; y' = 18 \;)
\end{aligned}
$$

FALSE

$$
\begin{aligned}
\vee \; & (\quad 18 > 12 \\
& \;\wedge\; y' = 18 - 12 \\
& \;\wedge\; x' = 12 \;)
\end{aligned}
$$

Behavior:

$[x = 12,\ y = 18]$

For Euclid's Algorithm

$Init: \quad (x = M) \land (y = N)$

$$
\begin{aligned}
Next: \quad & (\quad 12 > 18 \\
& \;\land\; x' = 12 - 18 \\
& \;\land\; y' = 18 \quad) \\
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\end{aligned}
$$

FALSE

Behavior:

$[x = 12,\ y = 18]$

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& \;\land\; x' = 12 \quad)
\end{aligned}
$$

FALSE

Behavior:

$[x = 12,\ y = 18] \rightarrow [x = 12,\ y = 6]$

For Euclid's Algorithm

$$Init: \quad (x = M) \land (y = N)$$

$$
\begin{aligned}
Next: \quad & (\quad x > y \\
& \;\; \land \; x' = x - y \\
& \;\; \land \; y' = y \;\;) \\
\lor \; & (\quad y > x \\
& \;\; \land \; y' = y - x \\
& \;\; \land \; x' = x \;\;)
\end{aligned}
$$

Behavior:

$[x = 12,\ y = 18] \ \rightarrow\ [x = 12,\ y = 6]$

For Euclid's Algorithm

$Init: \quad (x = M) \wedge (y = N)$

$$\begin{aligned} Next: \quad & (\quad 12 > 6 \\ & \quad \wedge\ x' = 12 - 6 \\ & \quad \wedge\ y' = 6 \quad) \\ \vee\ & (\quad 6 > 12 \\ & \quad \wedge\ y' = 6 - 12 \\ & \quad \wedge\ x' = 12 \quad) \end{aligned}$$

Behavior:

$$[x = 12,\ y = 18] \rightarrow [x = 12,\ y = 6]$$

For Euclid's Algorithm

$Init: \quad (x = M) \land (y = N)$

$$
\begin{aligned}
Next: \quad & (\ \boxed{12 > 6}\ \ \textbf{TRUE} \\
& \ \land\ x' = 12 - 6 \\
& \ \land\ y' = 6\ ) \\
\lor\ & (\ \boxed{6 > 12}\ \ \textbf{FALSE} \\
& \ \land\ y' = 6 - 12 \\
& \ \land\ x' = 12\ )
\end{aligned}
$$

Behavior:

$[x = 12,\ y = 18] \rightarrow [x = 12,\ y = 6]$

For Euclid's Algorithm

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$$
\begin{aligned}
Next: \quad & (\quad 12 > 6 \\
& \land \; x' = 12 - 6 \\
& \land \; y' = 6 \;) \\
\lor \; & (\quad 6 > 12 \\
& \land \; y' = 6 - 12 \\
& \land \; x' = 12 \;)
\end{aligned}
$$

FALSE

Behavior:

$[x = 12,\; y = 18] \;\rightarrow\; [x = 12,\; y = 6]$

For Euclid's Algorithm

$$Init: \quad (x = M) \land (y = N)$$

$$
\begin{aligned}
Next: \quad & (\quad 12 > 6 \\
& \;\land\; \boxed{x' = 12 - 6} \\
& \;\land\; \boxed{y' = 6 \;)} \\
\lor\; & (\quad 6 > 12 \\
& \;\land\; y' = 6 - 12 \\
& \;\land\; x' = 12 \;)
\end{aligned}
$$

FALSE

Behavior:

$$[x = 12,\ y = 18] \;\rightarrow\; [x = 12,\ y = 6] \;\rightarrow\; [x = 6,\ y = 6]$$

For Euclid's Algorithm

$$Init: \quad (x = M) \land (y = N)$$

$$\begin{array}{lll} Next: & & (\quad x > y \\ & & \;\;\land\; x' = x - y \\ & & \;\;\land\; y' = y \quad) \\ & \lor & (\quad y > x \\ & & \;\;\land\; y' = y - x \\ & & \;\;\land\; x' = x \quad) \end{array}$$

Behavior:

$$[x = 12,\; y = 18] \;\rightarrow\; [x = 12,\; y = 6] \;\rightarrow\; [x = 6,\; y = 6]$$

For Euclid's Algorithm

$$Init: \quad (x = M) \wedge (y = N)$$

$$\begin{aligned} Next: \quad & (\ \boxed{6 > 6} \quad \textbf{FALSE} \\ & \ \wedge\ x' = 6 - 6 \\ & \ \wedge\ y' = 6\ ) \\ \vee\ & (\ \boxed{6 > 6} \quad \textbf{FALSE} \\ & \ \wedge\ y' = 6 - 6 \\ & \ \wedge\ x' = 6\ ) \end{aligned}$$

Behavior:

$$[x = 12,\ y = 18] \ \rightarrow\ [x = 12,\ y = 6] \ \rightarrow\ [x = 6,\ y = 6]$$

For Euclid's Algorithm

$Init: \quad (x = M) \land (y = N)$

$$\begin{aligned} Next: \quad & (\ 6 > 6 \\ & \ \land\ x' = 6 - 6 \\ & \ \land\ y' = 6\ ) \\ \lor\ & (\ 6 > 6 \\ & \ \land\ y' = 6 - 6 \\ & \ \land\ x' = 6\ ) \end{aligned}$$

FALSE

FALSE

Behavior:

$$[x = 12,\ y = 18] \ \rightarrow\ [x = 12,\ y = 6] \ \rightarrow\ [x = 6,\ y = 6]$$

For Euclid's Algorithm

$Init: \quad (x = M) \wedge (y = N)$

$$
\begin{array}{lll}
Next: & & (\quad 6 > 6 \\
& & \ \wedge \ x' = 6 - 6 \\
& & \ \wedge \ y' = 6 \quad) \\
& \vee & (\quad 6 > 6 \\
& & \ \wedge \ y' = 6 - 6 \\
& & \ \wedge \ x' = 6 \quad)
\end{array}
$$

FALSE

FALSE

NO NEXT STATE

Behavior:

$$[x = 12,\ y = 18] \ \rightarrow \ [x = 12,\ y = 6] \ \rightarrow \ [x = 6,\ y = 6]$$

Euclid’s Algorithm

.

Euclid's Algorithm

For any values of x and y, there are unique values of x' and y' that make $Next$ true.

.

Euclid's Algorithm

For any values of x and y, there are unique values of x' and y' that make $Next$ true.

Euclid's algorithm is deterministic.

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To Model Nondeterminism

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To Model Nondeterminism

Allow multiple next states for a current state.

Euclid's Algorithm

For any values of x and y, there are unique values of x' and y' that make $Next$ true.

Euclid's algorithm is deterministic.

To Model Nondeterminism

Allow multiple next states for a current state.

Multiple assignments of values to primed variables that make $Next$ true for a single assignment of values to unprimed variables.

What About Formal Specs?

What About Formal Specs?

Need them only to apply tools.

What About Formal Specs?

Need them only to apply tools.

Require a formal language.

The Language: TLA$^+$

The Language: TLA$^+$

This

$$Init: \quad (x = M) \land (y = N)$$

$$\begin{aligned} Next: \quad & \ (\quad x > y \\ & \qquad \land\ x' = x - y \\ & \qquad \land\ y' = y \quad) \\ & \lor\ (\quad y > x \\ & \qquad \land\ y' = y - x \\ & \qquad \land\ x' = x \quad) \end{aligned}$$

The Language: TLA+

becomes this

$$Init \triangleq (x = M) \land (y = N)$$

$$\begin{aligned} Next \triangleq \quad & (\quad x > y \\ & \;\land\; x' = x - y \\ & \;\land\; y' = y \;) \\ \lor\; & (\quad y > x \\ & \;\land\; y' = y - x \\ & \;\land\; x' = x \;) \end{aligned}$$

The Language: TLA$^+$

plus declarations

CONSTANTS M, N

VARIABLES x, y

$$Init \triangleq (x = M) \land (y = N)$$

$$\begin{aligned} Next \triangleq \quad & (\quad x > y \\ & \quad \land\ x' = x - y \\ & \quad \land\ y' = y \quad) \\ \lor\ & (\quad y > x \\ & \quad \land\ y' = y - x \\ & \quad \land\ x' = x \quad) \end{aligned}$$

The Language: TLA$^+$

plus some boilerplate.

MODULE *Euclid*

EXTENDS $Integers$

CONSTANTS M, N

VARIABLES x, y

$$Init \triangleq (x = M) \land (y = N)$$

$$\begin{aligned} Next \triangleq \quad & (\quad x > y \\ & \;\land\; x' = x - y \\ & \;\land\; y' = y \quad) \\ \lor \; & (\quad y > x \\ & \;\land\; y' = y - x \\ & \;\land\; x' = x \quad) \end{aligned}$$

The Language: TLA$^+$

You type

```
------------------ MODULE Euclid --------------------
  EXTENDS Integers

  CONSTANTS M, N

  VARIABLES x, y

  Init == (x = M) /\ (y = N)

  Next ==     (   x > y
               /\ x' = x - y
               /\ y' = y   )
            \/ (   y > x
               /\ y' = y - x
               /\ x' = x )
======================================================
```

You can model check TLA^+ specs.

You can model check TLA$^+$ specs.

– Checks all executions of a small model.

You can model check TLA$^+$ specs.

– Checks all executions of a small model.

– Extremely effective and fairly easy.

You can model check TLA$^+$ specs.

You can write formal correctness proofs
and check them mechanically.

You can model check TLA$^+$ specs.

You can write formal correctness proofs
and check them mechanically.

– Hard work.

You can model check TLA^{+} specs.

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But math works only for toy examples.

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To model real systems, you need a real language
with types, procedures, objects, etc.

You can model check TLA$^+$ specs.

You can write formal correctness proofs
and check them mechanically.

~~But math works only for toy examples.~~

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with types, procedures, objects, etc.~~

Wrong

Chris Newcombe
Amazon Engineer

[W]e have used TLA$^+$ on 10 large complex real-world systems.

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Chris Newcombe
Amazon Engineer
November, 2013

The XBox 360 Memory System

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Writing a TLA$^+$ spec caught a bug that would not otherwise have been found.

The XBox 360 Memory System

Writing a TLA$^+$ spec caught a bug that would not otherwise have been found.

That bug would have caused every XBox 360 to crash after 4 hours of use.

You can learn about TLA$^+$ on the web.

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Today, I’ll talk about informal specs, starting with an example.

TLATEX — the TLA$^+$ pretty-printer

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The input:

```
Foo => /\ a   = b
       /\ ccc = d
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The naive output:

$$\begin{array}{l} Foo \;\Rightarrow\; \land\, a = b \\ \qquad\quad \land\, ccc = d \end{array}$$

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The user probably wanted these aligned.

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TLATEX — the TLA$^+$ pretty-printer

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/\ aaa + bb = c
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$$\begin{aligned}&\land\ aaa + bb = c\\&\land\ iii = jj * k\end{aligned}$$

The user probably didn't wanted these aligned.

There is no precise definition of correct alignment.

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We can’t specify mathematically what the user wants.

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Not knowing what a program should do means
we have to think even harder.

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We can't specify mathematically what the user wants.

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Wrong.

Not knowing what a program should do means
we have to think even harder.

Which means that a spec is even more important.

My Spec

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6 rules plus definitions (in comments).

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Example:

A left-comment token is LeftComment aligned with its covering token.

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Example:

A left-comment token is LeftComment aligned with its covering token.

Defined term.

Why did I write this spec?

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It was a lot easier to understand and debug 6 rules than 850 lines of code.

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The few bugs in implementing the rules were easy to catch.

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It was a lot easier to understand and debug 6 rules than 850 lines of code.

I did a lot of debugging of the rules, aided by debugging code to report what rules were being used.

The few bugs in implementing the rules were easy to catch.

Had I just written code, it would have taken me much longer and not produced formatting as good.

Why not a formal spec?

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Getting it right not that important.

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Getting it right not that important.

It didn't have to work on all corner cases.

Why not a formal spec?

Getting it right not that important.

It didn't have to work on all corner cases.

There were no tools that could help me.

What is Typical About This Spec

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No method or tool for writing better code would have made the spec unnecessary.

It says nothing about how to write code.

You write a spec to help you think about the problem before you think about the code.

What is Not Typical About This Spec

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A set of rules/requirements/axioms is usually a bad spec.

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It's quite subtle.

95% of the code most people write requires less thought; specs that are shorter and simpler suffice.

It's a set of rules.

A set of rules/requirements/axioms is usually a bad spec.

It's hard to understand the consequences of a set of rules.

Specifying How to Compute a Function

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Figuring out how to implement it efficiently is hard
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Specifying what a sorting program should do is easy.

Figuring out how to implement it efficiently is hard
(if no one has shown you).

It requires thinking, which means writing a specification.

An example: Quicksort

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A divide-and-conquer algorithm for sorting an array $A[0], \ldots, A[N-1]$.

An example: Quicksort

A divide-and-conquer algorithm for sorting an array $A[0], \ldots, A[N-1]$.

For simplicity, assume the $A[i]$ are numbers.

It uses a procedure $Partition(lo, hi)$.

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It chooses $pivot$ in $lo \ldots (hi - 1)$, permutes $A[lo], \ldots, A[hi]$ to make $A[lo], \ldots, A[pivot] \leq A[pivot + 1], \ldots, A[hi]$, and returns $pivot$.

It uses a procedure $Partition(lo,\, hi)$.

It chooses $pivot$ in $lo \ldots (hi - 1)$, permutes $A[lo], \ldots, A[hi]$ to make $A[lo], \ldots, A[pivot] \leq A[pivot + 1], \ldots, A[hi]$, and returns $pivot$.

For this example, we don't care how this procedure is implemented.

Let's specify Quicksort in pseudo-code.

procedure $Partition(lo, hi)$ {

Pick $pivot$ in $lo \ldots (hi - 1)$;

Permute $A[lo], \ldots, A[hi]$ to make
$A[lo], \ldots, A[pivot] \leq A[pivot + 1], \ldots, A[hi]$;

return $pivot$; }

procedure $Partition(lo, hi)$ {

Pick $pivot$ in $lo \ldots (hi - 1)$;

Permute $A[lo], \ldots, A[hi]$ to make
$A[lo], \ldots, A[pivot] \leq A[pivot + 1], \ldots, A[hi]$;

return $pivot$; }

procedure $QS(lo, hi)$ { **if** $(lo < hi)$ { $p := Partition(lo, hi)$;
$QS(lo, p)$;
$QS(p + 1, hi)$; } }

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main { $QS(0, N - 1)$; }

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Informal: no formal syntax, no declarations, ...

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Informal: no formal syntax, no declarations, ...

Easy to understand.

procedure $Partition(lo, hi)$ {

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Permute $A[lo], \ldots, A[hi]$ to make
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procedure $QS(lo, hi)$ { **if** $(lo < hi)$ { $p := Partition(lo, hi)$;
$QS(lo, p)$;
$QS(p + 1, hi)$; } }

main { $QS(0, N - 1)$; }

But is it really Quicksort?

It’s the way Quicksort is almost always described.

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But recursion is not a fundamental part of Quicksort.

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It’s just one way of implementing divide-and-conquer.

It's the way Quicksort is almost always described.

But recursion is not a fundamental part of Quicksort.

It's just one way of implementing divide-and-conquer.

It's probably not the best way for parallel execution.

Problem: Write a non-recursive version of Quicksort.

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Almost no one can do it in 10 minutes.

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Almost no one can do it in 10 minutes.

They try to “compile” the recursive version.

Solution:

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Maintain a set U of index ranges on which $Partition$ needs to be called.

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Initially, U equals $\{\langle 0, N-1 \rangle\}$

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Initially, U equals $\{\langle 0, N-1 \rangle\}$

We could write it in pseudo-code,
but it's better to simply write $Init$ and $Next$ directly.

$$
\begin{aligned}
Init\text{:}\quad & A = \text{any array of numbers of length } N \\
\wedge\ & U = \{\langle 0, N-1 \rangle\}
\end{aligned}
$$

$$\begin{aligned} Init: \quad & A = \text{any array of numbers of length } N \\ & \land\ U = \{\langle 0, N-1 \rangle\} \end{aligned}$$

Before writing $Next$, let's make a definition:

$$Init: \quad A = \text{any array of numbers of length } N$$
$$\land \; U = \{\langle 0, N-1 \rangle\}$$

Before writing $Next$, let's make a definition:

$Partitions(B, pivot, lo, hi) \triangleq$
the set of arrays obtained from B by permuting
$B[lo], \ldots, B[hi]$ such that ...

$Init$: A = any array of numbers of length N
$\wedge$ $U = \{\langle 0, N-1 \rangle\}$

Before writing $Next$, let's make a definition:

$Partitions(B, pivot, lo, hi) \triangleq$
the set of arrays obtained from B by permuting $B[lo], \ldots, B[hi]$ such that ...

$Next$:

A relation between old values of A, U and new values A', U'.

$Next$:

$Next$:

$$U \neq \{\}$$ Stop if $U = \{\}$

$\wedge$

$Next$:

$$U \neq \{\}$$

$\land$ Pick any $\langle b, t \rangle$ in U :

$Next$:

$U \neq \{\}$

$\wedge$ Pick any $\langle b, t \rangle$ in U :

IF $b \neq t$

THEN

ELSE

$Next$:

$U \neq \{\}$

$\wedge$ Pick any $\langle b, t \rangle$ in U :

IF $b \neq t$

THEN Pick any p in $b \,..\, (t-1)$:

ELSE

$Next$:

$U \neq \{\}$

$\wedge$ Pick any $\langle b, t \rangle$ in U :

IF $b \neq t$

THEN Pick any p in $b \,..\, (t-1)$:

$A' =$ Any element of $Partitions(A, p, b, t)$

ELSE

$Next$:

$U \neq \{\}$

$\wedge$ Pick any $\langle b, t \rangle$ in U :

IF $b \neq t$

THEN Pick any p in $b \,..\, (t-1)$:

$A' =$ Any element of $Partitions(A, p, b, t)$

$\wedge$ $U' = U$ with $\langle b, t \rangle$ removed and $\langle b, p \rangle$ and $\langle p+1, t \rangle$ added

ELSE

$Next$:

$$
\begin{array}{l}
\quad U \neq \{\} \\
\land \ \text{Pick any } \langle b, t \rangle \text{ in } U: \\
\qquad \text{IF } b \neq t \\
\qquad\quad \text{THEN Pick any } p \text{ in } b \,..\, (t-1): \\
\qquad\qquad\quad A' = \text{Any element of } Partitions(A, p, b, t) \\
\qquad\qquad \land \ U' = U \text{ with } \langle b, t \rangle \text{ removed and} \\
\qquad\qquad\qquad\quad \langle b, p \rangle \text{ and } \langle p+1, t \rangle \text{ added} \\
\qquad\quad \text{ELSE} \quad A' = A
\end{array}
$$

$Next$:

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$\wedge$ Pick any $\langle b, t \rangle$ in U :

IF $b \neq t$

THEN Pick any p in $b \,..\, (t-1)$:

$A' =$ Any element of $Partitions(A, p, b, t)$

$\wedge$ $U' = U$ with $\langle b, t \rangle$ removed and $\langle b, p \rangle$ and $\langle p+1, t \rangle$ added

ELSE $A' = A$

$\wedge$ $U' = U$ with $\langle b, t \rangle$ removed

Why can (almost) no one find this version of Quicksort?

Why can (almost) no one find this version of Quicksort?

Their minds are stuck in code.

Why can (almost) no one find this version of Quicksort?

Their minds are stuck in code.

They can't think at a higher level.

$Next$:

$U \neq \{\}$

$\wedge$ Pick any $\langle b, t \rangle$ in U :

IF $b \neq t$

THEN Pick any p in $b \,..\, (t-1)$:

$A' =$ Any element of $Partitions(A, p, b, t)$

$\wedge$ $U' = U$ with $\langle b, t \rangle$ removed and $\langle b, p \rangle$ and $\langle p+1, t \rangle$ added

ELSE $A' = A$

$\wedge$ $U' = U$ with $\langle b, t \rangle$ removed

Easy to write this as a formula.

$Next$:

$$
\begin{array}{l}
U \neq \{\} \\
\land \text{ Pick any } \langle b, t \rangle \text{ in } U: \\
\quad \text{IF } b \neq t \\
\quad \text{THEN Pick any } p \text{ in } b \,..\, (t-1): \\
\qquad\qquad A' = \text{Any element of } Partitions(A, p, b, t) \\
\qquad\quad \land\ U' = U \text{ with } \langle b, t \rangle \text{ removed and} \\
\qquad\qquad\qquad \langle b, p \rangle \text{ and } \langle p+1, t \rangle \text{ added} \\
\quad \text{ELSE} \qquad A' = A \\
\qquad\quad \land\ U' = U \text{ with } \langle b, t \rangle \text{ removed}
\end{array}
$$

Pick an arbitrary value

$Next$:

$$
\begin{array}{l}
\quad U \neq \{\} \\
\wedge \;\; \exists \langle b, t \rangle \in U : \\
\qquad \text{IF } \; b \neq t \\
\qquad\quad \text{THEN } \; \text{Pick any } p \text{ in } b \,..\, (t-1) : \\
\qquad\qquad\qquad A' = \text{Any element of } Partitions(A, p, b, t) \\
\qquad\qquad \wedge \;\; U' = U \text{ with } \langle b, t \rangle \text{ removed and} \\
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\qquad\quad \text{ELSE } \qquad A' = A \\
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\end{array}
$$

Pick an arbitrary value is existential quantification.

$Next$:

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Or sometimes

$Next$:

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$$

Or sometimes even simpler.

$Next$:

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\qquad\qquad\qquad A' \in Partitions(A, p, b, t) \\
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\\
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And so on.

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\qquad\qquad A' \in Partitions(A, p, b, t) \\
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$$

A TLA$^+$ formula.

$Next$:

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If you prefer pseudo-code...

PlusCal

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Like a toy programming language.

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Like a toy programming language.

Algorithm appears in a comment in a TLA$^+$ module.

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Constructs for nondeterminism.

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Compiled to an easy to understand TLA$^+$ spec.

PlusCal

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Algorithm appears in a comment in a TLA^+ module.

An expression can be any TLA^+ expression.

Constructs for nondeterminism.

Compiled to an easy to understand TLA^+ spec.

Can apply TLA^+ tools.

Programs that Run Forever

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I've been talking about programs that compute a function.

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Programs that run forever usually involve concurrency:

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Few people can get them right by just thinking (and writing).

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I’m not one of them.

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We need tools to check what we do.

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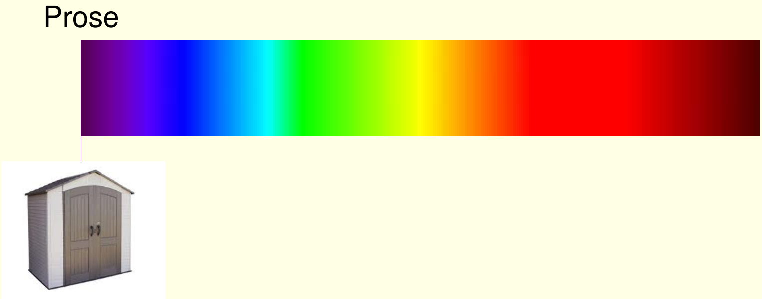
We need tools to check what we do.

Use TLA$^+$/ PlusCal.

The Other 95%



Most code is really simple.

The Other 95%

```
/*************************************************************
* CLASS ResourceFileReader                                  *
*                                                           *
* A ResourceFileReader returns an object for reading a      *
* resource file, which is a file kept in the same           *
* directory as the tlatex.Token class.  The constructor     *
* takes a file name as argument.  The object's two public   *
* methods are                                               *
*                                                           *
*   getLine() : Returns the next line of the file as a      *
*               string.  Returns null after the last line.  *
*                                                           *
*   close()   : Closes the file.                            *
*************************************************************/
```

Why did I write that spec?

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To be sure I knew **what** the code should do before writing it.

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General rule:
A spec of **what** the code does should say everything
that anyone needs to know to use the code.

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To be sure I knew **what** the code should do before writing it.

Without writing a spec, I only thought I knew what it should do.

Later, I didn't have to read the code to know what it did.

How the code worked was too simple to require a spec.

What programmers should know about thinking.

What everyone should know about thinking.

What everyone should know about thinking.

Everyone thinks they think.

What everyone should know about thinking.

Everyone thinks they think.

If you don't write down your thoughts,
you're fooling yourself.

What programmers should know about thinking.

What programmers should know about thinking.

You should think before you code.

What programmers should know about thinking.

You should write before you code.

What programmers should know about thinking.

You should write before you code.

A spec is simply what you write before coding.

What code should you specify?

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Any piece of code that someone else might want to use or modify.

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Any piece of code that ~~someone else~~ you might want to use or modify in a month.

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An entire program or system.

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It could be:

- An entire program or system.
- A class.

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Any piece of code that ~~someone else~~ you might want to use or modify in a month.

It could be:

- An entire program or system.
- A class.
- A method.

What code should you specify?

you

Any piece of code that ~~someone else~~ might want to use or modify in a month.

It could be:

- An entire program or system.
- A class.
- A method.
- A tricky piece of code in a method.

What should you specify about the code?

What should you specify about the code?

What it does.

What should you specify about the code?

What it does.

Everything anyone needs to know to use it.

What should you specify about the code?

What it does.

Everything anyone needs to know to use it.

Maybe: how it does it.

What should you specify about the code?

What it does.

Everything anyone needs to know to use it.

Maybe: how it does it.

The algorithm / high-level design.

How should you think about / specify the code?

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Above the code level.

How should you think about / specify the code?

Above the code level.

In terms of states and behaviors.

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Mathematically, as rigorously / formally as necessary.

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Above the code level.

In terms of states and behaviors.

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Perhaps with pseudo-code or PlusCal if specifying **how**.

How do you learn to write specs?

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By writing formal specs.

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TLA$^+$ may not be the best language for your formal specification needs.

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TLA$^+$ may not be the best language for your formal specification needs.

But it's great for learning learning to think mathematically.

How do you connect the spec to the code?

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Comments connecting mathematical concepts and their implementation.

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Example:

Mathematical concept: graph

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Comments connecting mathematical concepts and their implementation.

Example:

Mathematical concept: graph

Implementation: array of node objects & array of link objects

What about coding?

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Nothing I have said implies anything about how you should code.

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You still have to think while you code.

What about coding?

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You still have to think while you code.

What you write while coding is code.

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I have nothing to say about how you should code.

Use any programming language you want.

What about coding?

Nothing I have said implies anything about how you should code.

You still have to think while you code.

What you write while coding is code.

I have nothing to say about how you should code.

Use any programming language you want.

Use any coding methodology you want:
test-driven development, agile programming, ...

You'll still have to test and debug.

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Writing specs is an additional step.

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It may save time by catching errors early,
when they're easier to correct.

You'll still have to test and debug.

Writing specs is an additional step.

It may save time by catching errors early,
when they're easier to correct.

It will improve your programming,
so you write better programs.

Why are programmers reluctant to write specs?

Why are programmers reluctant to write specs?

Writing is hard.

Why is writing hard?

Why is writing hard?

Writing requires thinking.

Why is writing hard?

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Thinking is hard.

Why is writing hard?

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It’s easier to think your thinking.

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Writing is like running.

- The less you do it, the slower you are.

Why is writing hard?

- Writing requires thinking.
  - Thinking is hard.
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Writing is like running.

- The less you do it, the slower you are.
- You have to strengthen your writing muscles.
- It takes practice.

It's easier to find an excuse not to.

What if the spec is wrong?

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Maybe you made a mistake.

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Maybe the requirements change,
or an enhancement is needed.

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The code will have to be changed,
maybe even before the program is finished.

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This eventually happens to all useful programs.

In an ideal world, a new spec would be written and the code completely rewritten.

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In the real world, the code is patched
and maybe the spec is updated.

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and the code completely rewritten.

In the real world, the code is patched
and maybe the spec is updated.

If this is inevitable, why write specs?

Reason 1

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Whoever has to modify the code will be grateful for every word or formula of spec you write.

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And “whoever” may be you.

Reason 1

Whoever has to modify the code will be grateful for every word or formula of spec you write.

And “whoever” may be you.

That’s why you should update the spec when changing the code.

Reason 2

Reason 2

Every time code is patched, it becomes a little uglier, harder to understand, and harder to maintain.

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The program starts out ugly, hard to understand, and hard to maintain.

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"No battle was ever won according to plan

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> "No battle was ever won according to plan, but no battle was ever won without one."
>
> Dwight D. Eisenhower

Some people will tell you that writing specs is a waste of time.

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In some situations it is.

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In some situations it is. Sometimes there's no need to think about what you're doing.

Some people will tell you that writing specs is a waste of time.

In some situations it is. Sometimes there’s no need to think about what you’re doing.

But remember: when they’re telling you not to write a spec, they’re really telling you not to think.

Thinking doesn't guarantee that you won't make mistakes.

Thinking doesn’t guarantee that you won’t make mistakes.

Not thinking usually guarantees that you will.

To find out more about TLA$^+$, go to my home page and click on:

The TLA Web Page

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Thank You