Rate Adaptation Games in Wireless LANs: Nash Equilibrium and Price of Anarchy

Prasanna Chaporkar
Indian Inst. of Technology Bombay, India
chaporkar@ee.iitb.ac.in

Alexandre Proutiere
Microsoft Research, Cambridge, UK
Alexandre.Proutiere@microsoft.com

Božidar Radunović
Microsoft Research, Cambridge, UK
bozidar@microsoft.com

***Abstract*—In Wireless LANs, users may adapt their transmission rates depending on the radio conditions of their links so as to maximize their throughput. Recently, there has been a significant research effort in developing distributed rate adaptation schemes. Unlike previous works that mainly focus on channel tracking, this paper characterizes the optimal reaction of a rate adaptation protocol to the contention information received from the MAC. We formulate this problem analytically. We study both competitive and cooperative user behaviors. In the case of competition, users selfishly adapt their rates so as to maximize their own throughput, whereas in the case of cooperation they adapt their rates so as to maximize the overall system throughput. We show that the Nash Equilibrium reached in the case of competition is inefficient (i.e. the price of anarchy goes to infinity as the number of users increases), and provide insightful properties of the socially optimal rate adaptation schemes. We find that recently proposed collision-aware rate adaptation algorithms decrease the price of anarchy. We also propose a novel collision-aware rate adaptation algorithm that further reduces the price of anarchy.**

I. Introduction

Distributed scheduling and rate adaptation are two key parts of the IEEE 802.11 MAC layer. Users share the radio resources in a distributed manner using the mandatory contention-resolution scheme DCF (Distributed Coordination Function). This scheme specifies how users should adapt their channel access probability when they experience transmission failures. When the network is perceived as congested, under the DCF, users *cooperatively* decrease their access probability, which in turn limits the number of collisions and keeps the overall network efficiency at a satisfying level. DCF is time-critical and it is implemented in hardware. Users cannot modify it.

IEEE 802.11 standards support multiple transmission rates, eight or more different rates in the case of 802.11a/g and proprietary extensions, and more than twenty in case of 802.11n [1]. Each link should adapt its modulation and coding rate to identify the optimal trade-off between the transmission rate and the packet losses due to channel errors. The objective of rate adaptation algorithms is to estimate the channel quality and to find this optimal trade-off. These algorithms are typically implemented in software and can be modified. IEEE 802.11 standards do not specify any rate adaptation algorithm.

Today, the prevalent approach to rate adaptation is to track channel quality based on packet losses and to adapt the rate accordingly. This approach was initially proposed in ARF [2], but many other algorithms have been proposed in the literature to replace ARF, see e.g. [3], [4], [5]. One of the key limitation of the tracking approach is that the channel has to be constant during many packet transmissions so that the tracking algorithm is able to converge to the optimal rate. This can be alleviated using channel measurements to infer the optimal rate from the measured SNR (see e.g. [6], [7], [8], [9]). In all cases, the rate adaptation protocols optimize the transmission rate as if a link was in isolation and they do not consider the interaction of the rate adaptation with the scheduling.

An important challenge in designing rate adaptation schemes stems from the fact that transmitters may not be able to distinguish between the causes of transmission failures. Transmission failures are caused by collisions and/or channel errors. Without this *loss differentiation* capability, both DCF and rate adaptation schemes may make wrong decisions. Channel errors can lead to an unnecessary access probability decrease when the network is lightly loaded. When the network is congested, collisions may be interpreted as channel errors and lead users to decrease their transmission rates, which in turn increases the packet transmission durations and further exacerbates the network congestion. Researchers have proposed several ways of differentiating channel errors and collisions, and of exploiting this information in the design of rate adaptation schemes, see e.g. [10], [11], [5], [12], [8], [13]. Most often, the proposed rate adaptation algorithms are based on heuristic arguments and numerical experiments.

In this paper, *our aim is to provide understanding of the interaction between rate adaptation protocol and IEEE 802.11 DCF in a network with multiple nodes from an analytical viewpoint*. We assume that all nodes implement the standard DCF (with or without loss differentiation), but can modify their rate adaptation algorithms. We aim at characterizing how users should optimally select a transmission rate depending on their state in the DCF scheme (i.e., their back-off stage) representing the level of congestion in the network. We also study the impact of loss differentiation on the system performance.

Rate adaptation can be done in a cooperative or competitive manner. In the former scenario, the rate adaptation scheme is designed so as to maximize the total throughput of the network while guaranteeing a certain degree of fairness among users, thus achieving a social optimum. In the latter scenario, each user designs its rate adaptation strategy with the aim

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of maximizing its own throughput without accounting for its impact on the system performance. If there is a single greedy user, selfishly trying to adapt its rate, this user could potentially receive a higher throughput than that obtained in the social optimal. However, if all users compete, the system may evolve to an inefficient Nash Equilibrium (NE) where all users would receive a lower throughput than that in the social optimum. The performance gap between the social optimum and Nash Equilibria is called the *price of anarchy* [14].

The main contributions of this paper are as follows:

• We formulate the problem of designing optimal rate adaptation algorithms analytically in networks with slowly varying fading. We develop a generic framework that enables us to consider both cooperative and competitive user behaviors and the ability or inability of users to distinguish collisions from channel errors.

• We show that the competitive scenarios lead to inefficient Nash equilibria. In particular, when the number of links is large, competition yields zero network throughput.

• We find that, although collision awareness does not eliminate the starvation when the number of competing users is high, it increases the efficiency of the competitive equilibria. It does not increase the performance of the cooperative optimum.

• We propose a novel way of reacting to channel errors called *ROCE* (Return to 0 on Channel Errors). We show that it further reduces the price of anarchy, although it does not alleviate the starvation in large competitive networks.

The above findings are obtained through a mix of analytical and numerical results. Our paper is novel in two aspects: in [15], [16], the authors have proposed an analytical model for the interaction of DCF and rate adaptation schemes, but they haven't considered improved adaptation algorithms; the possible selfish behavior of users in adapting their rate is rarely considered (in [17], [18], the authors provide preliminary analysis of rate adaptation games in WLANs where transmission failures due to channel errors are not modeled).

The paper is organized as follows. Section II presents the models. We present a generic stationary analysis is given in Section III. Competitive scenarios are discussed in Section IV and cooperative scenarios in Section V. Numerical results are presented in Section VI. The proofs are in the appendix.

II. Models

We consider a network of $\mathcal{N} = \{1, \dots, N\}$ links. All links interfere with each other, and the corresponding transmitters always have packets to send.

Scheduling. All transmitters implement the same distributed random back-off scheduling mechanism to access the channel, e.g. DCF, that cannot be changed. This mechanism is modeled as follows. There are $I+1$ back-off stages: stage $i \in \{0, \dots, I\}$ indicates that i consecutive collisions or packet losses have been experienced. In stage i a node transmits a packet with a fixed probability $p(i)$ such that $p(i) \geq p(i+1)$ for every $i < I$ (in DCF $p(i) = 2^{-i} p(0)$). We optimize the rate adaptation algorithm for a fixed scheduling mechanism.

Rate Adaptation. The radio conditions for link n is characterized by the signal-to-noise ratio SNR_n at the receiver. We are interested in the interaction of the rate adaptation and scheduling, and we do not study how to track channel changes. Therefore, we focus on the rate adaptation protocols that can measure the channel SNR (see e.g. [6], [7], [8], [9]) and we assume that the SNR_n is known at the transmitter of link n. We also assume slow fading, such that the SNRs can be assumed constant for the duration of the analysis. This model is reasonable abstraction for networks in which user mobility is limited, e.g., office WLANs.

To send a packet, each transmitter n can select a rate from a set $\mathcal{R}$ using certain rate adaptation strategy ρ_n. Formally, a rate adaptation strategy ρ_n is a map from $\{0, \dots, I\}$ to $\mathcal{R}$, and thus $\rho_n(i)$ denotes the transmission rate for user n in the i^{th} back-off stage. A strategy ρ_n is said to be *constant* if $\rho_n(i) = \rho_n(i+1)$ for all $i < I$. When a packet is sent at rate $R \in \mathcal{R}$, the transmission duration is $T = \sigma / R$, where σ is the fixed packet size. We use rate $\rho_n(i)$ and transmission duration $T_n(i) = \sigma / \rho_n(i)$ interchangeably to denote the rate adaptation strategy.

The probability that a packet sent at rate R is lost due to channel error is a function $e(R, \mathrm{SNR})$ of the rate and the SNR. For brevity, we denote $e_n(R) = e(R, \mathrm{SNR_n})$. We use the model for channel loss from [19, Section II.C], $e_n(R) = 1 - \exp\left(-(e^{\gamma R} - 1)/\kappa_n\right)$, where κ_n depends on the link n quality (the received SNR) and γ depends on different system parameters (such as the bandwidth) but not on the link quality. Optimal operation points of a wireless network are in a low channel loss regime. Hence, we can simplify the above equation using the first order approximation:

$$e_n(R) = \frac{e^{\gamma R} - 1}{\kappa_n} \tag{1}$$

This model is also similar to the models used in [20]. We also verify that it fits well to the data measured in [9] (see Section VI). We also use the form $e_n(T) = (e^{\gamma \sigma / T} - 1)/\kappa_n$. Note that function $e_n(T)$ is convex in T.

Collisions. The duration of a collision T_{coll} is either the maximum duration of the packet transmissions involved in the collision when RTS/CTS is not used, or simply the duration of RTS/CTS signaling T_{RTS} when RTS/CTS is used. It is difficult to analyze the former, where T_{coll} depends on the duration of different transmissions. However, we observe that in a well designed system the collision rate should be small (e.g. less than 10% [21]) and we can ignore it. In this paper we will assume a constant duration of a collision, $T_{coll} = T_{RTS}$. This is an exact model in the case of networks with RTS/CTS and a reasonable approximation otherwise.

We study systems with and without loss differentiation.

Systems Without Loss Differentiation (WoLD). Here, collisions and channel errors can not be distinguished (like in all of the 802.11 standards). Thus, the back-off stage is incremented whenever a packet is lost.

Systems With Loss Differentiation (WLD). We also consider the case where collisions and channel errors can be differentiated.

We consider two families of collision-aware rate adaptation strategies:

• The first family, called WLDS (where S stands for *Standard*), includes the strategies proposed in the literature, e.g. in [10], [11], [5], [12], [8], [13]. If a transmission fails due to collision, then the back-off stage is incremented. If it fails due to channel error, then the back-off stage remains the same.

• We propose a second new family of rate adaptation strategies, referred to as ROCE (Return to 0 On Channel Error). Here, unlike WLDS, if a transmission fails due to a channel error, the back-off stage is reset to the minimum value ($i = 0$). The intuition behind this is that since the loss was not caused by collision, there is no reason to remain in a high-contention DCF state. If a transmission fails due to a collision, the back-off stage is increased, as in all other schemes.

Competition vs. Cooperation: In this paper, we aim at characterizing the performance of systems where users either tend to maximize their own throughputs (selfish) or they jointly optimize a network-wide performance metric (cooperative). The selfish behavior of transmitters is modeled as a pure strategic rate adaptation game. This means that each transmitter $n \in \mathcal{N}$ adopts a deterministic strategy ρ_n with the aim of maximizing its own throughput. We will investigate the existence and the properties of Nash Equilibria in this game. In the cooperative scenario we will consider Pareto optimal strategies as the most general way to model socially optimal strategies.

III. Stationary analysis

We start the analysis by studying the steady state behavior of systems where transmitter $n \in \mathcal{N}$ uses a given rate adaptation strategy ρ_n. This analysis will be used to examine the system performance in both competitive and cooperative scenarios. We denote by $\pi_n(i)$ the stationary probability that transmitter n is in the back-off stage i, by $p_n = \sum_i \pi_n(i)p(i)$ the average transmission probability of link n, and by $c_n = 1-\prod_{m\neq n}(1-p_m)$ the probability that link n will collide with any other link, given that it transmits. We also define $\boldsymbol{\pi} = (\pi_n)_{n\in\mathcal{N}}, \boldsymbol{p} = (p_n)_{n\in\mathcal{N}}, \boldsymbol{c} = (c_n)_{n\in\mathcal{N}}, \boldsymbol{\rho} = (\rho_n)_{n\in\mathcal{N}}$.

Average slot duration. We consider *virtual slots*, as defined in [21]: a slot may correspond to a slot where the channel is idle (no transmission occurs), to a successful transmission, or to a collision. All links "see" the same virtual slots. Denote by $S(\boldsymbol{\rho})$, $S^n(\boldsymbol{\rho})$, and $S^n_R(\boldsymbol{\rho})$ the expected slot duration when transmitters use rate adaptation strategies $\boldsymbol{\rho}$, unconditioned on n, given that transmitter n is silent, and given that transmitter n is transmitting a packet at rate R, respectively. We have

$$S(\boldsymbol{\rho}) = \sum_l (1-c_l)\sum_j \pi_l(j)p(j)T_{\rho_l(j)} + \prod_l (1-p_l) + \left(1-\prod_l(1-p_l)-\sum_l(1-c_l)p_l\right)T_{RTS},$$

$$S^n(\boldsymbol{\rho}) = \sum_{l\neq n}\frac{1-c_l}{1-p_n}\sum_j \pi_l(j)p(j)T_{\rho_l(j)} + \prod_l (1-p_l) + \left(1-\prod_l(1-p_l)-\sum_l(1-c_l)p_l\right)T_{RTS}.$$

$$S^n_R(\boldsymbol{\rho}) = T_{RTS} + T_R(1-c_n).$$

Link Throughput. From the average slot duration, we can compute $\phi_n(\boldsymbol{\rho})$ the stationary throughput of link n by:

$$\phi_n(\boldsymbol{\rho}) = \frac{\sum_i \pi_n(i)p(i)(1-e_n(\rho_n(i)))(1-c_n)}{S(\boldsymbol{\rho})}. \quad (2)$$

Stationary distributions and residual times. To compute the link throughputs, we need to evaluate the stationary distribution $\boldsymbol{\pi}$ and the collision probabilities $\boldsymbol{c}$. In order to characterize the best response of a user, we also define the *average residual time* $J_n(i,\boldsymbol{\rho})$ as the average time needed by link n to transmit a packet, given that it is in stage i and the rate adaptation strategies are defined by $\boldsymbol{\rho}$. Here we compute $\boldsymbol{\pi}, \boldsymbol{c}$ and $J_n(i,\boldsymbol{\rho})$, first for WoLD systems, and then for WLDS and ROCE systems.

1) WoLD systems: Given that transmitter of link n is in stage i, it can either successfully transmit and move to state 0 with probability $p(i)(1-c_n)(1-e_n(\rho_n(i)))$, or experience of transmission failure with probability $p(i)(1-(1-c_n)(1-e_n(\rho_n(i))))$ or remain idle with probability $1-p(i)$. Then we classically deduce that (for $0 < i < I$):

$$\pi_n(i) = \frac{p(0)\prod_{k=0}^{i-1}(1-(1-c_n)(1-e_n(\rho_n(k))))}{p(i)}\pi_n(0),$$

$$\pi_n(I) = \frac{p(0)\prod_{k=0}^{I-1}(1-(1-c_n)(1-e_n(\rho_n(k))))}{p(I)(1-c_n)(1-e_n(\rho_n(I)))}\pi_n(0).$$

The average residual transmission times are given by:

$$J_n(i,\boldsymbol{\rho}) = \frac{1-p(i)}{p(i)}S^n(\boldsymbol{\rho}) + S^n_{\rho_n(i)}(\boldsymbol{\rho}) + (1-(1-c_n)(1-e_n(\rho_n(i))))J_n(i+1,\boldsymbol{\rho}),$$

$$J_n(I,\boldsymbol{\rho}) = \frac{[(1-p(I))/p(I)]S^n(\boldsymbol{\rho}) + S^n_{\rho_n(I)}(\boldsymbol{\rho})}{(1-c_n)(1-e_n(\rho_n(I)))}.$$

where $\frac{1-p(i)}{p(i)}S^n(\boldsymbol{\rho})$ is the average time the link is idle before transmitting for the first time, $S^n_{\rho_n(i)}(\boldsymbol{\rho})$ is the average time the link transmits (regardless of the success of the transmission) and if the transmission is unsuccessful (with probability $(1-(1-c_n)(1-e_n(\rho_n(i))))$ the average time to transmit the packet is $J_n(i+1,\boldsymbol{\rho})$ as the link moves to stage $i+1$.

2) WLDS systems: Given that transmitter n is in stage i, it can either move to stage $i+1$ if it encounters a collision with probability $p(i)c_n$, or it can remain in stage i with probability $1-p(i)+p(i)(1-c_n)e_n(\rho_n(i))$ (either it remains silent or it encounters a channel error), or it can return to stage 0 with probability $p(i)(1-c_n)(1-e_n(\rho_n(i)))$. We have (for $0 < i < I$):

$$\pi_n(i) = \frac{p(0)c_n^i}{p(i)\prod_{k=1}^{i}(1-(1-c_n)e_n(\rho_n(k)))}\pi_n(0),$$

$$\pi_n(I) = \frac{[p(0)/p(I)]c_n^I \pi_n(0)}{(1-c_n)(1-e_n(\rho_n(I)))\prod_{k=1}^{I-1}(1-(1-c_n)e_n(\rho_n(k)))}.$$

and for the average transmission residual times:

$$J_n(i,\boldsymbol{\rho})(1-(1-c_n)e_n(\rho_n(i))) = \frac{1-p(i)}{p(i)}S^n(\boldsymbol{\rho}) + S^n_{\rho_n(i)}(\boldsymbol{\rho}) + c_n J_n(i+1,\boldsymbol{\rho}),$$
$$J_n(I,\boldsymbol{\rho}) = \frac{[(1-p(I))/p(I)]S^n(\boldsymbol{\rho}) + S^n_{\rho_n(I)}(\boldsymbol{\rho})}{(1-c_n)(1-e_n(\rho(I)))}.$$

3) ROCE systems: Given that transmitter n is in stage i, it can either move to stage $i+1$ if it encounters a collision with probability $p(i)c_n$, or it can remain silent with probability $1-p(i)$, or it can return to stage 0 with probability $p(i)(1-c_n)$ (regardless of channel errors). We then have (for $0 < i < I$):

$$\pi_n(i) = \frac{p(0)c_n^i}{p(i)}\pi_n(0), \qquad \pi_n(I) = \frac{p(0)c_n^I}{p(I)(1-c_n)}\pi_n(0).$$

Also note that unlike in WoLD and WLDS systems, the stationary distribution $\boldsymbol{\pi}$ does not depend on the rate adaptation scheme $\boldsymbol{\rho}$ in ROCE systems, nor on the link qualities (that is $\boldsymbol{\pi}_n = \boldsymbol{\pi}_m$ for all $m,n \in \mathcal{N}$). The average transmission residual times are given by:

$$J_n(i,\boldsymbol{\rho}) = \frac{1-p(i)}{p(i)}S^n(\boldsymbol{\rho}) + S^n_{\rho_n(i)}(\boldsymbol{\rho}) + c_n J_n(i+1,\boldsymbol{\rho}) + (1-c_n)e_n(\rho(i))J_n(0,\boldsymbol{\rho}),$$
$$J_n(I,\boldsymbol{\rho})(1-c_n) = \frac{1-p(I)}{p(I)}S^n(\boldsymbol{\rho}) + S^n_{\rho_n(I)}(\boldsymbol{\rho}) + (1-c_n)e_n(\rho(I))J_n(0,\rho).$$

In each of the cases, we use the above expressions to obtain the collision probabilities $\boldsymbol{c}$ as a fixed point of the following expressions: $p_n = \sum_{i=0}^{I}\pi_n(i)p(i)$, $c_n = 1-\prod_{m\neq n}(1-p_m)$. We next show that there exists a unique fix point of the above system (we give the result here only for WoLD, the other two cases are similar).

Proposition 1: Let us assume for all i, $e_i \leq e_{MAX}$, $p(i+1)/p(i) = a$, $p(0) \geq (1+2a+2a^2 e_{MAX})/(1-a)$ and $1+a-2a(1-e_{MAX}) < 0$. Then, for every N, $(SNR_n)_{n\in\mathcal{N}}$ and any given rate adaptation $\boldsymbol{\rho}$, there exists a unique fixed point $(\boldsymbol{p},\boldsymbol{c})$ in WoLD network.

Proof: The proof is an extension of the analysis in [22] and we shall use the same notation as in [22]. Let us denote with $\zeta_n(i) = \prod_{k=0}^{i-1}(1-(1-c_n)(1-e_n(\rho_n(k))))$ for $i \leq I$ and $\zeta_n(i) = \zeta_n(I)(1-(1-c_n)(1-e_n(\rho_n(k))))^{i-I}$ for $i > I$, and let b_i be the mean back-off duration in back-off stage i (as in [22]). We are then able to get rid of $\boldsymbol{\pi}$ and link directly $\boldsymbol{p}$ and $\boldsymbol{c}$ as

$$p_n = \Delta_n(c_n) = \frac{1+\sum_{i=0}^{\infty}\zeta_n(i)}{b_0+\sum_{i=0}^{\infty}b_{i+1}\zeta_n(i)}, \tag{3}$$
$$c_n = \Gamma_n(p_1,\ldots,p_n) = 1-\prod_{m\neq n}(1-p_m). \tag{4}$$

We are looking for a fixed point of the following system of the following system $c_n = \Gamma_n(\Delta_1(c_1),\ldots,\Delta_n(c_n))$. The mapping is continuous so a fixed point exists. From Lemma 2 we have that $\Delta_n(c)$ is monotonically decreasing and that $(1-c)(1-\Delta_n(c))$ is monotonous. Then from [22, Theorem 5.3] we have that the fixed point is unique. ■

Observe that the conditions of Theorem 1 are satisfied in the practical system. For example, in 802.11 we have $a=2$ and to satisfy the conditions of the theorem we need $e_{MAX} \leq 5/8$. A typical 802.11 link does not operate with a channel error rate larger than e_{MAX}.

IV. Competition

In this section, we analyze systems with selfish but rational users, who want to maximize their respective throughputs. We model this scenario as a rate adaptation game with the set of feasible strategies ρ_n for any user n, and the throughput it achieves being its pay-off for a given $\boldsymbol{\rho}$. We study the existence of pure Nash Equilibria for this game, and provide some structural properties of these equilibria. We restrict our attention to scenarios where users play pure strategies (do not randomized their rate adaptation decisions). We prove the following: (a) pure NEs exist; (b) we establish certain properties of NEs to gain insights into the system behaviour and to reduce the complexity of computing these equilibria for WoLD and WLDS systems; (c) we provide an explicit procedure to compute NEs in ROCE system.

Because of the analytical difficulties, we make the following assumptions in this section: (1) For WoLD and WLDS systems, we consider non-atomic game, i.e., the number of users is large so that the strategy change of one user does not change the system statistics, specifically it does not affect $\boldsymbol{c}$. For a precise definition of non-atomic games, refer to [23]. This assumption is not required for ROCE as under ROCE $\boldsymbol{\pi}$ and hence $\boldsymbol{c}$ does not depend on $\boldsymbol{\rho}$. (2) For all the systems, let $\mathcal{R} = [0, R_{\max}]$, where $R_{\max}$ is finite. We verify the results for an atomic setting (small number of nodes N) using simulations in Section VI.

A. Best Response Correspondence

Let $\boldsymbol{\rho}_{-n} = (\rho_m)_{m\neq n}$, and denote[1] by $\phi(\rho_n,\boldsymbol{\rho}_{-n})$ the throughput of user n when it uses strategy ρ_n and others use $\boldsymbol{\rho}_{-n}$. Let $\mathcal{A} = \mathcal{R}^{I+1}$, and let us define the best response for link n as $B_n : \mathcal{A}^{N-1} \to \mathcal{A}$ such that if $\rho_n \in B_n(\boldsymbol{\rho}_{-n})$, then $\phi(\rho_n,\boldsymbol{\rho}_{-n}) \geq \phi(\rho,\boldsymbol{\rho}_{-n})$ for every $\rho \in \mathcal{A}$. Observe that $\phi(\rho_n,\boldsymbol{\rho}_{-n}) = 1/J_n(0,\rho_n,\boldsymbol{\rho}_{-n})$ and thus $\rho \in B_n(\boldsymbol{\rho}_{-n})$ only when $\rho \in \arg\min_{\rho\in\mathcal{A}} J_n(0,\rho,\boldsymbol{\rho}_{-n})$. This shows that the best response can be obtained by minimizing the residual times. Next, we minimize the residual time using MDP formulation [24]. To this end, denote by $J_n^\star(i,\boldsymbol{\rho}_{-n})$ the minimum expected residual time from i^{th} back-off stage given that other users use rate adaptation strategies $\boldsymbol{\rho}_{-n}$. Now, it can be shown that under WoLD, WLDS (with non-atomic game assumption)

[1] Throughout this section we also represent $\boldsymbol{\rho}$ as a tuple $(\rho_n,\boldsymbol{\rho}_{-n})$

and ROCE that $J_n^\star(i,\boldsymbol{\rho}_{-n})$ can be obtained as

$$J_n^\star(i,\boldsymbol{\rho}_{-n}) = \min_{R\in\mathcal{R}} G_n(R,u_i) + \xi_i^n, \text{ where} \tag{5}$$

$$G_n(R,u) = (1-c_n)\left[\frac{\sigma}{R} + \frac{e^{\gamma R}u}{\kappa_n}\right]. \tag{6}$$

Intuitively, ξ_i^n is the average overhead of the packet transmission in stage i which is independent of the transmission rate R and u is the average time needed to send a packet if this transmission fails. Depending on the system under consideration u_i, and ξ_i^n have to be chosen appropriately.

For WoLD system $u_i = J_n^\star(k_i,\boldsymbol{\rho}_{-n})$, and $\xi_i^n = \frac{1-p_i}{p_i}S^n(\boldsymbol{\rho}) + T_{RTS} + (1-(1-c_n)(1+1/\kappa_n))J_n^\star(k_i,\boldsymbol{\rho}_{-n})$, where $k_i = i+1$ for $i < I$ and $k_I = I$. For WLDS system $u_i = J_n^\star(i,\boldsymbol{\rho}_{-n})$, and $\xi_i^n = \frac{1-p_i}{p_i}S^n(\boldsymbol{\rho}) + T_{RTS} + c_nJ_n^\star(k_i,\boldsymbol{\rho}_{-n}) - (1-c_n)J_n^\star(i,\boldsymbol{\rho}_{-n})/\kappa_n$. Note that only because of the non-atomic game assumption, c_n and $S^n(\boldsymbol{\rho})$ is independent of ρ_n, but can be completely characterized by $\boldsymbol{\rho}_{-n}$. Finally, for ROCE system $u_i = J_n^\star(0,\boldsymbol{\rho}_{-n})$, and $\xi_i^n = \frac{1-p_i}{p_i}S^n(\boldsymbol{\rho}) + T_{RTS} + c_nJ_n^\star(k_i,\boldsymbol{\rho}_{-n}) - (1-c_n)J_n^\star(0,\boldsymbol{\rho}_{-n})/\kappa_n$.

Let us define $R^\star(u) = \min_{R\in\mathcal{R}} G_n(R,u)$. Then, we note that $\rho \in B_n(\boldsymbol{\rho}_{-n})$ only when $\rho(i) = R^*(u_i)$ for each i. Thus, it suffices to study the properties of $G_n(R,u)$ in order to study the properties of the best response.

Lemma 1: $R^\star(u)$ is unique for every u, and is a monotone decreasing function of u. Moreover, $G_n(R^\star(u),u)$ is monotone increasing function of u.

Proof: Note that for a given $u > 0$, $G_n(R,u)$ is a strictly convex function of R in the positive half plane. Thus, it has a unique minimum (say $R'(u)$) in the positive half plane. Now, $R^\star(u)$ is uniquely determined as $\min\{R'(u), R_{\max}\}$.

We next need that $R^\star(u)$ is a monotone decreasing function of u. Let $F(R) = \gamma R + r\log(R)$. We have that $R'(u)$ is the unique solution of $\frac{\partial G_n(R,u)}{\partial R} = 0$, i.e., it is the solution of $F(R) = \log(\sigma\kappa_n/u\gamma)$. Note that $F(R)$ is monotone increasing, with $F(0) = -\infty$ and $F(\infty) = \infty$. Now, if $u_1 > u$, then $\log(\sigma\kappa_n/u\gamma) > \log(\sigma\kappa_n/u_1\gamma)$. Thus, $F(R)$ will cross level $\log(\sigma\kappa_n/u_1\gamma)$ before crossing level $\log(\sigma\kappa_n/u\gamma)$. Hence, $R'(u_1) < R'(u)$, which proves the required.

For the last part, note that $G_n(R,u) \le G_n(R,u_1)$ whenever $u < u_1$ for every R. Thus, $G_n(R^\star(u),u) \le G_n(R^\star(u_1),u) \le G_n(R^\star(u_1),u_1)$. This concludes the proof. ■

B. Existence of Nash Equilibrium

It is well known that a mixed strategy NE always exists (for finite action space see [25], and for compact action space see [26]). However, a mixed strategy would imply that each user implements one rate adaptation strategy for a long time, and then switch to another one with some probability, run it for a long time, and so on. This is not feasible from the system point of view. Hence, we are interested in a pure strategy NE, where each n implements constant strategy ρ_n.

A mixed strategy NE does not always exist, but we prove that it exists our case in an arbitrary network. Unlike [25], we need to establish a fixed point in the space of pure strategies and not in the space of mixed strategies.

Proposition 2: In WoLD, WLDS (with non-atomic game assumption) and ROCE systems, the rate adaptation games have a pure NE.

Proof: To prove the existence of pure NE, it suffices to prove that the correspondence $B(\cdot)$ has a fixed point. We use Kakutani's fixed point theorem to prove the required. First note that by construction, $\mathcal{A}^N$ is a compact, convex and non-empty subset of the finite dimensional Euclidean space. Moreover, $B_n(\boldsymbol{\rho}_{-n})$ contains a single point as shown in Lemma 1, hence $B(\cdot)$ is non-empty and convex. Hence, it suffices to show that $B(\cdot)$ has a closed graph, i.e., if $(\rho_n^\ell, \boldsymbol{\rho}_{-n}^\ell) \to (\rho_n, \boldsymbol{\rho}_{-n})$ such that $\rho_n^\ell \in B(\boldsymbol{\rho}_{-n}^\ell)$ for every ℓ, then $\rho \in B(\boldsymbol{\rho}_{-n})$. To show this, it suffices to show that (1) $\phi(\rho_n,\boldsymbol{\rho}_{-n})$ is continuous in ρ_n for any given $\boldsymbol{\rho}_{-n}$ and (2) $\phi(\rho_n,\boldsymbol{\rho}_{-n})$ is continuous in $\boldsymbol{\rho}_{-n}$ for any given ρ_n.

Both (1) and (2) are straightforward in ROCE system as both $\boldsymbol{\pi}$ and $\boldsymbol{c}$ do not depend on $\boldsymbol{\rho}$, rather only the packet transmission times and the transmission failure probabilities vary continuously with the rate adaptation strategy. Such statement is not true for WoLD and WLDS system in general, and hence we need the non-atomic game assumption in this case. Because of the assumption, change in ρ_n does not changing $\boldsymbol{\pi}$ or $\boldsymbol{c}$. Thus, as in ROCE, (1) holds for WoLD and WLDS as well. Now, we show (2) in these cases. Note that $\boldsymbol{\pi}$ and $\boldsymbol{c}$ are given as a unique fixed point of (i) $p_n = \sum_i \pi_n(i)p(i)$ and (ii) $c_n = 1 - \prod_{m\neq n}(1-p_m)$, where $\pi_n(i)$ is a function of c_n. Note that the graph for (ii) is a monotone continuous and is independent of $\boldsymbol{\rho}$, but that of (i) depends on $\boldsymbol{\rho}$ causing the fixed point to vary with $\boldsymbol{\rho}$. We need to show that fixed point changes continuously with $\boldsymbol{\rho}$. This will holds if for any given $\boldsymbol{c}$, $\boldsymbol{\pi}$ varies continuously with $\boldsymbol{\rho}$. But this follows as, for a given $\boldsymbol{c}$, π_m can be computed using a linear transform from the transition probabilities that vary continuously with ρ_m. ■

C. Structural property of NE

We now give a structural property of the symmetric NEs for WoLD and WLDS systems. Although this property does not fully characterize NEs, it gives a useful insight into the system behaviour and greatly simplifies the complexity of computing these equilibria.

Proposition 3: Let $\boldsymbol{\rho}$ be a pure NE in WoLD or WLDS non-atomic systems. Then, $\rho_n(i) \ge \rho_n(i+1)$ for every i, n.

Proof: Fix $\widehat{\boldsymbol{\rho}}$ and the corresponding $(\boldsymbol{\pi},\boldsymbol{c})$ and let $\rho_n' = B_n(\widehat{\boldsymbol{\rho}}_{-n})$. Then, we show that $\rho_n'(i) \ge \rho_n'(i+1)$ for every $i < I$ in WoLD and WLDS systems (Lemma 3 in Appendix and Lemma 1). Hence, the result follows. ■

This result illustrates the following incentive: being in a high back-off state is expensive due to a large back-off. In the case of WoLD and WLDS it depends on the transmission rate whether the link will return to state 0 (successful transmission) or not. Hence, links have an incentive to use lower (otherwise inefficient) rates in the high back-off state to increase the chance to return to state 0.

Next, we consider ROCE systems. Here, we do not need the rate adaptation game to be non-atomic. First, we characterize the properties of NEs in this setting, and provide an explicit algorithm to compute a NE.

Proposition 4: Let $\rho_n = B_n(\boldsymbol{\rho}_{-n})$. Then, ρ_n is constant, i.e., $\rho_n(i) = \rho_n(i+1)$ for each $i < I$. Moreover, if $\boldsymbol{\rho}^\star$ is a NE, than $\boldsymbol{\rho}^\star$ is constant.

Proof: The fact every best response is constant follows immediately from (5) and (6): given $\boldsymbol{\rho}_{-n}$, the $\rho_n(i)$ depends only on u_i, and u_i is the same for every i under ROCE; consequently $R^\star(u_i)$ is the same for each i. Since every best response is constant, every NE has to be constant as well. ■

In the ROCE case, unlike in the WoLD and WLDS cases, rate adaptation strategy does not impact the back-off state, and the rates are the same in all the stages. This makes the selfish ROCE approach more efficient than the selfish WoLD and WLD, as we illustrate in Section VI.

D. Convergence to A Nash Equilibrium

Here, we look at the convergence of an arbitrary competitive network to a Nash equilibrium. In the following proposition we prove that any network converges to a Nash equlibrium if all the nodes initially use the highest rates.

Proposition 5: Let us consider an arbitrary network. Let $\boldsymbol{\rho}^{(0)}$ denote an initial rate adaptation strategies, and let $\boldsymbol{\rho}^{(m)}$ be computed iteratively as $\rho_n^{(m+1)} = B_n(\boldsymbol{\rho}_{-n}^{(m)})$ for every n. Now, if $\boldsymbol{\rho}^{(0)}$ is constant with $\rho_n^{(0)}(i) = R_{\max}$ for every i and n, then $\lim_{m\to\infty} \boldsymbol{\rho}^{(m)}$ exists (say $\boldsymbol{\rho}^\star$) and $\boldsymbol{\rho}^\star$ is a NE.

Proof: We prove the required by showing that $\boldsymbol{\rho}^{(m)}$ is a monotone decreasing sequence (Lemma 4 in Appendix). ■

We also numerically verify that a network always converges to a unique Nash equilibrium, regardless of the initial state, in a large number of random configurations we have evaluated.

E. Inefficiency of Competition

Next, we look at the performance of the network when the number of nodes is large and we show that the selfish approaches become highly inefficient. We first need a technical assumption that $\sigma/R_{max} - T_{RTS} = \epsilon > 0$, which is typically true because a packet transmission includes an exchange of RTS/CTS signaling messages. We then have

Proposition 6: Consider an arbitrary ROCE network with N links. As N goes to infinity, all links will use the minimum available rate in the set $\mathcal{R}$. In the case when $\min \mathcal{R} = 0$, the sum of all rates in the system $\sum_{n\in\mathcal{N}} \phi_n$ tends to zero.

Proof: Since from Proposition 4 all the rates are the same and the network is symmetric, eq. (2) simplifies to

$$\phi_n = \frac{p(1-c)(1-e(\rho_n))}{S(\boldsymbol{\rho})},$$
$$S(\boldsymbol{\rho}) = p(1-c)(T_n + \sum_{m\neq n} T_m) + (1-p)^N + (1-(1-p)^N - Np(1-c))T_{RTS}.$$

Then the best response $\rho = B(\boldsymbol{\rho}_{-n})$ has to satisfy $-\frac{\partial e}{\partial T} = \Phi(\rho, \boldsymbol{\rho}_{-n})$ and by derivating $e(T)$ we obtain

$$T^2 = \frac{\gamma\sigma e^{\gamma\sigma/T} S(\rho, \rho_{-n})}{\kappa_n p(1-c)(1-(e^{\gamma\sigma/T}-1)/\kappa_n}.$$

When N is large we have $p = O(1/N)$ [21]. Also, by the assumptions we have $T_m - T_{RTS} > \epsilon$, and hence

$$T^2 = \frac{\gamma\sigma e^{\gamma\sigma/T}}{\kappa_n p - e^{\gamma\sigma/T} - 1}\left(T + \sum_{m\neq n} T_m - NT_{RTS} + O(N)\right) = \frac{\gamma\sigma e^{\gamma\sigma/T}}{\kappa_n p - e^{\gamma\sigma/T} - 1}\left(T + O(N)\right)$$

Clearly, the solution of the above equation tends to $T = \infty$ when N goes large, if the minimum rate is zero, or to the minimum rate if the rates are bounded. If the minimum rate is zero, then $\lim_{N\to\infty} \sum_{n\in\mathcal{N}} \phi_n = 0$. ■

Similar argument can be made for other types of networks (although the derivation becomes more complex). The basic observation is that as the number of nodes in the network increases, a single node no more affects the average slot duration and it tends to optimize its own performance by decreasing the error probability, and hence the rate, to zero.

F. A Note on Distributed Implementation

One of our main motivation for studying competitive strategies was their amicability for distributed implementation. We note that for the distributed implementation, each user will be required to compute the best response to strategies used by other users. Now, it may seem that to compute the best response, a user has to know the strategies of all the other users. We, however, like to point out that the best response computation only requires a user to know $S^n(\boldsymbol{\rho})$ and c_n, both of which can be estimated locally by the user. Thus, each user can separately estimate the required quantities for certain duration (say T time units), and based on the estimates computes its rate adaptation strategy. Though, the convergence of this procedure is not guaranteed for WoLD and WLDS systems, our simulations verify that the procedure indeed converges (at least in all the cases we considered).

V. COOPERATION

In this section, we derive a structural property of the socially optimal rate adaptation strategy that will help us calculate it. The problem is tractable only for ROCE systems because in that case the stationary distribution $\boldsymbol{\pi}$ does not depend on the rate adaptation $\boldsymbol{\rho}$.

Proposition 7: Consider an arbitrary ROCE network. Any Pareto optimal rate allocation is a constant rate allocation ($\rho_n(i) = \rho_n(j)$ for all i, j, n).

Proof: Suppose that $\boldsymbol{\rho}$ is Pareto optimal and suppose that $\rho_n(i) \neq \rho_n(j)$ for some i, j, n. Let us construct a new rate allocation policy $\boldsymbol{\rho}'$ such that

$$T'_n(i) = T'_n(j) = \frac{\pi_n(i)p(i)T_n(i) + \pi_n(j)p(j)T_n(j)}{\pi_n(i)p(i) + \pi_n(j)p(j)}$$

and $\rho'_n(k) = \rho_n(k)$ for all $k \neq i, j$ and $\rho'_{-n} = \rho_{-n}$. It is easy to verify that $S(\boldsymbol{\rho}') = S(\boldsymbol{\rho})$ hence the rates of all other links but n are unchanged ($\phi_m(\boldsymbol{\rho}') = \phi_m(\boldsymbol{\rho})$ for $m \neq n$).

Next, due to convexity of $e_n(T)$ we have that

$$e(T'_n(i)) < \frac{\pi_n(i)p(i)e_n(T_n(i)) + \pi_n(j)p(j)e_n(T_n(j))}{\pi_n(i)p(i) + \pi_n(j)p(j)}$$

and from (2) we see that $\phi_n(\boldsymbol{\rho}') > \phi_n(\boldsymbol{\rho})$, which contradicts the assumption that $\boldsymbol{\rho}$ is Pareto optimal. ■

We verify numerically that the same property approximately holds for WoLD and WLDS as well.

VI. Numerical experiments

In this section we present numerical experiments to illustrate and validate the theoretical results derived in previous sections, and to compare the performance of the cases we were not able to treat analytically. In order to obtain realistic results, we take the error probability function $e(R, SNR)$ from the results of measurements presented in [9, Figure 2] and we fit the data to our model $e_n(R) = \left(e^{\gamma R} - 1\right)/\kappa_n$. The data fits well and we obtain $\gamma = 0.17$ and $\kappa = \{29, 350, 5.2 \cdot 10^3, 1.3 \cdot 10^5\}$ for SNR $= \{5, 10, 15, 20, 25\}$ dB. We assume that the rate set $\mathcal{R} = [0, \infty)$ is continuous, and we verify that all the rates are in the region where (1) holds.We fix the number of back-off stages to 7 ($I = 6$) and we take the standard RTS/CTS signaling parameters to compute T_{RTS}.

We use a gradient descent method to compute the co-operatively optimal rate adaptation. We seek to maximize the sum of log of rates (proportional fairness). We cannot theoretically prove that it always converges but we verified that it converged in all the cases we analyzed. To calculate the competitive equilibria (NE), we iterate over the best responses. In Proposition 5 we have proven that the process converges in all cases for ROCE, and in non-atomic cases for WoLD and WLDS. We observe numerically that this process converges for all N even without the non-atomicity assumption. Thus we are able to obtain the numerical results for all the cases analyzed in the paper.

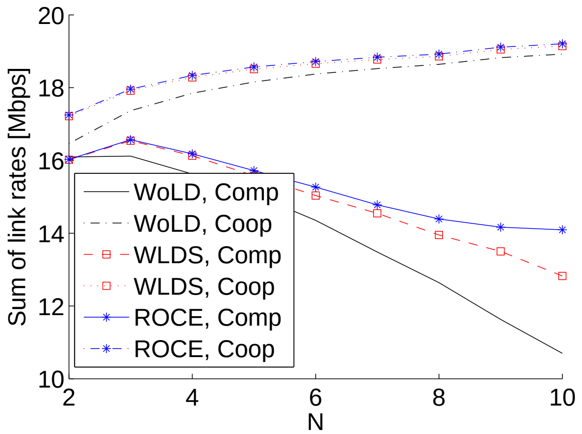

Fig. 1. The system throughput for a symmetric topology (all links have the same SNR) achieved with the optimal rate adaptation, when SNR= 15 dB.

Type	$\rho(i)$
All, cooperative	23.3, 23.3, 23.3, 23.3, 23.3, 23.3, 23.3
WoLD, competitive	16.0, 13.9, 11.9, 10.0, 8.3, 8.3, 8.3
WLDS, competitive	18.6, 18.1, 17.4, 16.3, 15.0, 13.9, 13.9
ROCE, competitive	18.6, 18.6, 18.6, 18.6, 18.6, 18.6, 18.6

Fig. 2. The optimal rate adaptation scheme for a symmetric network with $N = 4$ users, SNR$_n$ = 15 dB for all n. The optimal rate adaptation is constant in all cases of the cooperative scenario (WoLD, WLDS and ROCE).

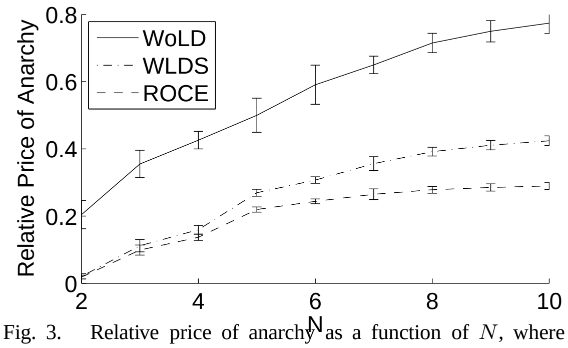

Fig. 3. Relative price of anarchy as a function of N, where the relative price of anarchy is the difference between the cooperative and the competitive optima, divided by the cooperative optimum. For each N we average over 10 different topologies and plot the mean and the confidence interval.

A. Cooperation: Optimal rate adaptation and maximum throughput

We start by analyzing cooperative scenarios where users tune their rate adaptation scheme so as to achieve proportional fairness. As an illustration, we first consider a symmetric topology where SNR$_n$ $= 15$ dB for various N. The maximum throughput and the corresponding optimal rate adaptation schemes are presented in Figure 1 and Figure 2. We make the following observations: **(1)** The throughputs obtained with the proportionally fair rate adaptation scheme in various types of systems are similar. This indicates that, surprisingly, the capability of differentiating collisions from channel errors does not bring significant improvements in the cooperative scenario when the SNR is constant and known at the transmitters. On the contrary, loss differentiation helps the estimation of the channel quality and may improve the system performance when the SNR is unknown as shown in [10], [11], [5], [12], [8], [13]. **(2)** The socially optimal rate adaptation scheme is always almost constant and it is almost the same in all the cases (WoLD, WLDS, ROCE).

Also, note that ROCE can be disadvantageous if a MAC is not well design to cope with congestion. For example, if a packet is lost due to a channel error and a network is very congested, it may be better not to return to the zero back-off stage and thus ease the congestion. However, this remedy works only if the channel incurs losses, and implies that MAC itself is not well designed (e.g. for reliable channels with no losses). In our numerical evaluation, using 802.11 DCF parameters, we do not observe this problem and ROCE performs at lease as well than WoLD and WLDS. This implies that 802.11 MAC is well designed to deal with congestion, as claimed in previous works (see e.g. [21]).

B. Competition: Symmetric NE and Price of Anarchy

We next consider various heterogeneous networks. For each N we construct 10 networks of N links, each link having a randomly chosen SNR from the set of $\{5, 10, 15, 20, 25\}$ dB. For each of these network we calculate the cooperative optimum, the competitive NE, and the relative price of anarchy. The results are depicted in Figure 3.

We make the following observations: **(1)** We verify the finding from Proposition 6 that the price of anarchy increases with N. **(2)** The price of anarchy is the largest for WoLD. Therefore, having loss differentiation helps in the competitive scenario. **(3)** ROCE exhibits a smaller price of anarchy than that in WLDS, and the difference between the two grows with N. We also verify (see Figure 2) that the optimal strategy yields rates that decrease with the back-off stage, as shown in Proposition 3.

VII. Conclusions

In this work we analyzed the interaction between medium access and rate adaptation protocols in a cooperative and a competitive scenario, for three types of rate adaptation protocols: WoLD, WLDS and a newly proposed ROCE protocols. In the competitive scenario we proved the existence of a Nash equlibrium, we characterized structural properties and proved convergence in some cases. We also gave structural properties of the cooperative scenarios. We showed that the competitive scenario yields high inefficiency, but this inefficiency is smaller in the case of ROCE than the other two types of protocols.

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Appendix

A. Supporting Lemma for Proposition 1

Lemma 2: Function $\Delta_n(c)$, as defined in (3), is monotonically decreasing and $F_n(c) = (1-c)(1-\Delta_n(c))$ is monotonous.

Proof: Let $E_i = \sum_{k=0}^{i}(1-e_k)/(1-(1-c_n)(1-e_k))$ (where $e_k = e_I$ for $k > I$). We have that $\frac{\partial \zeta_n(i)}{\partial c_n} = \zeta_n(i)E_i$. To prove that $\Delta_n(c)$ is decreasing we need to show that the first derivative is negative, that is

$$\left(\sum_{i=0}^{\infty}\zeta_i E_i\right)\left(b_0 + \sum_{i=0}^{\infty} b_{i+1}\zeta_i\right) < \left(\sum_{i=0}^{\infty} b_{i+1}E_i\zeta_i\right)\left(1+\sum_{i=0}^{\infty}\zeta_i\right)$$

or equivalently

$$\sum_{i=0}^{\infty}(b_{i+1}-b_0)\zeta_i E_i + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\zeta_i\zeta_j E_i(b_{i+1}-b_{j+1}) > 0.$$

Since the back-off time increases with the stage, we have $b_{i+1} > b_0$ and the first sum is positive. We can rewrite the second sum as

$$\sum_{i=0,j=0}\zeta_i\zeta_j E_i(b_{i+1}-b_{j+1}) = \sum_{i=0,j>i}\zeta_i\zeta_j(E_j-E_i)(b_{j+1}-b_{i+1}).$$

Again, $b_{j+1} - b_{i+1}$ is positive because the back-off increases with the stage, and the sequence E_i is increasing by construction, hence the second sum is also positive which proves that $\Delta_n(c)$ is decreasing.

We next prove the monotonicity of $F_n(c)$. We have $\zeta_i E_i \leq (i+1)c_n^i$. We also have $e_k \leq e_{MAX}$, hence $\zeta_i \geq c^i(1-e_{MAX})^i$. Extending the arguments from [22, Lemma 5.3] we have

$$\begin{aligned}|\Delta_n'(c)| &\leq \frac{\left(\sum b_{i+1}\zeta_i E_i\right)\left(1+\sum \zeta_i\right)}{\left(b_0 + \sum b_{i+1}\zeta_i\right)^2} \\ &\leq \frac{\left(\sum b_{i+1}(i+1)c_n^i\right)\left(1+\sum c_n^i\right)\left(b_0+\sum b_{i+1}c_n^i\right)^2}{\left(b_0+\sum b_{i+1}c_n^i\right)^2\left(b_0+\sum b_{i+1}c_n^i(1-e_{MAX})^i\right)^2} \\ &\leq \frac{2a}{b_0}\frac{1-a(1-e_{MAX})}{1-a}.\end{aligned}$$

Now the derivative $F_n'(c) = -1 + \Delta_n(c) - \Delta_n'(c)(1-c)$ and since $\Delta_n(c) \leq \frac{1}{b_0}, \Delta_n'() \leq 0, (1-c) \geq 0$ we have

$$F_n'(c) \leq -1 + \frac{1+a-2a^2(1-e_{MAX})}{b_0(1-a)}.$$

Consequently, $F_n(c)$ is monotone if $1+a-2a^2(1-e_{MAX}) < 0$ and $b_0 > (1+a-2a^2(1-e_{MAX}))/(1-a)$. ■

B. Supporting Lemma for Proposition 3

Lemma 3: Fix any given $\boldsymbol{\rho}$, and let $(\boldsymbol{\pi}, \boldsymbol{c})$ denote the steady state probability and the collision probabilities, respectively, when the users use rate adaptation profile $\boldsymbol{\rho}$. Then, $J_n(i, \boldsymbol{\rho}_{-n}) \leq J_n(i+1, \boldsymbol{\rho}_{-n})$ for every $i \geq 0$ in WoLD and WLDS system.

Proof: We focus of WoLD systems. The proof for WLDS systems follows similarly. With some abuse of notation, let us define $J_n(i, R, \boldsymbol{\rho}_{-n}) = G_n(R, u_i) + \xi_i^n$, and thus $J_n^\star(i, \boldsymbol{\rho}_{-n}) = \min_{R\in\mathcal{R}} J_n(i, R, \boldsymbol{\rho}_{-n})$.

We prove the required by induction. Let R_i be such that $J_n(i, R_i, \boldsymbol{\rho}_{-n}) = J_n^\star(i, \boldsymbol{\rho}_{-n})$ for every i. Now, note that

$$\begin{aligned}&J_n(I, R_I, \boldsymbol{\rho}) - J_n(I-1, R_I, \boldsymbol{\rho}) \\ &\quad= \xi_I^n - \xi_{I-1}^n, \\ &\quad= \left(\frac{1-p_I}{p_I} - \frac{1-p_{I-1}}{p_{I-1}}\right) S^n(\boldsymbol{\rho}).\end{aligned}$$

Since $p_I \leq p_{I-1}$, we conclude that $J_n(I, R_I, \boldsymbol{\rho}_{-n}) \geq J_n(I-1, R_I, \boldsymbol{\rho}_{-n})$. Consequently, $J_n^\star(I, \boldsymbol{\rho}_{-n}) = J_n(I, R_I, \boldsymbol{\rho}_{-n}) \geq J_n(I-1, R_I, \boldsymbol{\rho}_{-n}) \geq J_n^\star(I-1, \boldsymbol{\rho}_{-n})$.

By induction hypothesis, let $J_n^\star(i, \boldsymbol{\rho}_{-n}) \leq J_n^\star(i+1, \boldsymbol{\rho}_{-n})$ for every $i \geq j$. Now, consider

$$\begin{aligned}&J_n(j, R_j, \boldsymbol{\rho}) - J_n(j-1, R_j, \boldsymbol{\rho}) \\ &\quad= \frac{(1-c_n)(e^{\gamma R_j}-1) + \kappa_n c_n}{\kappa_n} \times \\ &\qquad (J_n(j+1, R_j, \boldsymbol{\rho}_{-n}) - J_n(j, R_j, \boldsymbol{\rho}_{-n})) \\ &\qquad + \left(\frac{1-p_j}{p_j} - \frac{1-p_{j-1}}{p_{j-1}}\right) S^n(\boldsymbol{\rho}).\end{aligned}$$

Again note that $p_j \geq p_{j-1}$, and also $J_n(j+1, R_j, \boldsymbol{\rho}_{-n}) \geq J_n(j, R_j, \boldsymbol{\rho}_{-n})$ by induction hypothesis. Thus, it follows that $J_n^\star(j, \boldsymbol{\rho}_{-n}) \geq J_n^\star(j-1, \boldsymbol{\rho}_{-n})$. ■

C. Supporting Lemma for Proposition 5

For a given $\boldsymbol{\rho}^{(m)}$, we compute the $J_n^\star(i, \boldsymbol{\rho}_{-n}^{(m)})$, and thereby compute $\boldsymbol{\rho}^{(m+1)}$. In this, for brevity, we define

$$G_n'(R, u) = (1-c_n)\left[\frac{\sigma}{R} + \frac{\left(e^{\gamma R}-1\right)u}{\kappa_n}\right].$$

Clearly, $G_n'(R, u)$ also has the same properties as that of $G_n(R, u)$ which are described in Lemma 1. Moreover, $\min_{R\in\mathcal{R}} G_n(R, u) = \min_{R\in\mathcal{R}} G_n'(R, u)$ for every u. Now, note that for ROCE system,

$$\begin{aligned}J_n^\star(i, \boldsymbol{\rho}_{-n}) = \min_{R\in\mathcal{R}} G_n'(R, J_n^\star(0, \boldsymbol{\rho}_{-n})) + \frac{1-p_i}{p_i} S^n(\boldsymbol{\rho}) \\ + T_{RTS} + c_n J_n^\star(k_i, \boldsymbol{\rho}_{-n}).\end{aligned}$$

Recall that $k_i = i+1$ for $i < I$ and $k_I = I$. This follows from (5) and (6). Now, let us consider the value iteration method for computing $J_n^\star(i, \boldsymbol{\rho}_{-n}^{(m)})$ [24]. Let, for every i, $J_{n,\ell}(i, \boldsymbol{\rho}_{-n}^{(m)})$ denote the value of J-function in ℓ^{th} iteration for user n in stage i. Then, $J_{n,\ell+1}(i, \boldsymbol{\rho}_{-n}^{(m)})$ is computed using the following recursion.

$$\begin{aligned}J_{n,\ell+1}(i, \boldsymbol{\rho}_{-n}^{(m)}) = \min_{R\in\mathcal{R}} G_n'(R, J_{n,\ell}(0, \boldsymbol{\rho}_{-n}^{(m)})) + T_{RTS} \\ + \frac{1-p_i}{p_i} S^n(\boldsymbol{\rho}^{(m)}) + c_n J_{n,\ell}(k_i, \boldsymbol{\rho}_{-n}^{(m)}). \quad (7)\end{aligned}$$

From results in MDP theory, $\lim_{\ell\to\infty} J_{n,\ell}(i, \boldsymbol{\rho}_{-n}^{(m)}) = J_n^\star(i, \boldsymbol{\rho}_{-n})$ for every i starting from any intial condition. Moreover, if we let $\boldsymbol{\rho}^{(m)}(\ell)$ to be the rate adaption policy in the ℓ^{th} iteration, then $\lim_{\ell\to\infty} \boldsymbol{\rho}_\ell^{(m)} = \boldsymbol{\rho}^{(m)}$.

Lemma 4: Let $\boldsymbol{\rho}^{(0)}$ be the constant such that $\rho_n(i) = R_{\max}$ for every n and i. Then, $\boldsymbol{\rho}^{(m)} \geq \boldsymbol{\rho}^{(m+1)}$ for every m.

Proof: The proof is by induction. Clearly, $\boldsymbol{\rho}^{(0)} \geq \boldsymbol{\rho}^{(1)}$. By induction hypothesis, we assume that $\boldsymbol{\rho}^{(0)} \geq \boldsymbol{\rho}^{(1)} \geq \cdots \geq \boldsymbol{\rho}^{(m)}$. Now, we show that $\boldsymbol{\rho}^{(m)} \geq \boldsymbol{\rho}^{(m+1)}$. To show this, by Lemma 1, it suffices to show that $J_n^\star(0, \boldsymbol{\rho}_{-n}^{(m-1)}) \leq J_n^\star(0, \boldsymbol{\rho}_{-n}^{(m)})$. We show this by showing that $J_{n,\ell}(i, \boldsymbol{\rho}_{-n}^{(m-1)}) \leq J_{n,\ell}(i, \boldsymbol{\rho}_{-n}^{(m)})$ for every i, n and ℓ starting from the initial condition $J_{n,0}(i, \boldsymbol{\rho}_{-n}^{(m-1)}) = J_{n,0}(i, \boldsymbol{\rho}_{-n}^{(m)}) = 0$ for all i, n. Clearly, the required holds for $\ell = 0$. By induction hypothesis, let the required hold until ℓ^{th} iteration. Now, we consider the $(\ell+1)^{\text{th}}$ iteration and observe that for every i

$$\begin{aligned}&J_{n,\ell+1}(i, \boldsymbol{\rho}_{-n}^{(m)}) - J_{n,\ell+1}(i, \boldsymbol{\rho}_{-n}^{(m-1)}) \\ &= \left[\min_{R\in\mathcal{R}} G_n'(R, J_{n,\ell}(0, \boldsymbol{\rho}_{-n}^{(m)})) - \min_{R\in\mathcal{R}} G_n'(R, J_{n,\ell}(0, \boldsymbol{\rho}_{-n}^{(m-1)}))\right] \\ &+ \frac{p_i}{1-p_i}\left(S^n(\rho^{(m)}) - S^n(\rho^{(m-1)})\right) \\ &+ c\left[J_{n,\ell+1}(k_i, \boldsymbol{\rho}_{-n}^{(m)}) - J_{n,\ell+1}(k_i, \boldsymbol{\rho}_{-n}^{(m-1)})\right].\end{aligned}$$

Note that $J_{n,\ell}(0, \boldsymbol{\rho}_{-n}^{(m)}) \geq J_{n,\ell}(0, \boldsymbol{\rho}_{-n}^{(m-1)})$ by the induction hypothesis on ℓ. Thus, the first term in the above expression is non-negative by Lemma 1. The second term is also non-negative as $\boldsymbol{\rho}^{(m)} \leq \boldsymbol{\rho}^{(m-1)}$ by the induction hypothesis on m. Finally, the third term is also non-negative by induction hypothesis of ℓ. Thus, the required follows. ■