Transactions Briefs

Electronically Controlled Active-*C* Filters and Equalizers with Operational Transconductance Amplifiers

HENRIQUE S. MALVAR

Abstract—**Current-controlled filters and equalizers that employ only operational transconductance amplifiers (OTA's) and capacitors are presented. Those circuits can be integrated in bipolar technology and are characterized by good linearity between controlled parameters and their respective control currents.**

I. Introduction

Programmable filters and equalizers are required in the areas of instrumentation, audio signal processing, music synthesis and others. Generally, wide tuning range and/or independent frequency response shape control are desired. Although structures based on bipolar operational amplifiers (OA's) may have good S/N characteristics, they are not adequate when high-order or multiple filters must be integrated into a single chip (due to the large silicon area required to implement typical OA–*RC* circuits). By using operational transconductance amplifiers (OTA's) as basic active elements, instead of OA's, it is possible to realize tunable filters in active-*C* topologies, i.e., without resistors, and since the internal circuitry of an OTA requires just a few or even no resistors, active-*C* OTA filters have a high potential for integration. Further, the tuning range of OTA filters is much wider than that of OA-multiplier or OA–FET circuits [7].

This paper describes a canonical biquad structure (section II) that is capable of realizing various second-order transfer functions simply by changing input, output and grounded nodes, and thus making a versatile building block. In Section III, equalizer circuits suitable for audio applications are presented. Finally, dynamic range and other practical issues are considered in Section IV.

II. A Filter Building Block

A basic topological structure for active filter design is the two-integrator-loop [1], also called the biquad [2], due to its low

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The author is with the Research Laboratory of Electronics, M.I.T., Cambridge, MA 02139, on leave from the Department of Electrical Engineering, University of Brasilia, 70910 Brasilia, DF Brazil.

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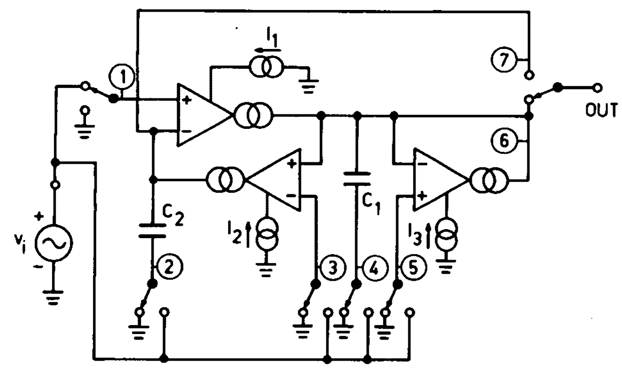

Fig. 1. Basic biquad structure that realizes the transfer functions in Table I.

sensitivity to component variations. In somewhat different forms, it is the basis for active-ladder (or "leapfrog") filters [3] and for switched-capacitor ladder filters [4]. Another advantage of that structure is the linear dependence between resonance frequency and integrator gains, a characteristic already used a decade ago [5] for linear programmability of the filter frequency response.

When the two-integrator-loop is built with OTA's [6], [7], a linear tuning range covering more than four decades is easily attained. Although the OTA filters described in [6], [7] use resistors for attenuation purposes, since traditional OTA configurations cannot handle more than a few millivolts input signal amplitude, the development of improved OTA structures [8], [9] which can accept higher amplitude signals allows the realization of active-C OTA filters with good dynamic range (around 60 dB).

A two-integrator-loop structure using only OTA's and capacitors is shown in Fig. 1. That circuit is embedded into the filters in [6], [7], but those were only able to realize low-pass and bandpass responses because the capacitors were grounded. With the extra degree of freedom of not necessarily grounding C_1 and C_2, it is possible to realize a larger number of useful transfer functions. Supposing that nodes 1–5 are connected to ground or the input voltage source (by means of switches S_1 to S_5) and the filter output is derived from node 6 or 7, as shown in Fig. 1, the voltages at nodes 1 to 5 can be written as

$$v_j = k_j v_i, \qquad j = 1,2,\cdots,5 \tag{1}$$

where v_i is the input voltage and each k_j is zero or one.

Neglecting the input impedance of the OTA's, the voltages at nodes 6 and 7 are given, in the frequency domain, by

$$\frac{V_6(s)}{V_i(s)} = \frac{N_6(s)}{D(s)} \qquad \frac{V_7(s)}{V_i(s)} = \frac{N_7(s)}{D(s)} \tag{2}$$

where

$$D(s) = s^2 + s\frac{g_{m3}}{C_1} + \frac{g_{m1}g_{m2}}{C_1C_2} \tag{3}$$

$$N_6(s) = s^2k_4 + \frac{s}{C_1}\left[g_{m1}(k_1 - k_2) + g_{m3}k_5\right] + \frac{g_{m1}g_{m2}}{C_1C_2}k_3 \tag{4}$$

$$N_7(s) = s^2k_2 + \frac{s}{C_1}\left[g_{m3}k_2 + g_{m2}(k_3 + k_4)\right] + \frac{g_{m2}}{C_1C_2}\left[g_{m1}k_1 + g_{m3}(k_3 + k_5)\right]. \tag{5}$$

From (3) the pole magnitude ω_0 and the bandwidth B are obtained as

$$\omega_0 = \left[\frac{g_{m1}g_{m2}}{C_1C_2}\right]^{1/2} \qquad B = \frac{g_{m3}}{C_1}. \tag{6}$$

TABLE I

FILTER TYPE	NUMERATOR POLYNOMIAL	INPUT NODES, k=1	OUTPUT NODE	GROUNDED NODES, k=0
LOWPASS I	$\frac{g_{m1}g_{m2}}{C_1C_2}$	3 or 1	6 7	1,2,4,5 2,3,4,5
LOWPASS II	$\frac{g_{m2}g_{m3}}{C_1C_2}$	5	7	1,2,3,4
BANDPASS I	$\frac{s\,g_{m1}}{C_1}$	1 or 4	6 7	2,3,4,5 1,2,3,5
BANDPASS II	$\frac{s\,g_{m3}}{C_1}$	5	6	1,2,3,4
HIGHPASS	s^2	4	6	1,2,3,5
NOTCH	$s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}$	3,4 or 1,2,3,4	6 6	1,2,5 5

Since transconductances g_{m1} to g_{m3} are proportional to currents I_1 to I_3 [6], [7], respectively, it is seen that the resonance frequency can be linearly controlled if $I_2 = I_1$ and the bandwidth is linearly and independently controlled by I_3. Each particular combination of switch positions in Fig. 1 will generate a different numerator polynomial in (2). The useful choices for the k_j's in (1) and for the output node are shown in Table I. Frequency responses are classified as type I or II according to the gain at resonance, which is inversely proportional to the bandwidth in the former and unity in the latter. The realization of a type II high-pass response would require at least a fourth OTA.

The circuit in Fig. 1 was built with CA3080 OTA's and $C_1 = C_2 = 130$ nF. Although the 3080 has some imperfections (which are discussed in Section IV), it is a low-cost device that can be useful for some applications where single-chip implementation is not a must. Measured frequency responses are shown in Fig. 2. Emphasis is given on the effect of the bandwidth control current I_3 since the linearity of the tuning by I_1 and I_2 over more than four decades was already verified in an early work [7]. The currents I_1 and I_2 were fixed at 40 uA (corresponding to $g_{m1} = g_{m2} = 1.54$ mmho [7]) whereas I_3 was varied in octaves, from 5 μA to 80 μA (corresponding to quality factors Q varying from 1/2 to 8).

There are two nonideal characteristics that can be verified from Fig. 2: 1) for type I filters, the higher Q curve show a little tilt to the left and 2) for type II filters the gain at resonance or the notch depth decreases for smaller bandwidths. Both problems are addressed to Section IV.

According to (3) current I_3 determines independently the bandwidth. There are applications where $Q = \omega_0/B$ must be independently controlled. The biasing circuit in Fig. 3, based on the Gilbert cell [10], can be used in those cases. The output

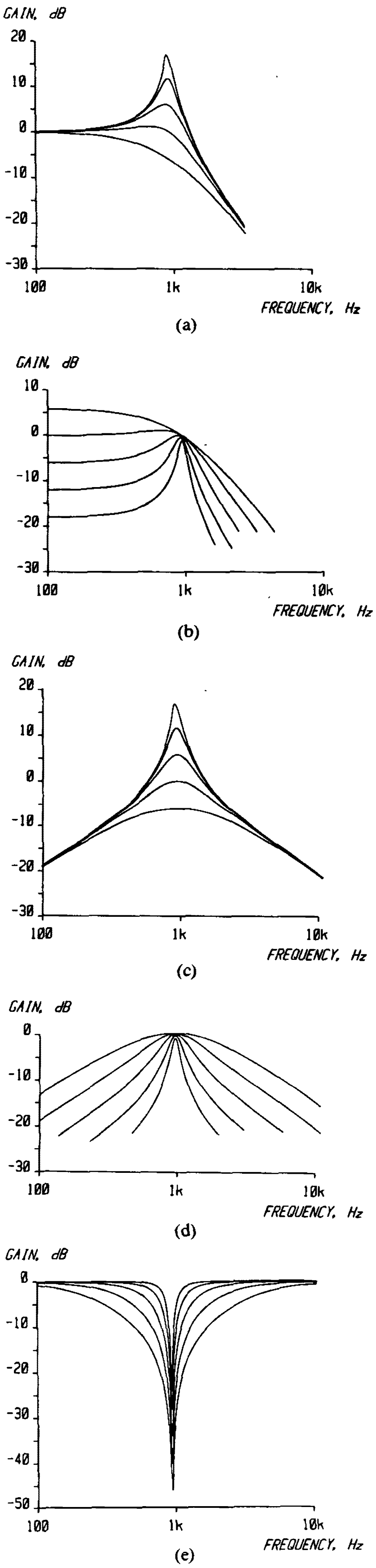

Fig. 2. Measured magnitude responses of the circuit in Fig. 1. For all cases, $I_1 = I_2 = 40$ μA and $I_3 = 5$, 10, 20, 40, and 80 μA. (a) Lowpass I. (b) Lowpass II. (c) Bandpass I. (d) Bandpass II. (e) Notch.

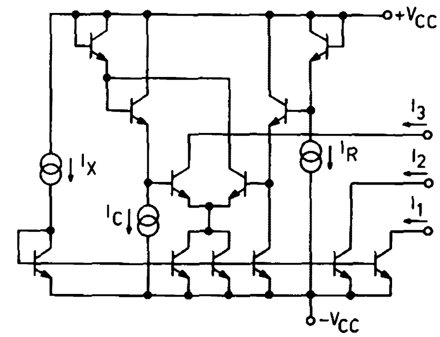

Fig. 3. Biasing circuit that allows independent and linear control of the quality factor Q.

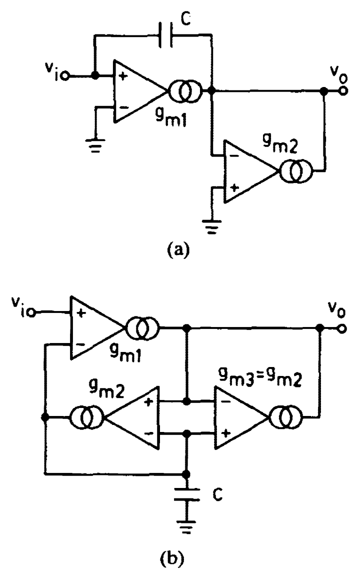

Fig. 4. First-order audio equalizers. a) Bass. b) Treble.

current I_3 satisfies

$$I_3 = \frac{I_R I_X}{I_C}, \qquad I_3 < 2I_X. \tag{7}$$

Using output currents I_1 to I_3 to bias OTA_1 to OTA_3 in Fig. 1, respectively, it follows from (6) that

$$Q = \frac{\omega_0}{B} = \frac{I_C}{I_R}, \qquad I_C > \frac{I_R}{2} \tag{8}$$

and so I_C controls Q linearly.

Programmable filters of higher order can be realized by cascading blocks like the one in Fig. 1. An alternative approach, preferred in applications where programmability is restricted to tuning, is to use the leapfrog technique [3]. The implementation of leapfrog filters with transconductance amplifiers is discussed in [11].

III. Equalizer Circuits

Simple first-order equalizer for "bass" and "treble" audio tone shaping can be realized with the circuits in Fig. 4. Their frequency responses are

$$\text{bass:} \qquad \frac{V_0(s)}{V_i(s)} = \frac{sC + g_{m1}}{sC + g_{m2}} \tag{9}$$

$$\text{treble:} \qquad \frac{V_0(s)}{V_i(s)} = \frac{(sC + g_{m2})g_{m1}}{(sC + g_{m1})g_{m2}}. \tag{10}$$

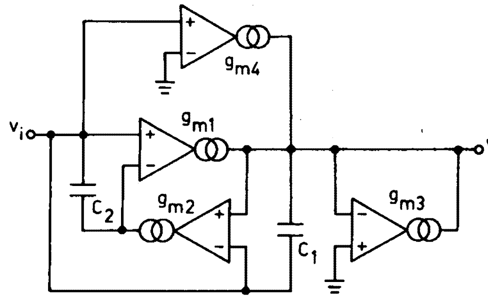

Fig. 5. An active-C bump equalizer. By interchanging the inputs to OTA, it can also work as a delay equalizer.

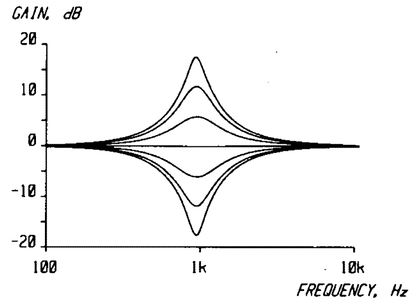

Fig. 6. Measured magnitude response of the bump equalizer in Fig. 5, with $I_1 = I_2 = 40$ μA, $I_3 + I_4 = 80$ μA, and $I_4/I_3 = 1/8$ (bottom), 1/4, 1/2, 1 (middle), 2, 4, and 8 (top).

If the sum $g_{m1} + g_{m2}$ is kept constant by the biasing circuitry, the ideal Baxandall-type responses are obtained. By cascading the equalizers in Fig. 4, a complete audio tone shaping unit is obtained. In [8] a tone shaping circuit with the same number of OTA's and capacitors is presented, but requiring more complicated biasing circuits to obtain symmetrical boost-cut responses.

A bump equalizer [12] can be realized with the circuit in Fig. 5, which results from adding a fourth OTA to the notch filter version of the basic structure in Fig. 1. Neglecting input conductances, it follows

$$\frac{V_0(s)}{V_i(s)} = \frac{s^2 + s\frac{g_{m4}}{C_1} + \frac{g_{m1}g_{m2}}{C_1C_2}}{s^2 + s\frac{g_{m3}}{C_1} + \frac{g_{m1}g_{m2}}{C_1C_2}} \tag{11}$$

where it is seen that the bump height, i.e., the gain at ω_0, is given by g_{m4}/g_{m3}. If the biasing circuitry keeps $g_{m3} + g_{m4}$ constant, symmetrical boost-cut responses are obtained, with the value of $g_{m3} + g_{m4}$ defining the bandwidth of the equalizer. Measured frequency responses for the circuit in Fig. 5 are shown in Fig. 6. Those curves were obtained from a prototype built with CA3080 OTA's and $C_1 = C_2 = 130$ nF. They show a good agreement with (11).

By interchanging the input terminals of OTA in Fig. 5, and making $g_{m3} = g_{m4}$, the roots of the numerator in (11) will have the same resonance frequency and bandwidth of the poles, but will be in the right-half s-plane. So, an all-pass (delay equalizer) transfer function will be obtained. The time delay at the resonance frequency will be approximately equal to the peak delay and will be given by

$$D = -\frac{d}{d\omega}\arg\left(\frac{V_0(j\omega)}{V_i(j\omega)}\right)\bigg|_{\omega=\omega_0} \simeq \frac{4C_1}{g_{m3}} \tag{12}$$

and so the bias currents for OTA_3 and OTA_4 control the peak delay without affecting the resonance frequency.

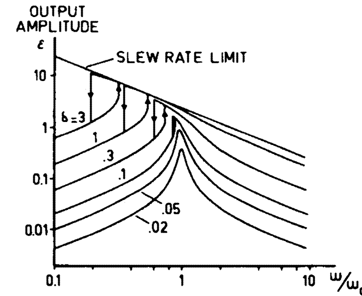

Fig. 7. Output amplitude (fundamental component) versus frequency for a sinusoidal input of fixed amplitude. Jump resonance occurs if $\epsilon > 1$.

IV. Nonidealities

If OTA's like the CA3080 are used to build the filters in Section II, dynamic range problems are expected, due to the limited linear range of the OTA transconductance [7]. By applying the describing function technique [13] to analyze the configuration in Fig. 1 with an accurate nonlinear OTA model [14], it is possible to determine the effect of high signal amplitudes over the frequency response. Although the detailed nonlinear analysis would take too much space to be described here, it is worthwhile to consider some of its results. In Fig. 7 it is shown the magnitude response for the bandpass I version of the circuit in Fig. 1 for $Q = 10$, where δ is the ratio of the input amplitude to the thermal voltage V_T ($V_T = kT$, where T is in degrees Kelvin and $k = 86.2$ μV/K) and ϵ the ratio of the fundamental component of the output signal to V_T. For $\epsilon > 1$, roughly, the jump resonance effect occurs. At $\epsilon \simeq 1$ there is a tilt to the left in the peak response, which was observed experimentally in Section II [see Fig. 2(a) and (c)]. For $\epsilon < 1/2$ the nonlinear effects are negligible.

As observed in Section II, the input impedance of the OTA may have a significant effect over the filter response. It is possible to predict accurately that influence, but for practical purposes it is much better to buffer the OTA outputs with Darlington pairs, an idea already incorporated in a new OTA chip.[1] The temperature influence on the OTA transconductance [7] should also be taken into account. This can be compensated by using current sources proportional to the absolute temperature [15].

From the above paragraphs it can be concluded that OTA's like the CA3080 may not be adequate for some applications. In those cases, improved OTA configurations should be adopted and transconductance reduction techniques [9] applied if low capacitance values are desired, as when fully integration is a must.

V. Conclusions

This paper has described some filter and equalizer circuits that are suitable for integration in bipolar technology since they are active-C, i.e., resistorless. Emphasis was given to the description of the basic circuit configurations, and some implementation problems were discussed. For applications were dynamic range is not a stringent factor, low-cost tunable filters can be built using the CA3080 OTA. In more sophisticated applications, the same circuit configurations can be used but built with improved OTA's.

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