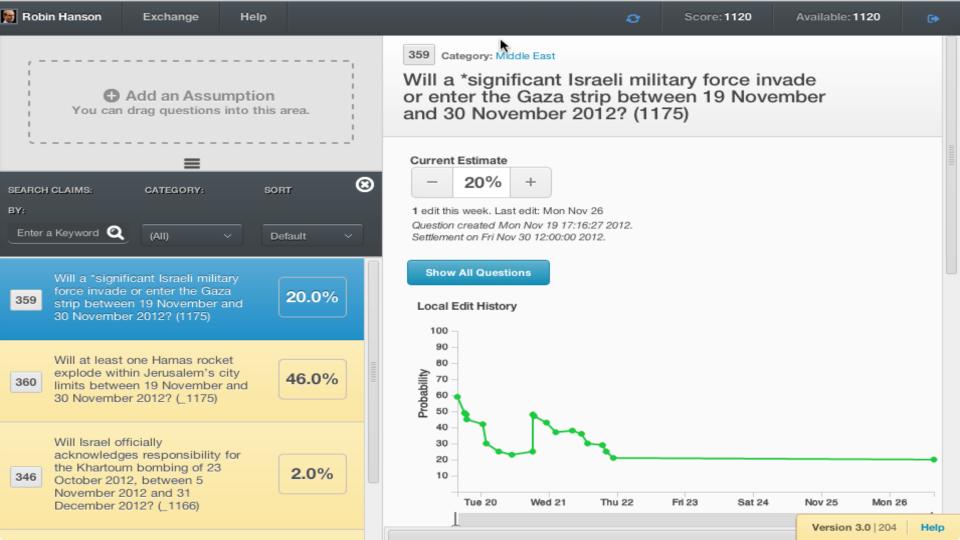
Bayes Net Based Combinatorial Prediction Markets

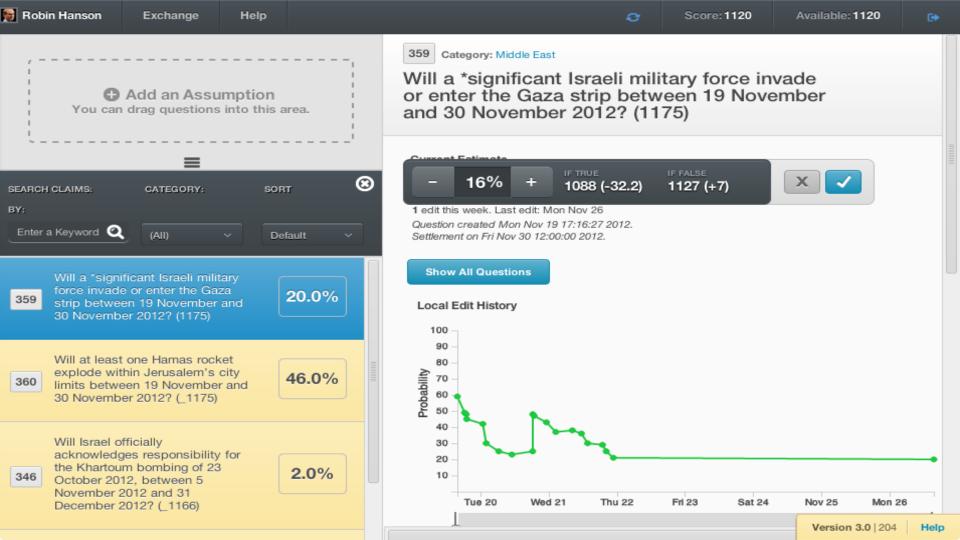
Robin Hanson

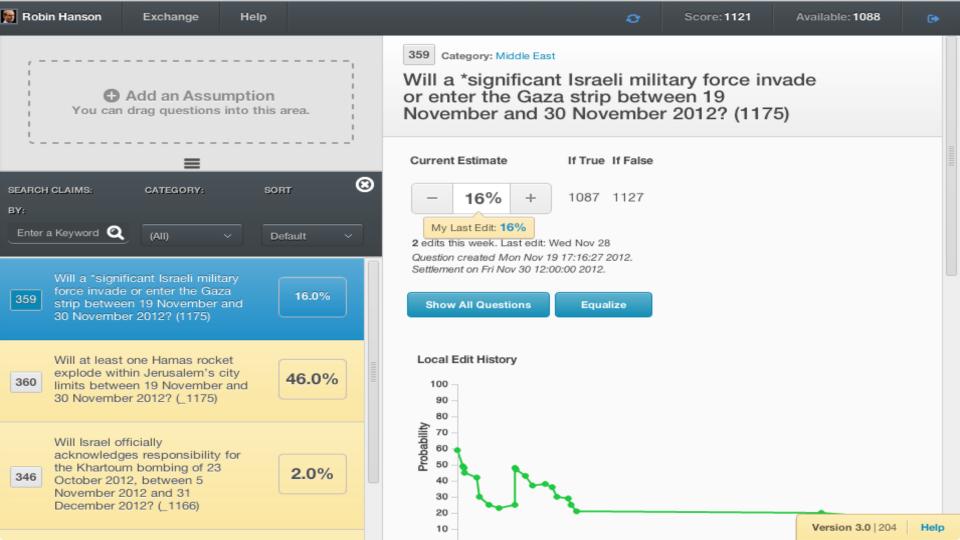
George Mason University,

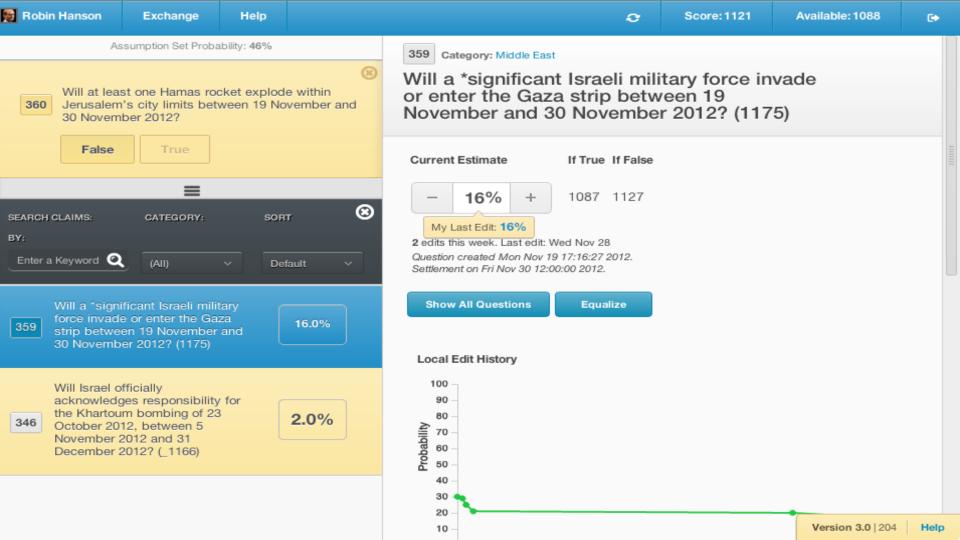
Consensus Point

With: Charles Twardy, Kathryn Laskey, Wei Sun, Shou Matsumoto









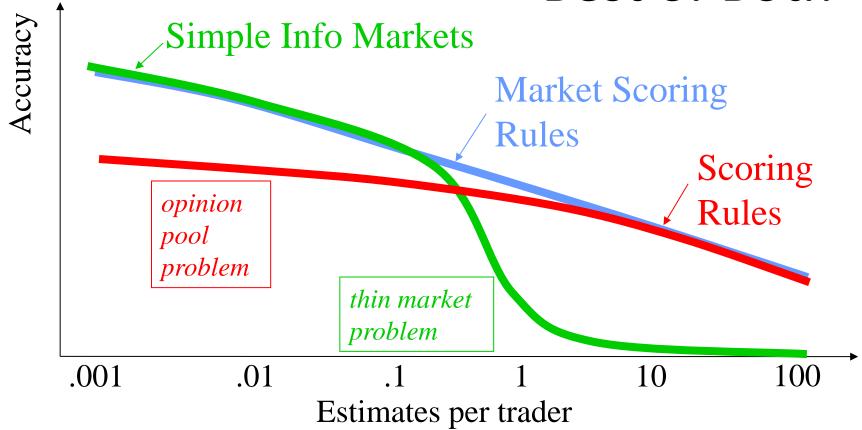
DAGGRE vs. SciCast

Same funder: IARPA

- DAGGRE
- On foreign events
- Live 9/'11 to 5/'13
- ~100 claims at a time
- ~300 users/mo.
- Users paid \$3000/mo.
- No performance pay

- SciCast:
- On science, technology
- To go live ~11/'13
- ~1000 claims at a time
- ~3000 users/mo.
- No user pay

Best of Both



Market Scoring Rule (MSR)

- Scoring rule: if report **r**, state is x, get s_x(**r**)
 - Proper if: **p** in argmax_r Σ_x p_x s_x(**r**)
- MSR: user t gets change $\Delta s_x = s_x(\mathbf{p}^t) s_x(\mathbf{p}^{t-1})$ "Anyone can use scoring rule if pay off prior user"
- Invert $s_x(\mathbf{p})$ for inventory market maker $p_x(\mathbf{s})$:
 - Tiny sale e_x if x fee: e_x fee: e_x (e_x (e_x e_x)
 - Big sale $s_{s(1)-s(0)}$ fee: $\int_0^1 \Sigma_x p_x(s(t)) s_x(t) dt$

Log Market Scoring Rule

- Log MSR: $s_x(r) = \ln(r_x)/\alpha$
- With log, cost bounded, changes uniquely modular
- Compute: state is probs p_x, assets S_x^u per user u
 - If u edits p_x -> p'_x , do $S'_x{}^u = S_x{}^u + ln(p'_x/p_x)/α$, if all ≥ 0
 - Helps to show market value of portfolio: $\underline{S}^u = \Sigma_x p_x S_x^u$
 - PROBLEM: If many vars, way too many states x!

Prediction Market Issues

- **Problem**: What we know depends on context
- Solution: Let tell relational, conditional info
- Problem: Too many combos to store/update
- Solution: Bayes nets store/update probs well
- **Problem**: Also need store/update assets, find expected assets, ensure assets not go negative
- <u>Solution</u>: In Bayes net LMSR, ways to store/update/find-min for probs also does assets

Edit-Based Combo System Needs

- 1) User u chooses assumptions A, target event T
- 2) Find & show to user u (who has assets S^u):
 - a) Current consensus p(T|A) Win If raise p
 - b) Now long/short? Via: $E_p[S^u|A\&T]$ - $E_p[S^u|A\¬T]$
 - c) Limits [min,max] of new p' (T|A), to ensure $S^u \ge 0$
- 3) User u aborts or picks a p'(T|A) in [min,max]
- 4) Update p to reflect $p(T|A) \rightarrow p'(T|A)$
- 5) Update assets Su to reflect bet for p' over p
- 6) Periodically show how $\underline{S}^u = E_p[S^u]$ vary with u

Reusing Assets

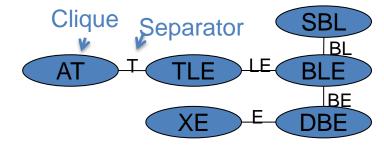
Belief: Supporting Trade: \$1 if B&A1 \$x if not A1 P(B|A1) > xP(B|A2) > x\$1 if B&A2 \$x if not A2 \$x P(B|A3) > x\$1 if B&A3 \$x if not A3 \$x \$1 if B&A9 \$x if not A9 P(B|A9) > x\$x

\$9x

\$1 if B

\$8x

Bayes/Markov Nets

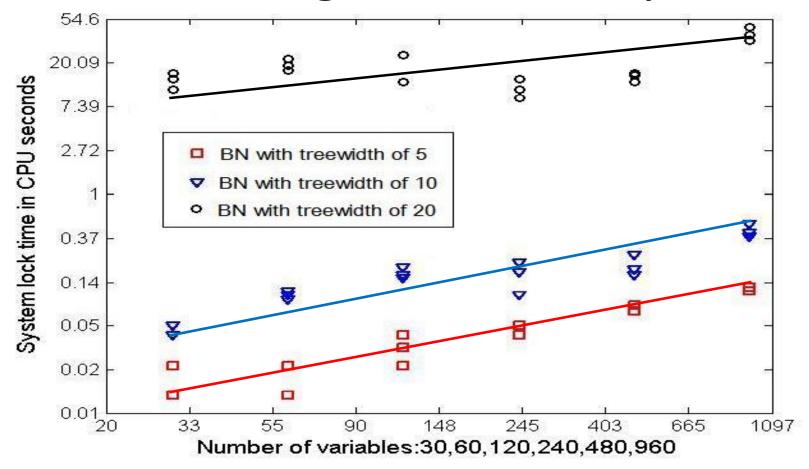


P(Clique | Rest of Net) = P(Clique | Its Separators)

$$X = \langle X_A, X_B, X_C, \dots \rangle$$

- $p_x = \prod_c p_c(x_c) / \prod_s p_s(x_s)$ lets update p(x), find mind, via JT alg
- Let $q_x^u = \exp(S_x^u/b)$, so $q_x'/q_x = p_x'/p_x$, $q_x^0 = constant$
- $q_x = \prod_c q_c(x_c) / \prod_s q_s(x_s)$, so can update q(x), find min, via JT alg
- Implies $S_x = \Sigma_c S_c(x_c) \Sigma_s S_s(x_s)$, $\underline{S} = \Sigma_c \underline{S}_c \Sigma_s \underline{S}_s$
- If edit p(T|A) -> p' (T|A), need T,A in same clique
- $[min,max] = [p/min_{(x in A\¬T)}q_x, 1-((1-p)/min_{(x in A\&T)}q_x)]$

Markov Engine Scalability Test



Edit-Based Combo System Needs

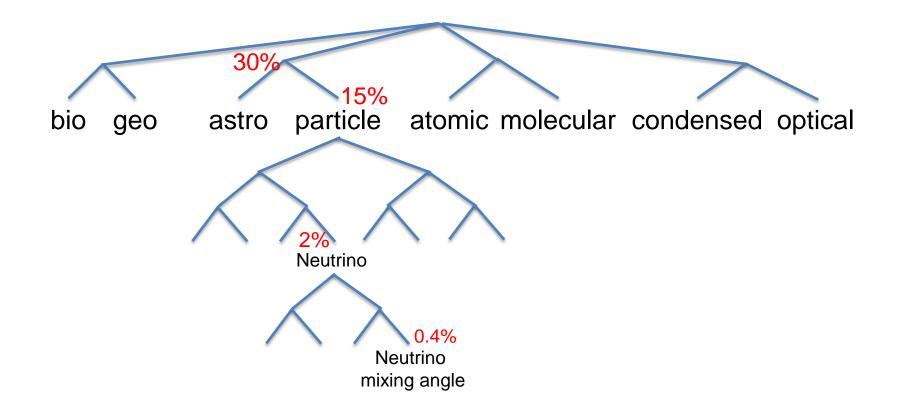
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DAGGRE vs. SciCast

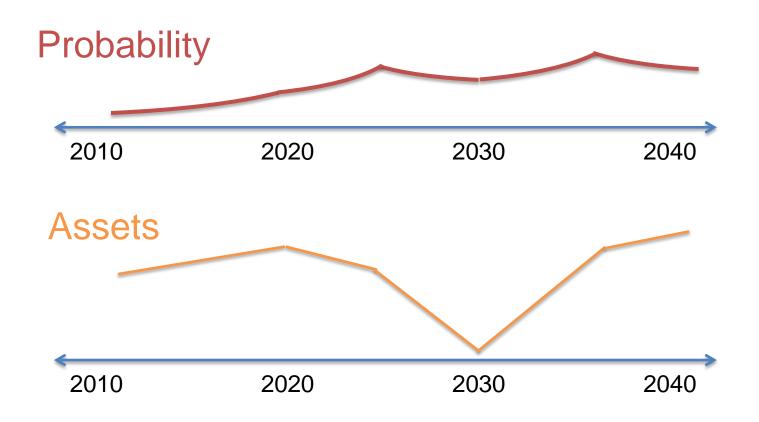
- DAGGRE
- ~100 claims at a time
- Network treewidth ~5
- Show expect long/short
- Only local edits allowed
- < 5 values per variables

- SciCast
- ~1000 claims at a time
- Network treewidth 50?
- Show min long/short
- Allow arbitrary edits?
- Value trees, cont. distr.

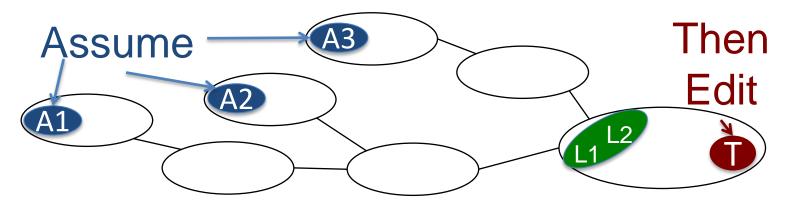
What Field 2013 Physics Nobel?



When 1000 qubits?

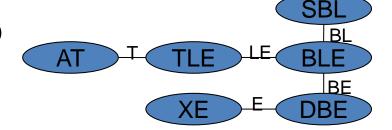


What If A Is Far from T?



- Option 1: Find nearest changes to ideal LMSR edit of P(T|A) that fit network constraints.
- Option 2: Translate far assumptions A into local clique assumptions L, let user edit P(T|L).

Can Users Edit Links?



- Add link => bigger cliques
 - Costs system more space/time to store/update
 - Allow if users willing to make big supporting edit?
- Delete link => some old assets can't be sold
 - Allow if edit creates conditional independence?
- How control compute costs while allowing structure changes?
 - Require combine add, delete links so same cost?